PLANE SURVEYING

RAYMOND

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FOR USE IN THE CLASSROOM AND FIELD

BY

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PREFACE TO THE SECOND EDITION

THIS book has not been changed since it was first written eighteen years ago. When a revision was proposed the author was advised by friends to put the book in pocket book form. The publishers being willing to do this, advantage has been taken of the necessity for resetting to rewrite almost the entire book and to rearrange parts of it for greater convenience in use.

Such errors as have been discovered have been corrected. The description of the adjustment of instruments has been wholly rewritten, and it is believed that the collimation adjustment of the transit is correctly explained for the first time in a textbook. The number of examples and exercises has been increased and these have been included in the articles they are intended to illustrate.

Authors differ as to what should be included in a text on surveying. This author feels that minute details of various survey operations are likely to prove cumbersome and confusing in a text. Some things the teacher should expect to tell the student, some things the student should discover for himself, and no surveyor is worthy the name who needs to have the details of every kind of survey outlined in a reference book. But the author does think that all principles should be carefully explained, and that exercises should be planned to show the student, by his own experience, not only the possibilities, but also the limitations of instruments, methods, and individuals. An attempt has been made to do these things.

In the author's judgment Crockett's logarithmic tables are the best for the use of surveyors. These were adopted for the former edition, and the same form, slightly modified to improve them still further for the surveyor's use, has been used in this revised edition.

The chapter on Mine Surveying has been carefully read by Mr. Robert M. Black, Professor of Mining Engineering in Pennsylvania State College. Mr. F. C. Young, Instructor in Civil

Engineering in the State University of Iowa, has given valuable suggestions, while Mr. John H. Dunlap, Assistant Professor of Hydraulics and Sanitary Engineering in the same institution, has given the entire manuscript a thorough critical reading to its great improvement. To these the author expresses his indebtedness. He is also under obligation to the following instrument makers for permission to use cuts from their catalogues:

Messrs. W. & L. E. Gurley, Troy, N.Y.; Keuffel & Esser Company, New York; Eugene Dietzgen Company, Chicago, Ill.; C. L. Berger & Sons, Boston, Mass.; and the Universal Drafting Machine Company, Cleveland, Ohio.

Other indebtednesses are noted in the text.

WILLIAM G. RAYMOND.

Iowa City, Iowa,
January, 1914.

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INTRODUCTION

1. Preliminary conceptions. An ellipse of axes AB and CD (Fig. 1), being revolved around its shorter axis CD, will generate the surface of an oblate spheroid of revolution.

If we imagine the sea to extend underneath the surface of the earth so that the visible solid portions of the earth will be, as it were, floating on a ball of water, the shape of that ball will be, approximately, that of an oblate spheroid of revolution. The surface of this ball is called the mean surface of the earth.

The visible solid parts of the earth above the mean surface and the invisible solid parts below, make up a very irregular body. It is customary to speak of the visible parts of the earth's surface, both fluid and solid, as the "surface of the earth."

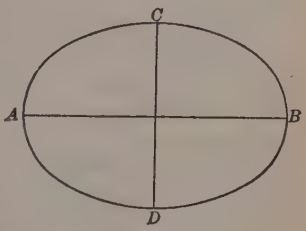


FIG. 1.

The shorter axis of the mean surface is that connecting the poles; the longer is the diameter of the circle called the equator. In the case of the earth these two axes do not differ much in length, and, hence, the earth is usually spoken of as a "sphere slightly flattened at the poles." It may seem strange to the student that a difference of twenty-seven miles should be spoken of as a slight difference. But when it is said that this difference is about one third of one per cent of the length of the longer axis, the meaning is clearer.

The lengths of the two axes according to the latest determinations[1] are:

Shorter or polar axis	41,709,790 feet.
Longer or equatorial axis . . .	41,852,404 feet.

If a plane is passed through an oblate spheroid of revolution, parallel to its shorter axis, it will cut an ellipse from the

[1] Clarke's spheroid of 1880. The values as found for Clarke's spheroid of 1866 are those generally used by geodesists. They are: shorter, 41,710,242 feet; longer, 41,852,124 feet.

spheroid. If passed parallel to the longer axis, it will cut a circle. So with the earth: a plane passed parallel to the polar axis cuts an ellipse from the mean surface of the earth, while one passed parallel to the equator cuts a circle. Hence, meridians of longitude are ellipses, and parallels of latitude are circles.

The surface of the sea, or that surface extended as before mentioned, forms what is called a level surface, and a line lying in this surface is a level line.

A line perpendicular to this surface at any point is a vertical line *for that point*. (Subject to slight deviations at some points, negligible by the surveyor, a plumb line at any point is a vertical line for that point. Except at a few points, the plumb line extended does not pass through the geometric center of the earth.)

A line perpendicular to a vertical line is a horizontal line.

A tangent to the earth's mean surface at any point is perpendicular to the vertical line at that point, and hence is a horizontal line for that point.

An inclined line is a straight line that is neither vertical nor horizontal.

A vertical plane at any point is a plane including the vertical line at that point.

A horizontal plane at any point is a plane perpendicular to the vertical line at that point.

A vertical angle is an angle formed by lines in a vertical plane.

A horizontal angle is an angle formed by lines in a horizontal plane.

If water collects upon the earth's surface in some depression above the mean surface, as in a lake or pond, or even as in a small glass, and if the water is still, its surface will be nearly parallel to that portion of the mean surface of the earth that is vertically below it; hence it will be a level surface, and a line drawn on it will be a level line. Such a line will be longer than the corresponding line drawn on the mean surface of the earth between the verticals through the extremities of the upper line.

Query. At any point on the earth's surface how many vertical lines may there be? How many horizontal lines? How many horizontal planes? How many vertical planes?

2. Surveying defined. Surveying is the art of finding the contour, dimensions, position, etc., of any part of the earth's surface, and of representing on paper the information found.

The operations involved are the measurements of distances, — level, horizontal, vertical, and inclined, — and of angles, — horizontal, vertical, and inclined; and the necessary drawing and computing to represent properly on paper the information obtained by the field work.

The drawn representation is called a *map*. It may be a map showing by conventional signs the shape of that part of the earth's surface that has been measured; or it may be simply an outline showing the linear dimensions of the bounding lines, together with the angles that they make with the north and south line, or with each other, and sometimes the position within the tract of structures, roads, or streams.

A map of the former kind is called a *topographical map*, and the operations necessary to its production constitute a *topographical survey*.

A map of the latter kind is a *land map*, and the operations necessary to produce it constitute a *land survey*.

Either one of these surveys is a *geodetic survey* if the tract is so large that the curvature of the earth's surface must be taken into account. This limit is supposed to be reached when the tract is greater than one hundred square miles, but many surveys of tracts of much greater area than this are made without considering the mean surface to be other than plane. Such surveys are of course inaccurate, but may be sufficiently correct for the purpose they are to serve.

A *plane survey* is one made on the assumption that the mean surface of the earth is a plane, above which is the irregular visible surface broken by hills and valleys. Almost all land surveys are plane surveys.

The operation of leveling is considered usually under plane surveying, though the work itself is done with reference to a level surface rather than to a plane surface.

In geodetic surveying the mean surface of the earth is assumed to be a sphere or spheroid according to the magnitude of the survey and the precision required. The irregular portion above the mean surface is supposed to be projected down vertically on the mean surface, and the lines measured on the actual surface are reduced to the lengths of their projections on

the mean surface. There is no such reduction in plane surveys.

It must be borne in mind that no physical measurements are exact. The art of surveying makes it possible to determine that a field of land contains a certain area, *more or less;* that a mountain is so many feet high, *more or less;* that a mine is so many feet deep, *more or less;* etc. That is to say, it is physically impossible to measure *exactly* either distance or angles. The precision attainable or desirable in any surveying operations will be discussed elsewhere in this book.

PLANE SURVEYING

CHAPTER I

MEASUREMENT OF LEVEL AND HORIZONTAL LINES

3. The line to be measured. The distance between two points on the surface of the earth is the length of the level line joining the verticals through the points. If one of these points is much higher than the other (further from the mean surface), confusion may arise as to which of several lines is meant by the above definition. In geodetic surveying it is customary to reduce the distance, when measured, to the length of the level line lying in the mean surface and contained between the verticals through the points. The distance as measured will always be approximately the length of the level line lying midway as to altitude between the two points. In plane surveying the approximate length of this line is obtained by measuring a series of short practically horizontal lines; the sum of these lines approximates to the length of the required level line somewhat as the regular polygon of a large number of sides approximates to the circle.

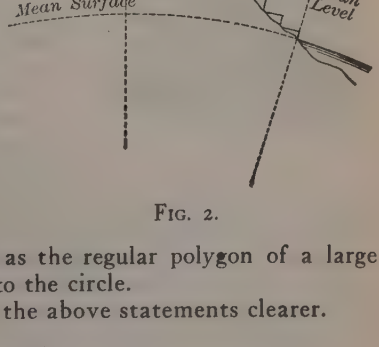


FIG. 2.

Figure 2 will serve to make the above statements clearer.

INSTRUMENTS USED

4. Chains. The instruments used for the direct measurement of distances are chains, tapes, and wooden or metallic rods. Chains are of two kinds: Gunter's chain and the engineer's chain. These chains are alike in form, but vary in the length

of the links and the length of the entire chain. In Gunter's chain the links are 7.92 inches long, and in the engineer's chain they are 12.00 inches, or one foot, long. With this exception, one description will apply to both.

A chain consists of one hundred "links" made of iron or steel wire. Number 12 steel wire is best. Figure 3 shows the form of the links. A link includes one of the long pieces and two or three rings, according as there are two or three rings used to connect the long pieces. The rings are introduced to enable one to handle the chain more readily. Brass tags with the proper number of points mark the ten-link divisions from each end toward the center, and a round tag marks the center of fifty-link division. The handles are of brass, and are usually made adjustable, so that slight changes in the length of the chain may be corrected. A special form of handle is sometimes used, having a knife edge on which a notch is filed indicating the zero of the chain for the day, the chain being compared daily with a standard kept for the purpose.

FIG. 3.

The Gunter's chain, having 100 links of 7.92 inches each, is 66.00 feet long, and the engineer's chain is 100.00 feet long. The former was devised by Mr. Edward Gunter, for the purpose of facilitating the computations of areas that have been measured. Its length was so taken that 10 square chains make one acre. It is the chain referred to in the table of surveyor's measure, which should be carefully memorized.

There are 7.92 inches in one link
100 links in one chain
80 chains in one mile
10 square chains in one acre
640 acres in one square mile
16½ feet in one rod, perch, or pole
160 square rods in one acre.

In all surveys of the public lands of the United States the Gunter's chain is used, and all descriptions of land, found in deeds or elsewhere, in which the word "chain" is used, are based on this chain. It is not convenient for use in connection with engineering works, such as railroad construction, canal building, bridge building, etc., where the unit of measure is the foot, and hence, in such work, the engineer's chain was formerly used, but has been almost entirely superseded by the steel tape.

Examples

1. How many rods are there in a Gunter's chain?
2. How many square feet are there in an acre?
3. What is the length of the side of a square acre in feet? In Gunter's chains?
4. What is the length of the side of a square ten-acre plot in feet? In chains?
5. What is the difference in area between two plots respectively ten chains square and ten square chains?
6. Reason as Gunter probably did in devising his chain and show that the length of his chain was correctly determined for its purpose.

5. Tapes and Reels. Steel tapes are better than any kind of chain for engineering work and for all fine surveying. These tapes are made in various forms, from thin ribbons half an inch wide to flat wires about one eighth of an inch wide and one fiftieth of an inch thick. The ribbon tapes are graduated on the front to feet, tenths, and hundredths of a foot, or to feet, inches, and eighths, and on the back to links of 7.92 inches. They come usually in box reels and are from twenty-five feet to one hundred feet long. They are suitable for very nice work of limited extent, and particularly for measurements for structures, such as bridges and buildings, both in the shop and in the field. They are not suitable for ordinary field operations of surveying, because they are easily broken. For such work the narrower, thicker tapes are preferable. These may be obtained in any lengths up to a thousand feet or more; but the lengths usually kept in stock are fifty feet and one hundred feet. They are graduated variously, but may be graduated to suit the purchaser. For surveying work the following graduation is recommended:

A graduation mark at every foot; every fifth foot numbered

FIG. 4.

from one end of the tape to the other and not from each end to the middle; the tape one foot longer next the zero end than its nominal length, the extra foot divided into tenths or tenths and hundredths. If the graduation of the end foot is to tenths and it is required to make measurements to hundredths, a pocket steel tape from three to five feet long graduated in feet, tenths, and hundredths may be used for the fractional portion. In country surveys hundredths can usually be estimated with sufficient precision, but in city surveys, and other surveys requiring accuracy, the hundredths must be measured.

In addition to the steel tapes, linen and "metallic" tapes are used for rough work. The ordinary linen tape is well known to every one. The metallic tape is a linen tape with a few strands of fine brass wire woven through it. The linen tape is subject to great change in length with changes of moisture in the atmosphere, is soon stretched, and is easily worn out. The metallic tape is not so subject to change in length with change

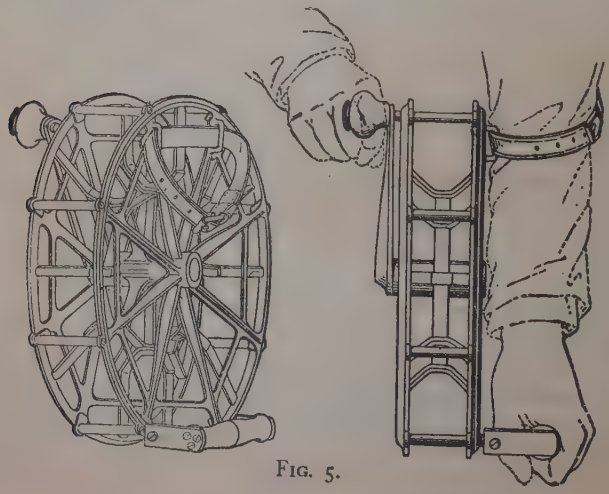


FIG. 5.

of atmospheric conditions; it is soon stretched, but is not nearly so soon worn out as is the linen tape. Both these tapes, being easily stretched, soon become quite inaccurate for any but the commonest kinds of work where the measurements are short and need not be closer than to the nearest tenth of a foot. They are graduated in feet, tenths, and half-tenths, and on the reverse side in links of 7.92 inches. Sometimes they are graduated in inches. They are sold in paper or leather box reels.

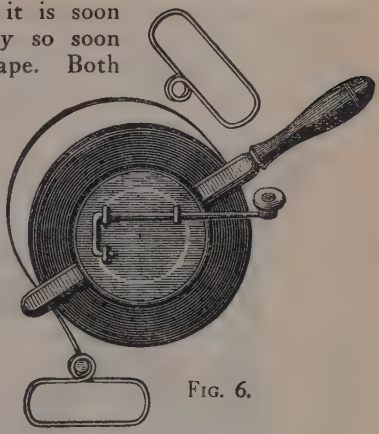


FIG. 6.

For the narrow steel tapes there are a number of patterns of reels, most of them aiming to furnish an open reel, of form convenient to go in the pocket when not in use, and so constructed as to enable the surveyor to reel the tape easily. There is but one reel that has come to the author's attention that combines all three of these requisites. This is shown in Fig. 6, and a modified form in Fig. 4. Other forms of reels are shown in Figs. 5, 7, 8, and 9. Figure 5 is a reel for a tape from 300 feet to 1000 feet in length.

FIG. 7.

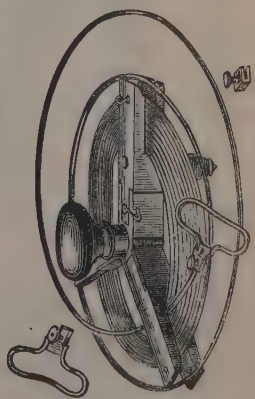


FIG. 8.

Figure 9 shows a tape fitted with a spring balance for measuring the pull on the tape when in use, a level to show when the tape is held horizontal, and a thermometer to give the temperature. The necessity for these attachments will appear

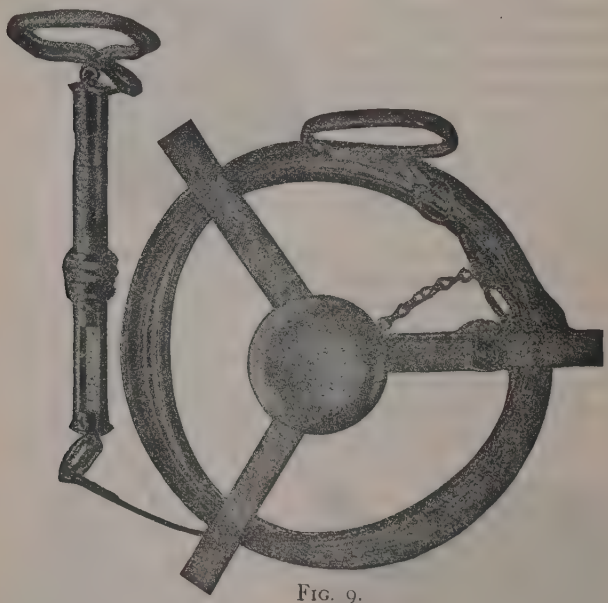


FIG. 9.

hereafter. Such a tape is used for land surveys in the city of New York.

Tapes should always be kept dry, and if wet by use, should be wiped dry and rubbed with a cloth or leather that has the smallest possible quantity of mineral oil on it.

EXAMPLES

1. Show how to apply a tape graduated as recommended when measuring a distance that proves to be 39.82 feet long. How would it be applied if the distance is 51.27 feet?

2. If the extra foot graduations are numbered, should they be numbered from the zero of the tape toward the end or in the opposite direction?

6. Rods. While some rough measurements are made with the ordinary ten-foot pole or a similar arrangement, no other surveying measurements are made now with wooden or metallic rods, except measurements of base lines in connection with important geodetic surveys; in these the rods, usually metallic, are arranged with other devices into a very elaborate piece of apparatus. It is believed that the narrow steel and brass tapes will entirely supersede the elaborate base apparatus now in use. So-called "invar" tapes made of an alloy of nickel and steel are now largely used in base line work.

FIG. 10.

7. Pins. These are used with the tape for the purpose of marking tape lengths. They are about fourteen inches long, made of steel or iron wire somewhat less than a quarter of an inch thick (No. 4 to No. 6 wire gauge), with a ring at one end into which a strip of cloth is fastened to insure ready finding of the pin when stuck in tall grass or brush. The other end is pointed. For nice work and work in the open, smaller pins from 6 inches to 9 inches long are used. Eleven of these pins constitute a set. They may be carried on a hook attached to the belt or loose in the hand. They are sometimes carried in a leather quiver or pouch (Fig. 10).

FIG. 11.

8. Range poles. Poles are used to range out the line to be measured. They are usually of wood, round or hexagonal, six to eight feet long, tapering from the bottom to the top, shod with a pointed iron shoe, and painted red and white in alternate strips one foot long. Gas pipe is sometimes used, but is not recommended, because, while it does not break, and while from its weight such a pole is easily balanced on its point, it is also very easily bent, and very difficult to

straighten, and is too heavy to be handled with ease. A good pole (Fig. 11) for nice work in cities or on railway surveys is made of hexagonal steel about three-eighths to five-eighths of an inch thick, painted like the wooden poles, and pointed at one end.

METHODS

9. Ranging a line. Ranging a line frequently means no more than placing a pole or flag at the end of the line toward which the measurement is to proceed. If not directed by an instrumental line of sight, the forward tapeman is kept in the

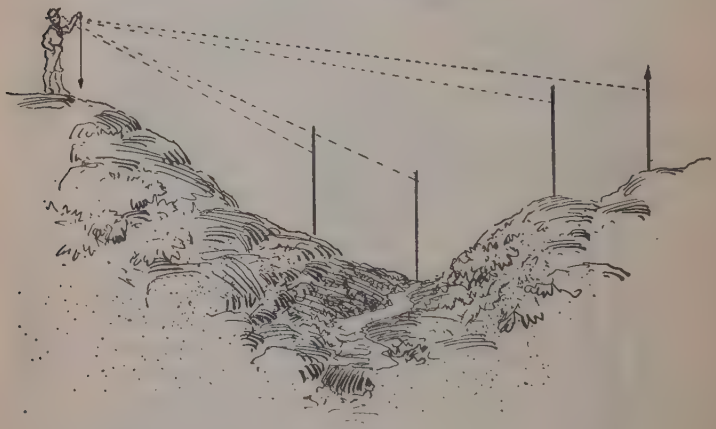


FIG. 12.

proper line by the rear tapeman, who "lines him in" with the forward flag when he has drawn out the tape and is ready to make a measurement. If the forward tapeman carries a pole, it is the pole that is put in line.

Sometimes it is necessary to range a line across a valley, hollow, or ravine between points on the two bounding ridges. It is impossible for the eye to carry a vertical plane down into a valley, particularly if the valley has a fairly steep slope across the line to be ranged. In such a case a plumb line is used as indicated in Fig. 12.

When a line is to be ranged over a hill intervening between the two ends of the line so that one end cannot be seen from the other, two men with flags station themselves as shown in Fig. 13; A lines B between himself and D; B then lines A between himself and C; they proceed similarly until no further movement of either is required; both are then in the line *CD*.

The same method is used when one wishes to place himself in line between two points and there is no one at either point to direct him. If alone, he may use two poles, or flags, placing first one and then the other until both are in the required line.

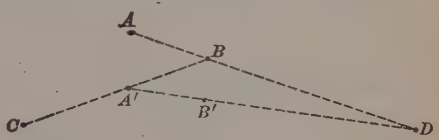


FIG. 13.

Query. To secure a true line, which is better, to use a foresight flag toward which the chaining is directed, or a back sight flag from which (after making the first measurement, which must be directed toward the distant point) the measurement proceeds?

10. Doing up and undoing the chain and tape. It will be noticed when the chain is received from the maker that it is so folded together as to be compact in the center of the bundle and somewhat bulky at the ends, in shape not unlike an hour glass. This results from doing up the chain as follows:

Take the two links at the center of the chain in the left hand, with the fifty-link tag on the left. Take the right hand ends of the pair of links next but one to those in the left hand, in the right hand, and lay the right hand pair and the intermediate pair in the left hand diagonally across the pair already there. In like manner proceed to the ends of the chain, being careful always to place the new links diagonally across the links already in the left hand and always diagonally the same way. It is better, however, to do up the chain from one end instead of from the middle. The method is the same except that the two end links are first taken in the left hand, the handle end to the right. A little more time is required, but the chain is more readily loosened for service.

To undo the chain the strap with which the chain is fastened is removed, and, if the chain has been done up from the middle,

the two handles are taken in the left hand of the forward chainman and the chain bundle in the right hand, allowing a few links next the handles to fall off. The chain bundle is then thrown out in a direction opposite to that in which the measurement is to be made, the chainman retaining the handles in his left hand. The chain should be thrown with sufficient force to straighten it out. If the chain is done up from one end, it should not be thrown out, but laid on the ground. The forward chainman then takes one end and carries it out, the rear chainman allowing the chain to slip through his hands and watching for kinks or bent links.

When, as is now usually the case, a steel tape is used, the tape is looped in the form of a figure eight and usually is kept in this way instead of on a reel. It falls naturally into this shape and is about of the right size if an amount of tape about equivalent to the easy stretch of a man's arms, or, say about five feet, is used for each full loop.

11. Measuring on level ground. It will be assumed that the ground on which the line is to be measured is comparatively level, and that the ends of the line are visible, one from the other. If there is no visible object to mark the farther end of the line, a range pole is placed there, toward which the measurement is to be made. If the rear end of the line is also marked by a pole, or the direction fixed by some other visible object, the head tapeman will be able, without difficulty, to put himself in approximate line, thus saving time. The forward man, usually called the *head tapeman*, takes the forward end of the tape and ten of the pins, and starts toward the farther end of the line, while the rear tapeman allows the tape to slip through his hands to see that it is not kinked or bent. If he finds any bends, he straightens them.

One pin is left with the rear tapeman. When the tape is almost out the rear man calls, " Chain." (The words, chainman and chain, are still quite commonly used even though all work is done with a tape.) The head man then stops, turns, and straightens the tape while being put into approximate line by the rear man. The tape being taut, the rear man, holding the zero point at the pin, by motions of head or hand, or the word " right " or " left," accurately aligns the pin held by the head man and cries " Stick." The head man then forces the pin

into the ground, taking care that it marks exactly the graduated end of the tape, and cries, "Stuck."

The rear man then, and not till then, draws his pin, keeping hold of the tape, and follows the head man, who moves on toward the forward end of the line, and the whole operation is repeated. After one pin has been placed, the head man, on being stopped by the call of the rear man, can quickly put himself in approximate line by sighting back over the pin last set to the flag left at the starting point or other object used for lining. The work thus proceeds till the farther end of the line is reached, when the head man notes the fraction of a tape between the last pin and the point. This added to the number of tape lengths gives the distance required. If the distance is more than ten tape lengths the head man, when he sticks his last or tenth pin, calls "Stuck out" or "Tally." He then waits by the pin till the rear man comes up with the pins he has collected, which, with the pin he started with, should be ten. He counts them, as does the head man as a check, and they note one "tally." At any time the number of tallies plus the number of pins in the rear tapeman's hands gives the distance that has been measured in tens of tapes and tapes.

When the head man sets his pin he sets it slanting across the line rather than plumb, the point on the pin where it enters the ground being directly under the proper graduation mark on the tape. The tape is removed from the reel for the day or the survey, and either metal handles or raw hide loops are used in the ends of the tape. The zero end of the tape should be carried by the head man.

12. Hints. The following hints may be of service to beginners:

The rear tapeman should not use the pin to brace himself. If a chain is used, he should hold the outside edge of the handle flush with the rear side of the pin, without moving the pin.

He should not stop the head tapeman with a jerk.

He should not sit down on the ground while holding the tape at the pin.

Motions and words should be sharp and distinct.

Motions and instructions should be proportionate to the distance that the pin is to be moved; for example, the arms should not be swung wildly when the pin is to be moved an inch.

The head tapeman should learn to walk " on line " and should move rapidly.

The head man should see that the rear man is looking when he tries to straighten the chain or tape.

The chain should not be jerked in straightening it; it should be straightened by an undulatory motion; the tape must be handled more carefully and never pulled unless it can be seen to be clear, as it may be kinked in the grass or weeds where a strong pull will break it.

In straining the chain or tape, the headman should pull steadily, an estimated proper amount learned by experience and discussed in Arts. 19 and 20.

Attention to these matters will greatly facilitate the work.

13. Measuring on slopes. In measuring up or down hill, one end of the tape is raised till both ends are as nearly as possible in a horizontal line.

If the slope is so steep that one end of a full tape cannot be raised enough to bring both ends in a horizontal line, the tape is " broken"; that is, the distance is measured by using a part of the tape at each measurement. To do this, the tape should be carried out as if a full length were to be used, the head man returning to such a point on the tape (preferably a ten-foot or link point) that the portion of tape between himself and the rear man may be properly leveled. A measurement is made with this portion, then with the next succeeding portion, and so on till the whole tape has been used. Care must be taken not to get the pin numbering confused. The rear man should have but one pin for the whole tape.

The high end of the tape is transferred to the ground in one of several ways, according to the precision desired. If the work is to be done with care, a plumb line is used. If an error of a tenth of a link or foot in each tape is not important, a pin may be dropped from the high end, and stuck in the ground where it is seen to fall. The pin should be dropped ring down. A small pebble will serve the purpose for rough work. In careful work the plumb bob should not be dropped and the pin placed in the hole made; but it should be noticed where the bob will drop, the ground should be made smooth with the foot, and the bob swung down till it is still and just clearing the ground; then it should be carefully lowered till it touches. The tape-

man should then lower his grasp on the string, hand over hand, keeping the bob steadily in its place, and place a pin in the ground at the point of the bob. The pin should be put in the ground in an inclined position across the line, so that the point where it enters the ground is that covered by the bob. The position should then be checked. In place of a pin a small wire brad may be used and left in the ground.

In measuring up hill, the rear man must hold the bob directly over the pin which has been set in an inclined position, and must at the same time align the head man and see that he sticks at a moment when the bob is directly over the point. It will be inferred at once that it is easier to measure down hill correctly than up hill. Therefore, where close work is required on inclined ground, the measurements always should be made down hill if possible.

Where close work is required, it is considered best to measure along the slope, keeping the tape or chain supported throughout its entire length, and making the necessary reductions when the line has been measured. The reduction can be made exactly by the use of a table of versed sines if the angle of slope is known. It may be approximately obtained from Table I, page 473, by interpolating for the small angles, or it may also be approximately obtained by the use of the following formula when the rise in a tape length or in the entire line, if it is of uniform slope, is known:

The square of the rise divided by twice the known side, be it base or hypotenuse, gives the difference between the base and hypotenuse.

Demonstration (Fig. 14): Let B be the base, H the hypotenuse, and R the rise; C being the difference between B and H. Then $B = H - C$ and $H = B + C$. Assuming H known, there is written

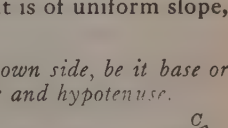


FIG. 14.

$$H^2 - (H - C)^2 = R^2;$$

whence

$$C = \frac{R^2 + C^2}{2H}.$$

Neglecting C^2 as a very small quantity, there results

$$C = \frac{R^2}{2H}.$$

Similarly, if B is known, there may be written

$$(B + C)^2 - B^2 = R^2,$$

$$C = \frac{R^2 - C^2}{2B};$$

and, as before, neglecting C^2, there results

$$C = \frac{R^2}{2B}.$$

Hence the rule already given.

ERRORS INVOLVED

14. Classes. The errors involved in the method of measuring just described, whether the work is done with a chain or a tape, are of two classes: (*a*) constant or cumulative errors, and (*b*) accidental or compensating errors.

(*a*) Cumulative errors are such as occur each time in the same direction. They are not equal necessarily, but may be so. Thus a line so long as to require that a tape one inch too short shall be applied to it ten times, will be recorded ten inches too long, the error of the tape being added each time the tape is applied. In this case the errors are equal.

(*b*) Compensating errors are such as tend to balance; that is, they are as likely to be in one direction as in another. Thus the error that may be made in setting the pin, if it is attempted to set it just right, will be a compensating error, for it will be set ahead of the true point about as often as it will be set behind it. Error in plumbing is of the same character.

Cumulative errors arise from four causes:

1. Failure to straighten the tape for each measurement;
2. Erroneous length of tape;
3. Sag of the chain or tape when not supported throughout its length (with a chain this is considerable);
4. Errors in judgment in making the tape horizontal in measuring up or down hill.

Compensating errors arise from accidental inaccuracies in setting the pin, from irregularities in the pull exerted on the chain or tape, and from erroneous reading of graduations. They are remedied by care.

15. Erroneous length of chain or tape. The chain may be too short or too long from any one of six causes:

1. One or more links may be bent, making the chain too short. The remedy is to see that the links are straight or to use a tape.

2. Mud or grass may get in the links and rings with the same effect. The remedy is obvious.

3. A bent link that has been straightened has been permanently lengthened, thus making the chain too long. The same effect results from straightening a kinked tape. The remedy is to compare the chain or tape frequently with a standard tape kept for this purpose. If the chain is found to be slightly too long, it may be adjusted by the nuts at the handle, or if such handle as is described in Art. 4 is used, the standard length of the chain for the day may be marked on the handle. If a tape is slightly too short or too long, allowance is made in the final record of the measurement.

4. The links and rings wear, thus making the chain too long. While the wear is but slight, it may be adjusted at the handle, but when it becomes excessive, it must be known and allowed for as hereafter described.

5. The chain or tape may be lengthened by being pulled too hard, but this does not often occur. The remedy is the same as in 3.

6. The chain or tape may be too short, or too long, according as the temperature is higher or lower than that for which the chain or tape is standard. The remedy is to know the temperature at which the chain or tape is standard and that at which the work is done, and make the necessary correction to the recorded measurements. Chain work is not corrected for temperature, since the precision of the work done does not warrant it. Fine tape work is corrected. Temperature effect is discussed in Art. 17.

In general it may be said that erroneous length of chain may be corrected by adjusting the handles, or by comparing the chain with a standard and correcting the records taken according to the errors found. Only the second method is used with tapes.

16. Effect of short or long tape. It should be noted carefully that in measuring the distance between two points a long tape

gives the distance too short and a short tape gives the distance too long, while in laying out a line of given length the errors are just reversed. Failure to appreciate this difference often causes confusion and error, and hence the student should thoroughly fix it in mind. Since similar figures are in area as the squares of their homologous sides, the erroneous area of a field, determined from measurements with an erroneous tape, will be to the true area as the square of the nominal length of the tape is to the square of its true length.

Examples

1. A line is measured with a chain that is afterwards found to be one link too long, and is found to be 10.36 chains long. What is its true length if the error is assumed to be distributed through the chain?

2. A line is measured with a 100-foot tape and found to be 723.36 feet long. The tape is afterwards found to be 0.02 of a foot short. What is the true length of the line?

3. A triangular field is measured with a chain that is afterwards found to be one link too long. The sides as measured are 6 chains, 4 chains, and 3 chains, respectively. What is the resulting area and what is the true area?

4. An irregular field is measured with a chain three links short. The area is found to be 36.472 acres. What is the true area?

17. Temperature. The coefficient of expansion of steel is about 0.0000065. A tape or chain will expand or contract sixty-five ten-millionths of its length for each Fahrenheit degree change of temperature. Thus, a line about ten chains long if measured in the summer with the chain at a temperature of, say, 80° F., the chain being standard at a temperature of 62° F., will be recorded 0.117 of a link too short; while the same line measured with the same chain in midwinter, with the chain at a temperature of 0° F., will be recorded 0.403 of a link too long, making a total difference of 0.52 of a link between the two measurements. This is an error of one in two thousand for the extreme difference in temperature of 80° F.

Thus it is seen that for all ordinary work the temperature correction may be neglected; but in city work where an inch in frontage may be worth several thousand dollars, it is very necessary that the temperature be determined and the standard temperature of the tape known. The tape shown in Fig. 9 is

adjustable for the effect of temperature. A scale numbered to correspond to the thermometer readings indicates the proper setting of the adjusting screw. The spring balance insures a constant pull.

Examples

1. A certain steel tape is of standard length at 62° F. A line is measured when the temperature of the tape is approximately 80° and found to be 323.56 feet long. What is its true length? Is it necessary to know the nominal or standard length of the tape to solve this problem?

2. A certain 100-foot steel tape, standard length at 62° F., is used to measure from the monuments to the point *A* (Fig. 15), in a line between lots 2 and 3 extended, when the temperature is 40° F. Assuming that the map distances are correct, what lengths must be measured from 7th Street and 8th Street monuments respectively to locate the point *A*, the monuments being in the center lines of the streets?

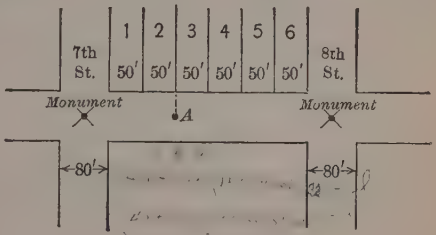


Fig. 15.

3. What change in temperature is necessary to change a 100-foot tape by 0.01 of a foot or 1 in 10,000?

18. Sag. The effect of the sag of the tape when unsupported is to make the measured distance appear too long. The distance measured is the chord of the sagging tape, while the distance recorded is the length of the curve or the full tape.

The difference between the length of the curve and the length of the chord is the correction to be applied to the measured distance to give the true distance. If the tape is applied several times — that is, if the measured line is several times as long as the tape — the total correction for the full line will be the sum of the several corrections for the several tape lengths.

The curve taken by the tape is called a *catenary*. It approaches quite closely to a parabola and also does not differ greatly from the arc of a circle. The depth of the sag, and consequently the difference between the chord and its arc, depends on the pull given the tape when making the measurement; the harder the pull the less the sag, and the nearer the

curve approaches to its chord. By mechanics, the depth of the sag v at the middle of the curve is given approximately by

$$v = \frac{wl^2}{8P},$$

in which w is the weight of a unit length of the tape, l is the length of the tape, P is the pull on the tape in the same units as w; *e.g.*, pounds. If the curve is assumed to be a parabola, the approximate difference between the chord and curve in terms of v may be shown to be (see any work on calculus)

$$x = \frac{8}{3}\frac{v^2}{l}.$$

Substituting the value of v gives

$$x = \frac{l}{24}\left(\frac{wl}{P}\right)^2 \quad (2)$$

as the difference between curve and chord in terms of the nominal length of the tape, the weight of the tape, and the pull exerted when the measurement is taken. It should be noted that wl within the parenthesis is the weight of the full tape or the unsupported part of it. It should also be noted that since x is a very small part of l, it is immaterial whether the l in the formula be chord or curve length, hence the nominal or graduated length is used because it is known.

If the tape is standard when unsupported and pulled with a force of P_0 pounds, and a measurement is made with the unsupported tape pulled with P pounds, the correction to be applied is the difference between the x as found by the formula for P_0 and that for P pounds, not the x from the formula for $(P - P_0)$ pounds. But if the tape is standard when supported throughout and pulled with P_0 pounds and a measurement is made with the unsupported tape pulled with P pounds, the sag correction is that found by the formula using P pounds. Usually the tape is standard for no pull more than enough to straighten it and for 62° F.

Examples

1. A 100-foot tape weighs 0.624 of a pound. What is the shortening due to sag when the tape is unsupported for its entire length and the pull is 10 pounds? 15 pounds? 20 pounds? 25 pounds?

2. If in the tape of Example 1 the shortening is 0.01 of a foot, which is an error of 1 in 10,000, what is the pull?

3. What is the rule for variation of the correction with variation of pull?

19. Pull. When the tape is pulled hard enough, it is stretched. A chain may also be stretched, the stretching being due chiefly to the elongation of the ring links.

There is no such thing as a perfectly elastic material. If there were, the amount that a given length of such material would be stretched by varying pulls would be proportional to the pulls. If a piece of such material be of unit cross section, as one square inch, the pull necessary to stretch it by its own length — *i.e.*, to double its length — is called the *modulus* of elasticity and is denoted by E.[1] Any other pull of P pounds would stretch the piece by $\frac{P}{E}$ (read P–Eths) of its length. If its cross section were two square inches, the stretch for the same pull would be half as much, since the pull per unit of area would be only half as much, so that if the area be S units, the stretch would be $\frac{1}{S}$ of $\frac{P}{E}$ of the original length. Thus, if a tape of length l, cross section S, and modulus of elasticity E be pulled with P pounds, the stretch is

$$y = \frac{Pl}{SE}.$$

For steel, E is variable, but, in the absence of more definite knowledge, may be taken at 28,000,000 for a square-inch section.

Examples

1. How much is a 100-foot tape with a cross section of 0.002 of a square inch lengthened by pulls of 10 pounds? 15 pounds? 20 pounds? 25 pounds?

2. How does the stretch vary with the pull?

[1] This is not the definition found in texts on Mechanics, but is a practical definition that does no violence to mechanics and seems to make the following discussion clearer.

3. If the tape of Example 1 is of standard length when fully supported for a pull of 10 pounds, and a level line is measured with a pull of 25 pounds, what correction must be applied per tape length and is it a plus or minus correction; *i.e.*, is the line recorded too short or too long?

4. What pull is necessary to stretch the tape of Example 1 by 0.01 of a foot, which is an error of 1 in 10,000?

20. Elimination of sag and pull. Since sag tends to shorten the distance between tape ends and pull to lengthen it, there is some pull at which these two effects are balanced. To find the pull that will just balance the effect of sag, the values of x and y of the preceding articles are equated and P is found. The solution gives

$$\frac{l}{24}\left(\frac{wl}{P}\right)^2 = \frac{Pl}{SE}; \quad \text{(Approx.)}^1$$

whence

$$P = \sqrt[3]{\frac{w^2 l^2 SE}{24}}. \quad \text{(Approx.)}$$

A good practical way to determine this value is as follows: Mark a standard[2] tape length on a smooth level floor with the tape supported its entire length and with only enough pull to straighten it. Raise the tape, and, supporting it only at the ends, with a spring balance measure the pull necessary to bring the ends over the marks on the floor. It will be best to have one end fastened in a firm hook in the wall for the test, and afterward to have both ends held by the chainmen, that they may see just the difficulties involved. The test should also be made for a whole chain, half chain, and quarter chain. The only way in which this work can be done with extreme nicety is by employing mechanical means to pull the chain or tape steadily, and a telescopic line of sight to transfer the floor marks upward to the tape ends. As in all work, except the measurement of base lines for geodetic surveys, or elaborate triangulation surveys of cities, the chain or tape is to be held in the hands of the chainmen, it will be unnecessary, except for comparisons, to resort to the nicer methods. Experiments of the kind noted above will demonstrate that the whole chain should never be

[1] This is a very close approximation, the nature of which the student should try to show.

[2] Tapes will be standardized for a nominal cost by the U. S. Bureau of Standards at Washington.

used unsupported, and that the tape is by far the most satisfactory measuring instrument. In rough work, where a precision of one in five hundred, or even one in five thousand, as a maximum limit, is sufficient, the chain *may* be used. In close work requiring a precision of one in five thousand and upward, the tape should invariably be used.

Examples

1. What pull is necessary to balance the effect of sag for the tape of Example 1, Art. 18? The cross section area in square inches is found by multiplying the weight per linear inch by 3.6, which is the number of cubic inches of steel in a pound.

2. If a tape were 300 feet long and of the same section as that of the preceding example, what pull would balance the sag?

21. Alignment. Errors due to inaccuracy of alignment of the tape usually are not great. In ordinary work no great pains need be taken to align the tape within an inch or two, except where stakes are to be driven on the line. In close work, of course, the tape should be correctly aligned.

Examples

1. What is the error, in a 1000-foot line measured with a 100-foot tape, arising from placing the end of the tape first six inches on one side of the line and then six inches on the other side throughout the measurement? Use the method of Art. 13.

2. How much must a 100-foot tape be out of line in order that the true line to be measured shall be in error by 0.01 of a foot, or 1 in 10,000?

22. Slope. In measuring on slopes, errors of judgment in making the tape horizontal are eliminated by the use of a level tube fastened to one end of the tape, which tube, if properly adjusted, will indicate when the tape is horizontal. Much can be done without such a level by having a third man stand on one side of the tape and compare the parallelism of the tape and the horizon, or the horizontal lines of some building. If there is no horizontal line visible, he can still judge better from the side as to the horizontality of the tape, than can the chainmen at the ends. It is almost always true that the lower end of the tape is not raised high enough, because a horizontal line on a hillside extending in the direction of the slope always

appears to dip into the hill. Hand levels (Arts. 57–60) carried by the chainmen are of great service in hilly country. The effect of neglecting the slope entirely, which is also the correction to be applied if the line has been measured on the slope instead of in horizontal lines, is given in Appendix, Table I, page 473.

It will be seen that the error caused by neglecting a slope of five in one hundred is about one in one thousand, while a slope of ten in one hundred, which is not unusual in hilly country, causes an error of one in two hundred. Fifty feet in one hundred is about the steepest slope met with in nature, aside from rock cliffs, and the error here is more than one in ten.

23. Summary of errors. The errors that are made in chaining with a tape are summed with their signs as follows, sign meaning the effect on a measured distance between two given points; + indicating result too long, − too short.

Tape too short or long	±	always same sign in one survey	Cumulative
Temperature	±	but always the same sign in a survey made under fairly constant conditions of temperature	Cumulative
Sag	+		Cumulative
Pull	∓	according as the pull is greater or less than standard	Compensating?
Alignment	+		Cumulative
Grade or slope	+		Cumulative
Error in setting pin or marking tape ends	±		Compensating
Error in reading tape on final measurement only	±		Compensating

The error in reading the tape is due to the fact that the measurement will not be likely to fall exactly at a graduation mark, and the nearest graduation will be used with or without an estimated fraction according to whether the graduations are fine or

coarse. Usually no measurement of less than 0.01 of a foot will be recorded. Thus it will be seen that this error is very small and affects the last tape length or fractional measurement only. The effect of pull, if really compensating in character, must be due to accidental variations on both sides of the standard pull. These are likely to be small and rather more likely to be all one way, unless a spring balance is used, than both ways. If a spring balance is used, the errors will be too small to consider. Thus it is seen that of all the possible errors the only one of any consequence that is really a compensating error, and hence may be supposed to follow the laws of chance as given in the theory of least squares, is the error of marking tape lengths and holding the graduations at the marks made.

24. Least squares theory. The theory developed by least squares shows that if a given measurement is made up of the sum of n measurements, as the length of a line is made up of the sum of n tape lengths, the probable error of each of which is r, the probable error of the given measurement is $r\sqrt{n}$. But this theory is based on the supposition that

(*a*) Small errors occur oftener than large ones.

(*b*) Positive errors occur as often as negative errors.

(*c*) Very large errors do not occur.

The errors of chaining cannot be said to follow the middle and perhaps most important supposition, and therefore least squares methods of discussing the reliability of the work are of doubtful value. It is thought to be true that errors in chaining in almost any surveying operations except those in which the greatest refinements are used, are in greater degree proportional to the length of the line measured than to the square root of that length. When work is done with all the refinement of geodetic base line measurements, so that all cumulative errors are allowed for or eliminated, and many measurements of each quantity are made, the method of least squares is applicable.

In addition to what has already been given, the theory reaches the following results which apply only to small, accidental, compensating errors:

If a quantity A is determined n times under similar conditions—

1. The most probable value of the quantity is the arithmetic mean of the n determinations. (This may be considered axiomatic.)

2. If v represents the difference between the mean and any one determination in general, and Σ is the sign for summation so that Σv^2 is the sum of the squares of all the different v's, the probable error of a single determination is

$$e = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}. \qquad (1)$$

The probable error of the mean or most probable value is

$$E = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}} = \frac{e}{\sqrt{n}}. \qquad (2)$$

If a quantity S is the sum of several independent quantities $Q_1, Q_2, Q_3, \cdots Q_n$, the probable errors of which are respectively $r_1, r_2, r_3 \cdots r_n$, the probable error of S is

$$E_S = \sqrt{r_1^2 + r_2^2 + r_3^2} = \sqrt{\Sigma r^2}. \qquad (3)$$

If the r's are all equal, this is

$$E_S = r\sqrt{n} \qquad (4)$$

as mentioned above.

The probable error is regarded as a measure of the precision with which the work is done. This precision is usually expressed as a ratio of $\frac{E}{\text{Length of line}}$. The probable error very likely does give a measure of the uniformity with which many measurements of the same quantity are made but this is about all that can be said for it. It is hardly proper to speak of it in connection with only two measurements of a quantity. Nevertheless, for want of a better measure, it will be used in following discussions.

It is defined by Merriman [1] as follows:

"In any series of errors the probable error has such a value that the number of errors greater than it is the same as the number less than it. Or, it is an even wager that an error taken at random will be greater or less than the probable error."

[1] "Method of Least Squares" by Mansfield Merriam.

Equation (2), page 34, shows that if e is constant, the probable error varies inversely with the square root of the number of observations, and that, therefore, to halve E, or to secure twice a given degree of precision, the number of observations or measurements must be four times as many as for the given precision. Thus, if 9 observations or measurements give a probable error of E for the arithmetic mean or most probable value of a quantity, 36 observations of equal care will be required to give a probable error of $\frac{1}{2}$ E, or, what is the same thing, a precision apparently twice as high.

25. Precision to be obtained. In measuring lines the degree of precision obtainable should be known by the surveyor. The author suggests the degrees of precision mentioned below as those that should be attained ordinarily before the surveyor can say he is doing good work. The figures given do not refer to the absolute lengths of the lines, involving a knowledge of the absolute length of the chain or tape, but merely to the probable error of the mean of two measurements of the same line made with the same tape under the same conditions. Not all conditions of work are covered; but only such as usually exist. The surveyor will be able to judge as to how closely the conditions under which he is working at any time correspond to those given. When only accidental compensating errors are considered and but two observations of a quantity have been taken, equations (1) and (2) of Art. 24 show that the probable error of either of the observations is about 0.48 D, or, roughly, $\frac{1}{2}$ D, D being the discrepancy between the two observations, and the probable error of the mean is about $\frac{1}{3}$ D. The student may show this. This supposes that all cumulative errors and mistakes have been eliminated by correction and that only accidental errors remain.

Assuming the work to be done with a chain in good, open, fairly level ground, good work will be represented by differences between two measurements of one in twenty-five hundred, and excellent work by differences of one in five thousand. These differences give the probable error of the mean value as $\frac{1}{7500}$ and $\frac{1}{15000}$ of the distance measured, and the probable error of a single determination rather better than $\frac{1}{3000}$ and $\frac{1}{10000}$ of that distance. On hilly ground, rough and covered with brush, one in one thousand might be considered good and one in five hun-

dred passable, where the land is not of great value. These differences give the probable errors of mean and single measurements as $\frac{1}{5000}$ to $\frac{1}{1500}$ and $\frac{1}{2000}$ to $\frac{1}{1000}$ of the distance measured respectively.

It should be remembered that the value of the land measured, or the object of the survey, is a better basis for judgment as to passable work than the conditions under which the work is done.

In work in large cities the author thinks that a precision of one in fifty thousand should be obtained. That is, it is thought that the probable error of the mean of two measurements should not be greater than one fifty-thousandth of the distance measured. This will require that the same line measured under totally different conditions as to weather should be recorded, after the necessary corrections for pull, grade, and temperature have been made, both times alike, within about one in seventeen thousand; or, in round numbers, three tenths of a foot in a mile.

The following are the requirements for securing a precision of one in five thousand and one in fifty thousand. For intermediate standards, the requirements will lie between those mentioned:

For a precision of one in five thousand, using a tape, no corrections for sag, grade, pull, or small changes of temperature need be made. The tape may be stretched by hand, the pull and horizontality being estimated by the tapemen. The plumb line will be used on uneven ground as in close chaining. For considerable changes in temperature the temperature of the air may be compared with that for which the tape is standard, and a corresponding correction deduced.

For a precision of one in fifty thousand, the temperature of the tape should be known within a degree or two Fahrenheit; the slope should be determined by measuring over stakes whose elevations have been determined by a level, or by measuring on ground whose slope is known. The pull should be known to the nearest pound, and hence should be measured with spring balances. If the tape is held on stakes, the sag correction must be considered or the normal pull used. The work may be done in any ordinary weather, but is best done on cloudy days, so that the temperature of the tape may be more constant. In sunny weather the mercurial thermometers attached to the tape may indicate a very different temperature

from that of the tape. If the absolute length of the tape is not known, of course the absolute length of the line is not determined.

Examples

1. To erect a perpendicular to a line. A triangle whose sides are in the ratio, 3, 4, 5, is a right triangle. Hence, using a chain, fasten one end and the forty-eight-link division at one point in the line and the sixteen-link division at another point. Carry the twenty-eight-link division out till the three portions of the chain used form a triangle. If the line is to be erected at a given point, the method will suggest itself to the student.

2. Using the tape and pins in the way a string and pencil or a pair of dividers would be used on the drawing board, perform the above problem in a different way.

3. To drop a perpendicular on a line from a given point: Run an inclined line from the point to the given line, erect a perpendicular to the latter, and produce it till it intersects the inclined line. The required perpendicular is parallel to the perpendicular erected, and the consideration of the similar figures will enable the student to solve.

4. To run a line parallel to a given line: Erect two equal perpendiculars to the given line. If the parallel is to pass through a given point, drop a perpendicular from the point to the line and erect a perpendicular to this at the point. Otherwise: The diagonals of a parallelogram bisect each other.

5. To measure an angle with a tape: In an isosceles triangle of legs a, included angle A, and base b, $2\ a \sin \frac{1}{2} A = b$.

6. To lay out a given angle: Reverse the process just suggested.

7. A 100-foot tape is standard length for a pull of 10 pounds, when supported its entire length, and at a temperature of 62° F. A line is measured and found to be 1000.00 feet long. The tape is of steel, has a cross section of 0.002 of a square inch, and weighs 0.00052 of a pound per inch. In the measurement it is at a temperature of 80° F., and is unsupported except at the ends. The pull is 25 pounds. What is the true length of the line?

8. A steel tape weighs 0.0061 of a pound per foot. It is 100 feet long and is standard for no pull when entirely supported, at 62° F. It has a cross section of 0.002 of a square inch. What pull must be exerted to keep the tape standard length if it is supported only at its ends, the temperature remaining unchanged?

9. A line is measured along the surface on a hillside. The first 300 feet has a vertical angle of 3°, the next 250 feet has a vertical angle of 4° 30′, and the last 700 feet has a vertical angle of 1°. What is the true length of the line?

10. Measure a line of about 1000 feet or more in length, at least twice with a chain and twice with a tape, and determine the difference of measurement. Do this, if possible, over two lines of about equal length, but offering very unequal difficulties to measurement, and note the differences.

CHAPTER II

VERNIER AND LEVEL BUBBLE

Before proceeding with a description of surveying instruments, it is necessary to describe two important attachments common to almost all such instruments. These are the vernier and the level bubble.

VERNIER

26. Vernier. This is a device for reading scales to a greater degree of precision than is possible with the finest convenient division of the scale. Thus, a scale graduated to read tenths of an inch may be read to hundredths of an inch by the aid of a vernier. This is done by making an auxiliary scale called a vernier, with divisions one one-hundredth of an inch smaller or larger than those of the main scale. If the divisions are larger than the main scale, the vernier is called a retrograde vernier; and if the divisions are smaller, it is called a direct vernier. The reason for this distinction will appear hereafter. In Fig. 16, S is a scale divided into inches and tenths. V is the vernier made by dividing a space equal to nine of the small divisions of the main scale into ten equal parts, thus making each division on the vernier one one-hundredth of an inch shorter than a division of the main scale. The first division line of the vernier falls one one-hundredth of an inch toward zero from the first division line of the main scale. If then the first division line of the vernier is made to coincide with the first line of the main scale, the vernier will have been moved one one-hundredth of an inch. Similarly, the second division of the vernier is two one-hundredths of an inch toward zero from the second line of the main scale, and, hence, if the vernier is moved along till the second line of the vernier coincides with the second division of the scale, the movement has been two one-hundredths of an inch, and so on. If the vernier is moved till the zero is opposite some other division of the scale than the zero division, the first

line of the vernier will be one one-hundredth short of the line of the main scale next ahead of the zero of the vernier; the second line of the vernier will be two one-hundredths short of the second line of the main scale, and so on. If the vernier is moved along a little farther until, say, the fourth line of the vernier has been brought into coincidence with the fourth line of the main scale ahead, the vernier has been moved a farther distance of four one-hundredths of an inch. Hence, to tell how far the zero of the vernier has moved from the zero of the main scale, note the inches and tenths on the scale from zero to the zero of the vernier, and get the fractional tenth expressed in hundredths by looking along the vernier and finding the division that coincides with a division of the main scale. This vernier is called direct, because it is read in the direction in which the scale is read.

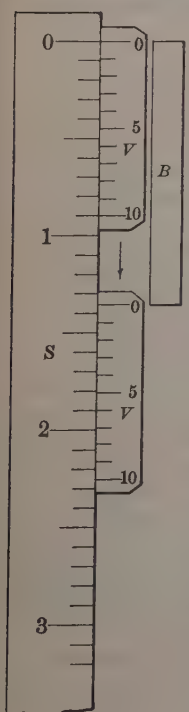


FIG. 16.

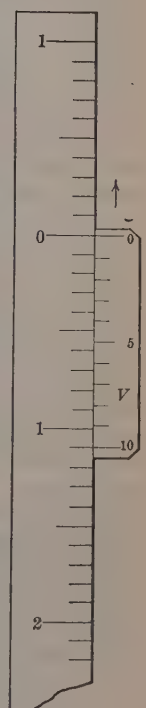


FIG. 17.

Let it be required to read the length of the bar *B*. Place one end of it opposite the zero of the main scale and vernier. It will be noticed that the other end is opposite a point on the main scale between one and three tenths inches, and one and four tenths inches. Move the vernier till the zero is opposite this end of the bar. To read the length of the bar, read on the main scale one and three tenths inches and look along the vernier and find that the sixth division coincides with a division of the scale, and that, therefore, the length of the bar is one and thirty-six one-hundredths inches. It will be observed that the divisions of the vernier are one tenth of one tenth of an inch smaller than the divisions of the scale. That is, the value of

the smallest division on the main scale divided by the number of divisions of the vernier gives the smallest reading that may be had with the vernier. This is called the least count.

In the retrograde vernier (Fig. 17), a space equal to a given number of divisions of the main scale is divided into a number one less on the vernier. Thus, for a vernier reading to hundredths of an inch with a scale graduated to tenths, eleven divisions of the scale will be divided on the vernier into ten spaces, making each division one tenth of one tenth of an inch longer than those of the scale. From the portion of the scale extended below the zero in the figure, it will be seen that the first line of the vernier is back of the first line of the scale by one one-hundredth of an inch, the second line by two one-hundredths, and so on. The principle of operation is the same as in the direct vernier, except that the vernier is read in the direction opposite to that in which the scale is read. It will be observed that the number of divisions on the vernier is the same for both classes, and that it is the corresponding number of divisions of the scale that varies.

A vernier to read angles is generally used when the angles are to be read to the nearest minute or less. The principle of construction is the same as for linear verniers. A vernier to read minutes will usually come with a circle graduated to read half degrees. If a space equal to twenty-nine of such divisions is divided on a vernier into thirty equal parts, each division of the vernier will be one thirtieth of thirty minutes, or one minute, less than a division of the main scale, and the instrument is said to read to minutes. If a circle is to be read to the nearest twenty seconds, it is usually divided into twenty minute spaces, and a vernier must then have sixty divisions, since

$$\frac{20'}{n} = \frac{1'}{3};$$

$$n = 60 \text{ divisions.}$$

That is, fifty-nine parts of the scale must be divided on the vernier into sixty parts, making each part of the vernier one sixtieth of twenty minutes, or one third of a minute, less than a division of the main scale. Figures 18, 19, and 20 show three double verniers. They are called double, because there are really two verniers in each figure, one on each side of the ver-

nier zero. They are arranged thus so that angles may be read in either direction, the circle graduations being numbered both ways for the same purpose. The student should determine

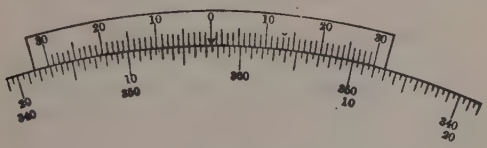


FIG. 18.

whether the first two are direct or retrograde, the least count of each, and their readings. The third is a peculiar pattern found ordinarily only on compasses. It is a double vernier,

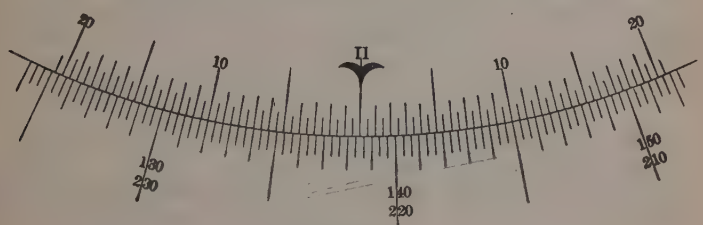


FIG. 19.

direct as to division (though it is sometimes made retrograde), and the lower left-hand and upper right-hand portions form one vernier. It is used where there is lack of space to make the

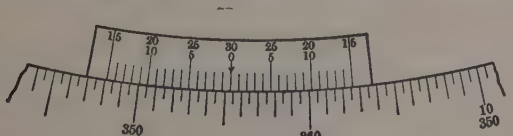


FIG. 20.

ordinary form. To read the angle indicated in the figure by the numbers increasing to the right, read on the scale to the last division before reaching the zero of the vernier, follow to the right along the vernier, noting the lower line of figures for a coinciding line, and if none is found, pass to the extreme left

end of the vernier and look along toward the right, noting the upper line of figures till a coinciding line is found. Thus, the reading of the vernier in the figure is 355° 20′, or 4° 40′ if read to the left.

A vernier to read decimals of a degree would be exactly like a linear vernier. If the main circle is divided into half degrees, and it is required to read hundredths of a degree with a direct vernier, forty-nine divisions of the circle must be divided into fifty divisions on the vernier, making each division of the vernier $\frac{1}{50} \times \frac{1}{2}° = \frac{1}{100}°$ shorter than a circle division. If the circle division be of quarter degrees, twenty-four parts of the circle will be divided into twenty-five parts on the vernier, and each vernier part will be $\frac{1}{25} \times \frac{1}{4}°$ shorter than a division on the circle. For simplicity in reading, the half degree division would be the better. If each vernier part be divided into two parts, the vernier would read to half hundredths or 18″, a little finer than a 20″ reading vernier.

Exercises

1. What are the readings of the verniers shown in Figs. 18 and 19?
2. Set a level rod target at random and read it.
3. Make the following settings with a level rod, using the target or back vernier as necessary:

1.264 ft.	4.807 ft.	5.036 ft.
7.060 ft.	8.008 ft.	10.210 ft.

4. Having a transit instrument set by the instructor, if the student does not know how, let the circle be read for several angles, particularly for angles near 360°, 0°, and 180°. Let this exercise be performed for each different instrument in the collection.
5. Make the following settings with a minute-reading instrument:

12° 22′	15° 44′	176° 52′
188° 18′	359° 28′	00° 36′

6. Make the following settings with a 30″ reading instrument:

18° 14′ 30″	22° 44′ 36″	171° 12′ 30″
187° 00′ 30″	259° 1′ 30″	00° 26′ 00″

7. If there is a decimal reading angle vernier, let the following setting be made:

0.02°	23.48°	174.72°
183.26°	359.89°	44.51°

8. With a radius of three inches draw an arc of about 30°, divide it into half degree spaces, and alongside make a direct and a retrograde vernier which will read to hundredths of a degree.

9. Explain the construction of the vernier in Fig. 20.

LEVEL BUBBLE

27. Description. The spirit level consists of a glass tube almost filled with ether, alcohol, or a mixture of these fluids. The remaining space is filled with air and vapor. The bubble of air and vapor will always seek the highest point in the tube. If the tube were perfectly cylindrical, the bubble would occupy the entire length of the tube when the tube is horizontal, and the same thing would be true if the tube were but slightly inclined to the horizon, thus making it impossible to tell when the tube is in a truly horizontal position. The tube is, therefore, ground on the inside so that a longitudinal section will show a circular arc. A line tangent to this circle at its middle point, or a line parallel to this tangent, is called the axis of the bubble tube. This axis will be horizontal when the bubble is in the center of its tube. Should the axis be slightly inclined to the horizon, the bubble will move toward the higher end of the tube, and if the tube is ground to the arc of a circle, the movement of the bubble will be proportional to the angle made by the axis with the horizon. Therefore, if the tube is graduated, being a portion of the circumference of a very large circle (so large in fact that the arc of a few seconds is quite an appreciable length), it will be possible to determine, within the limits of the tube, the angle that the axis may make at any time with the horizon, provided the angular value of one of the divisions of the tube is known. This is done by simply noting how many divisions the center of the bubble has moved from the center of the tube, and multiplying this number by the angular value of one division.

It will be evident that divisions of uniform length will cover arcs of less angular value as the radius of the tube increases, and also that the bubble with a given bubble space will become more elongated as the radius is increased. Therefore, the bubble is said to be sensitive in proportion to the radius of curvature of the tube, and this is also indicated by the length of the bubble. The length of the bubble, however, will change with change of

temperature, becoming longer in cold weather and shorter in warm weather. In the best class of tubes there is a partition near one end, with a small hole in it at the bottom, so that the amount of liquid in the main tube may be regulated, thus regulating the size of the bubble. This is necessary, because independently of the effect of long radius a longer bubble is more sensitive than a shorter one. Bubbles on ordinary surveying instruments have no such division. A bubble should settle quickly, but should also move quickly and easily.

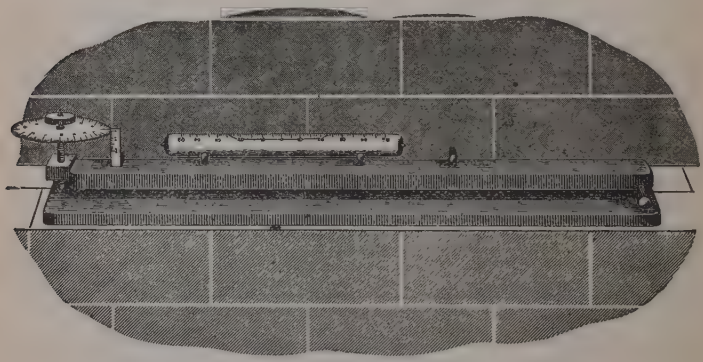


FIG. 21.

28. Determining the angular value of one division. There are several methods of determining the angular value of one division of the bubble tube, all essentially the same in principle.

The axis is moved through a small angle, and the movement of the bubble is recorded in divisions; then the angular value of one division is found at once by dividing the angle by the number of divisions through which the bubble has moved. It is not easy to measure the small angle exactly, but it is not very difficult if closely approximate results are sufficient.

In Fig. 21 is shown a level vial, as it is sometimes called, resting on a level trier. The construction of the level trier is perhaps sufficiently clear from the cut. It consists simply of a bar resting on a knife edge at one end, and capable of being raised or lowered at the other by means of a screw so divided as to tell the angle of inclination of the bar. The screw is

called a micrometer screw, because it will measure a very small movement. Suppose the pitch of the micrometer screw is one sixtieth of an inch. Then the divisions on the vertical scale attached to the movable bar will be one sixtieth of an inch; so that a single revolution of the screw will move the scale past the edge of the screw head by one division. If the circumference of the disk head of the screw is divided into one hundred parts, and the screw is turned only so much as will cause one division on the disk to pass the scale, the bar has been moved vertically, through one one-hundredth of one sixtieth of an inch. If now the length of the bar from pivot to screw is known, the angular movement of the bar may be computed. Thus, if the length of the bar is eighteen inches, and the bar is raised so that one division of the scale passes the micrometer head, and so that in addition ten divisions of the micrometer head pass the scale, the linear elevation of the end of the bar is

$$\tfrac{1}{10} \times \tfrac{1}{60} \text{ inch} = 0.018 + \text{inch}.$$

Remembering that the screw is perpendicular to the axis of the bar, a sketch will show the relation of the various parts, and if made to large scale will show that the straight-line elevation which is the tangent of the angular movement of the bar may be assumed to be equivalent to the arc measuring that movement. Since there are 206,265 seconds in an arc equal in length to radius, there results the proportion, in which x is the angle in seconds,

$$\frac{x}{206265} = \frac{0.018+}{18};$$

whence

$$x = 206.265 \text{ seconds}.$$

If a bubble tube were resting on the bar, and the run of the bubble were observed, for the above movement, to be ten divisions, the value of one division would be 20.6 seconds.

Other methods of finding the angular value of a division of the tube will be suggested in the problems in Chapters III and IV.

Many of the level vials found on compasses, and on the lower plates of many other instruments, are not graduated, are ground to short radii, and not uniformly, and hence are not fit for accurately leveling the instruments to which they are attached;

but such bubbles are cheaper than others, and when placed on a compass or other instrument not intended for high-class work, are sufficiently precise for the purpose for which they are used.

The plate bubbles of surveying instruments should have angular values of one division of from 20″ to 30″, or say from 0.006° to 0.01°, while the bubble on the telescopes of transits and levels should have angular values of one division of from 10″ to 20″ or from 0.003° to 0.006°.

Examples and Exercises

1. In the example in the above article it is found that the run of the bubble is one inch. Find the radius of curvature of the bubble tube.

2. There being 57.2958° in an arc equal to radius, find the angular value of one division of the bubble considered in the above article in degrees, using the radius found in Example 1.

3. With the assistance of an instructor find the angular value of one division of various bubbles on the instruments of the collection. The plate bubbles may best be tested on a level trier if one is owned. The telescope bubbles may be tested by reading a rod held at a known distance, first with the bubble near one end of its tube, then near the other. Both ends of the bubble must be visible and within the graduations, so that the position of its center may be found by averaging the readings of its ends. It is the movement of the center of the bubble that is required. The angle in radians is determined by dividing the movement of the line of sight over the rod by the distance, the corresponding run of the bubble in divisions is noted, and the angular value of one division is found.

Several determinations should be made and the mean value used.

29. Principle of reversion. Let a carpenter's level rest on a perfectly level support as in Fig. 22. If the bubble in the tube is not in the center, the axis of the bubble is not parallel to the base of the level; and if the level be turned in any direction on its absolutely level support, the bubble will remain in the same position in its tube, since the angle made by the axis of the tube with the horizontal does not change.

Let it now be supposed that the axis is parallel to the base of the level. This is the condition necessary to a proper use of the instrument. If the level rests on a surface not horizontal or level, the bubble will not be in the center of the tube, but if the surface is made level, the bubble will go to the center.

Again (Fig. 23), let it be supposed that the axis of the bubble is not parallel to the base and that the instrument is

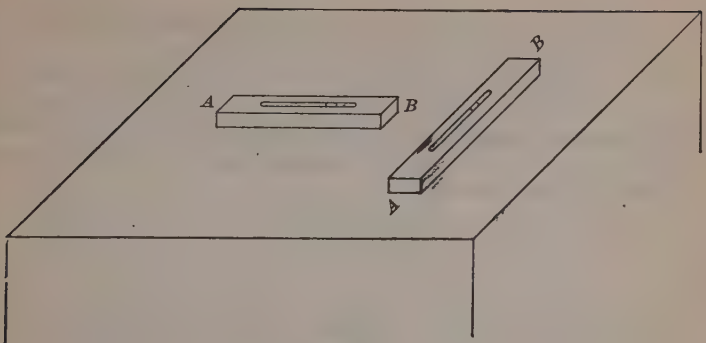


FIG. 22.

laid on a surface which is tipped until the bubble comes to the center of the tube. (In the figure the bubble is shown raised above the framework to show the lack of parallelism.) The surface on which the level rests will make an angle with the horizontal equal to that between the axis of the bubble tube and the base of the level. The two angles will be in opposite directions, thus balancing. If now (Fig. 24) the level be reversed end for end without disturbing the supporting surface, the two angles will be brought in the same direction, the axis of the bubble tube will be tipped to the horizontal by an angle twice that which it makes with the base of the level, and the bubble will move a corresponding distance from the center of its tube. If the lower end of the tube be raised, or the higher end lowered till the bubble moves halfway back to the center, the axis of the tube will be parallel to the base of the

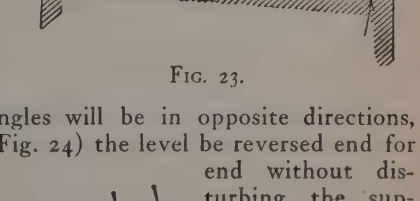


FIG. 23.

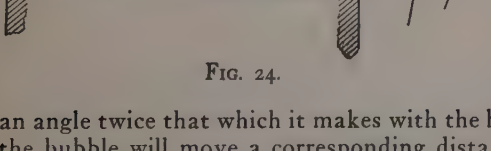


FIG. 24.

level, and if the support be tipped till the bubble is in the center of the tube, the support will be level.

All surveying instruments get out of adjustment in various ways, and this principle of reversion is used to make the errors of adjustment apparent so that they may be corrected. It will be noticed by the principle of reversion that the apparent error is double the real error.

CHAPTER III

MEASURING DIFFERENCES OF ALTITUDE, OR LEVELING

30. General principle. It will be evident that if by any means a line of sight may be made to revolve about a vertical axis to which it is perpendicular, it will describe a horizontal plane. Omitting consideration of the curvature of the earth, a rod graduated from the bottom up, and held at any point on the ground, will be cut by this horizontal plane at a distance above the ground equal to the height of the line of sight above the ground at the point where the rod is held. The distance above the bottom of the rod, as indicated by the graduations on the rod, is called the reading of the rod. If the elevation of the line of sight above some assumed base or datum, as sea level, is known, the rod reading subtracted from that elevation will give the elevation of the point where the rod is held, referred to the same base. Conversely, if the elevation of the point where the rod is held is known, and it is required to find the elevation of the line of sight, it is done by adding the rod reading to the elevation of the point. While there are many details to be considered, such as the curvature of the earth, the adjustment of instruments, atmospheric conditions, etc., the above contains the essential principle of leveling.

INSTRUMENTS

31. General description of level. Any instrument used for the purpose of securing a horizontal line of sight may be called a leveling instrument; or, as is more usual, simply a level. There are three comparatively common forms of levels, shown in Figs. 25–27.

Figure 27 is a precise level or level of precision. Figure 25 is called a Y level and Fig. 26 a dumpy level.

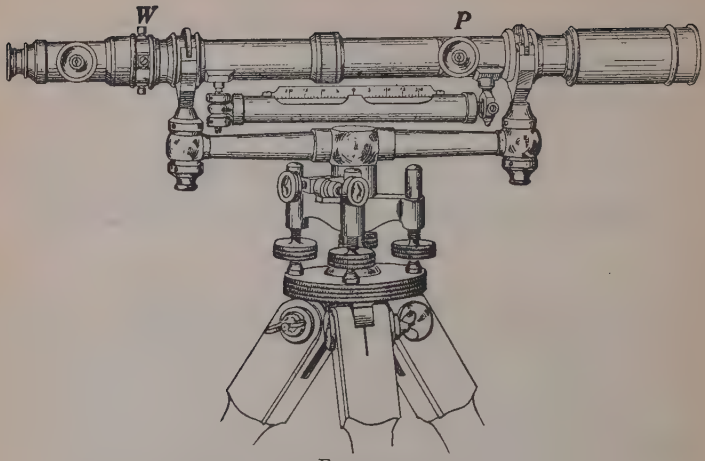


FIG. 25.

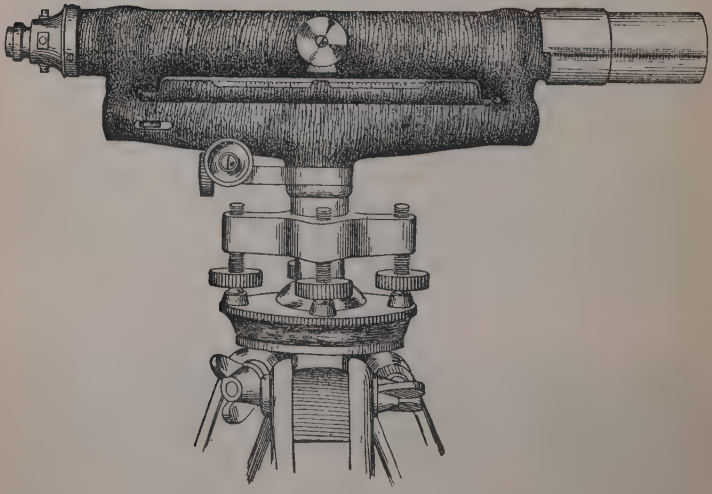


FIG. 26.

The precise level shown in Fig. 27 is the 1900 pattern of the U. S. Coast and Geodetic Survey. This instrument is used only in the most refined work, rarely coming to the engineer, and will not be described.[1]

The most common of the instruments is the Y level, so called because the telescope rests in Y-shaped supports. The instru-

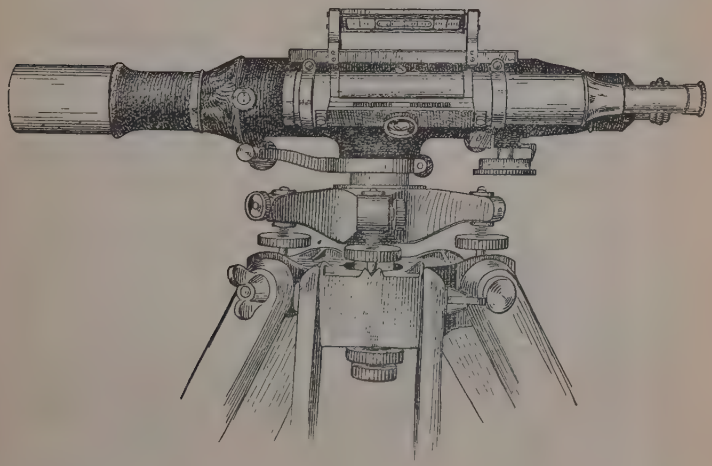


FIG. 27.

ment consists essentially of a telescopic line of sight, and an attached bubble tube whose axis may be made parallel to the line of sight by adjustment, combined with a leveling head which screws on to a tripod. A sectional view of a Y level is shown in Fig. 28.

From the standpoint of the optician the Y level is a more perfect instrument than the dumpy level because of its many easy adjustments; but this very feature is to some extent an undesirable one from the standpoint of the engineer who, for all ordinary work, needs an instrument with few parts to get out of adjustment.

The line of sight of the dumpy level cannot be so easily or

[1] It may be further studied in Johnson's "Theory and Practice of Surveying," in the Reports of the U. S. C. & G. Survey, and Trans. Am. Soc. C. E., Vol. XLV.

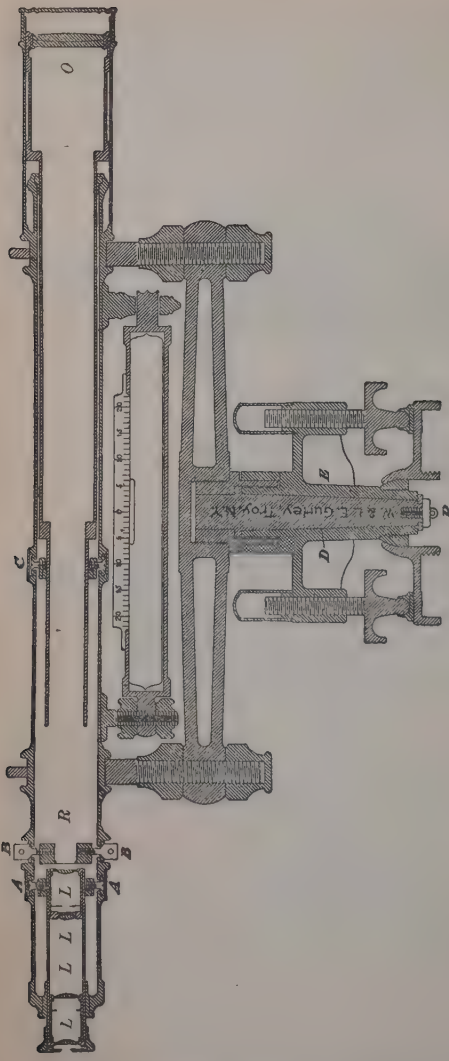


FIG. 28.

exactly adjusted as that of the Y level, but the instrument is sufficiently precise for all work not requiring a precise level and it is considerably cheaper than the Y level of the same make. It is used almost altogether by English engineers and was invented by an Englishman named Gravatt, whence the level is frequently called the Gravatt level. The term, dumpy level, comes from its short telescope with large aperture.

32. Telescope.[1] The telescope of the level consists of a barrel in which two tubes slide. One of these tubes is the eyepiece tube carrying the eyepiece lenses *LLLL* (Fig. 28), and the other is the objective tube carrying the objective, or object glass *O*. The objective tube is

[1] For a discussion of the principles of telescopes see any good book on Physics.

moved in and out by means of a pinion, which works in a rack attached to the sliding tube. The head of the pinion is shown at *P* in Fig. 25. The tube is made to move in the axis of the barrel by having it pass through the ring at *C*, which is accurately centered in the barrel. The eyepiece tube moves in and out of the barrel at the other end in a similar way, and is centered by the ring shown at *AA*. Instead of a rack and pinion movement, the eyepiece should be moved by twisting, a small screw extending through the barrel and into a helical slot in the eyepiece tube forcing the tube in or out as it is twisted. This is a better plan than the rack and pinion arrangement, because the eyepiece is but seldom changed and when once set should not be easily disturbed. Some instruments, however, have the rack and pinion movement, and it is preferred by some surveyors. In addition to these two tubes there is a ring at *R* (shown separately in perspective in Fig. 29) which carries two fine wires, one vertical and one horizontal. These wires are either spider lines or fine platinum wires. The spider lines are more common. This ring is centered in the barrel by means of the screws at *BB*. There are four of these screws, called capstan-headed screws, arranged as seen at *W* in Fig. 25. If the ring is to be moved to the right, the screw on the left is first loosened, and then the screw on the right is tightened, thus drawing the ring over to the right. Similarly for the vertical movement of the ring — if the ring is to be moved up, the lower screw is first loosened and the upper screw is then tightened, and *vice versa*. The difference between this telescope and the ordinary or Galilean telescope is in the introduction of these wires and a difference in eyepiece necessitated by them. In the ordinary telescope, such as is found in field glasses, there are no wires, and it is impossible to say at what particular point in the field of view the axis of the telescope is directed. Moreover, it is useless to put such wires in a field glass, since no image of the object looked at is formed in the tube of the telescope. With an angle-measuring instrument, or any instrument that must be pointed at a definite point, it is essential that the exact point at which the axis of the telescope is directed be known. It is inaccurate perhaps to say that the direction of the axis

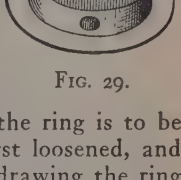


FIG. 29.

must be known, for any other fixed line in the telescope would do as well, provided the adjustments hereafter to be described could be made with that fixed line. It is more convenient to have the fixed line at least very close to the axis of the telescope, for reasons that will appear.

The imaginary line forming the optical axis of the object glass of the telescope is known as the *line of collimation*. It should, and will hereinafter be assumed to, coincide with the geometric axis of the telescope. The line joining the optical center of the objective and the intersection of the cross wires is the *line of sight*, and should coincide with the line of collimation.

The area seen at one time through the telescope, or rather the angle between the rays of light from the extreme edges of this area, measured at the instrument, is known as the *field of view*. This field of view is larger as the magnifying power of the telescope is smaller, and varies from about one and one half degrees to about fifty minutes. The former is for the commoner kinds of surveyor's transits, and the latter corresponds to a magnifying power of about thirty-five diameters, or about what is found in the better leveling instruments.

An image of an object within the field of view is formed at a point back of the object glass. There is a definite relation between the distances of the object from the lens and the lens from the image — the farther away the object, the nearer the image is to the lens.

The glass is moved in or out till the image of the object looked at falls in the plane of the wires. This is called focusing the objective. The point in the image covered by the intersection of the wires is that at which the line of sight is directed.

It would be practically impossible to tell when the focusing had been done, were it not for the eyepiece, which is nothing more nor less than a microscope with which to obtain a magnified view of the wires and the image formed by the objective. This is done by first focusing the microscope eyepiece till the wires are plainly seen. Then any other objects in the same plane with the wires will be seen at the same time and objects not in that plane cannot be seen distinctly, and hence it may be determined when the image formed by the objective is in the plane of the wires and also what point in the field of view is covered by the intersection of the wires.

The objective will need to be focused anew for each different

object looked at, unless, as will rarely occur, all the objects viewed are at the same distance. The eyepiece, on the other hand, since it has to be focused for only one distance, need be changed only for different individuals; and hence, if only one person is to use the instrument for a long time, the eyepiece may be focused once, and not again disturbed during the time it is used by this person.

The eyepiece shown in Fig. 28 has four lenses, and is commonly known as an erecting eyepiece. The image formed by the objective is inverted, and the eyepiece inverts the image so that the object appears right side up or erect. This eyepiece has been generally used in American surveying instruments because of a supposed difficulty in the use of one that shows the object inverted. Such an eyepiece, shown in Fig. 30, has two lenses less than the erecting eyepiece, and consequently absorbs less light and secures better definition of the object viewed. It is also possible to secure equal magnifying power with a shorter telescope, as is done in the dumpy level. Figure 25 would be recognized at once as an erecting instrument because of its long eyepiece reaching from near the plane of the cross wires to the eye end of the telescope, while Fig. 26 would be known at once as an inverting telescope because of its short eyepiece. It is to be preferred for all surveying instruments, and is well-nigh indispensable for some kinds of work; for instance, for stadia measurements, to be hereafter described. The inconvenience attending the use of a telescope that shows objects inverted is largely imaginary. A few hours with an inverting glass makes its use as natural as the use of an erecting glass. Dumpy levels and precise levels are almost always inverting instruments; while the Y level, the most commonly used in this country, is almost always erecting. As has already been stated, there now seems to be little reason for the existence of the Y level.

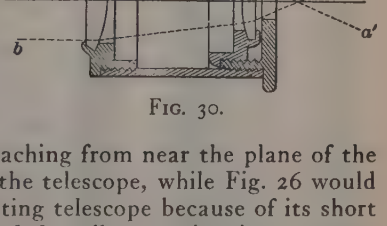


FIG. 30.

33. Leveling rods. There are three common patterns of leveling rods and an innumerable number of uncommon ones.

Fig. 31.

Fig. 32.

Fig. 33.

The three common patterns are shown in Figs. 31, 32, and 33. Each of these is made in two pieces about seven feet or less long. The New York rod (Fig. 31) and the Philadelphia rod (Fig. 32) differ in that the Philadelphia rod is so graduated as to be easily read at ordinary distances by the leveler, while with the New York rod the target must be set and the reading taken and called out by the rodman. The target of the New York rod is provided with a direct vernier, usually placed below the center of the target. This causes some confusion to the beginner, who has been taught to read the scale at the zero of the vernier and the fractional reading by looking along the vernier in the direction of motion for the coinciding line. There need be no confusion if the rodman remembers that it is the center of the target that is set by the leveler, and not the zero of the vernier. A little study of the vernier will show him that the main scale is read opposite the ten of the vernier, and the fractional reading by noting the number of the vernier line coinciding with a division of the scale. This rod is graduated to hundredths of a foot, and the vernier permits readings to thousandths. When a reading greater than 6.5 feet is required, the target is clamped

at 6.5 and the rod extended. On the side a second vernier will be found, which, when the rod is closed, reads 6.5 feet and which gives the readings when the rod is extended. This vernier is of the usual construction.

The original Philadelphia rod target had no vernier, but a tenth of a foot was divided into half-hundredths, permitting a direct reading of 0.005 of a foot and by estimation to 0.001 of a foot. The target is set at seven feet for greater readings than seven feet, and, if the rod is to be read by the leveler, it is extended to its full length, the graduation then being continuous from bottom to top. If the target is to be set, the rod is extended until the target is in the line of sight; it is then clamped, and the rod is read by the rodman by means of a scale on the back. The original rod was graduated only to tenths. The later rods are graduated to hundredths and may or may not have a vernier target.

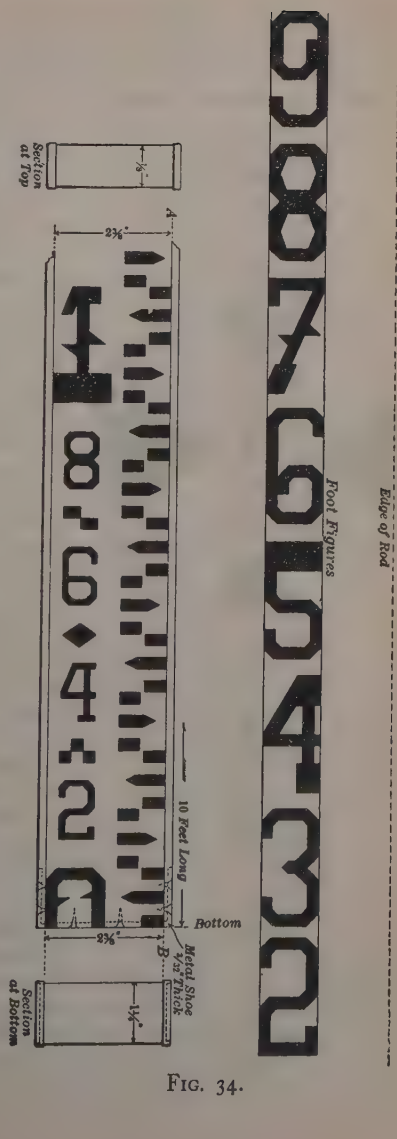


FIG. 34.

The target of the Boston rod (Fig. 33) is fixed to the rod. Readings are all obtained by extending the rod. It is held with the target end down, for readings less than 5.5 feet, and is inverted for readings greater than this. It is read altogether by vernier, the scales and vernier being on the sides. It is read to 0.001 of a foot. It is the lightest, neatest rod of the three, and the least used. The Philadelphia rod, which is the heaviest of the three, is the most used because of the fact that it may be quickly read by the leveler. In the great majority of sights, readings are taken to 0.1 of a foot only; and, on turning points (hereafter described), it is usual to read to 0.01 of a foot only. Readings to 0.001 of a foot are required in but a very small percentage of work done with the level, even on turning points or bench marks. For this reason many engineers prefer a "self-reading" rod without target, and made in one piece. Such a rod, which has been used in railroad work, is shown in Fig. 34.

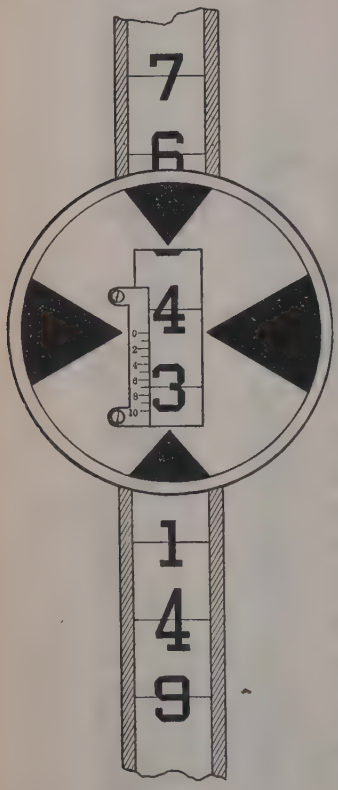


FIG. 35.

It will be noticed that the figures are so made as to mark the divisions into hundredths.

With a target rod, much better work may be done if the target is painted with diamond-shaped figures instead of with quadrants, as is customary. The target may be set more precisely if the wire has a sharp angle to bisect or sharp point to cover, than if it is to be made coincident with the edge of a dark surface. Professor Baker finds that at 300 feet the error of setting a quadrant target may be about 0.002 of a foot or more,

while that of setting a diamond-shaped target may be a little over 0.001 of a foot. Such a target is shown in Fig. 35.

Exercise

Read the several rods of the school collection for various settings. Make some of the settings so that the tenths shall be zero and some so that the hundredths shall be zero.

USE OF THE LEVEL

34. Adjustment. The level has been said to be an instrument for securing a horizontal line of sight. It will be evident from the construction of the ordinary leveling instruments that this may be accomplished with those instruments if the line of sight and bubble axis are made parallel; because, if this condition exists, and the bubble is brought to the center of the tube, the line of sight will be horizontal.

This introduces the idea of adjustment, and the adjustment of the level consists essentially in making the line of sight and the bubble axis parallel. There are other adjustments for convenience, but this is the only necessary one. The general discussion of the adjustments will be deferred till the use of the adjusted level in doing simple leveling has been explained.

35. Setting up. To "set up" the level is to place it in position for leveling, including making the line of sight horizontal. The level is an instrument that is rarely set "on line," except in making certain adjustments. It is placed in that position that will command the greatest possible number of points whose elevations are to be determined. To set up, plant the legs firmly in the ground with the *leveling plates approximately horizontal.* Focus the eyepiece on the wires. Bring the telescope and attached level over one set of diagonally opposite leveling screws and, by the screws, bring the bubble to the center of its tube. Perform the same operation over the other set of screws. This will disturb the former work to some extent. Therefore turn the telescope again over the first set of screws and relevel; then again over the second set, etc., till the instrument is level over both sets. If the instrument is in adjustment, the line of sight will now be horizontal in whatever direction it is pointed.

In using the leveling screws care must be taken not to bind them. If one set begins to bind, the other set should be loosened. The screws should always be set so that there is no lost motion, but also so that they are not tight. If this precaution is not observed, the screws soon wear so that they are unreliable. Opposite screws are turned at the same time, both thumbs in or out as the case may be. The beginner should practice until unconsciously he turns the screws the right way to move the bubble in the required direction. Time is saved if the first leveling over each set of screws is made only approximately, since it is to be disturbed by later operations.

Exercises

1. Set up the level several times on fairly level ground timing each set-up.
2. Set up the level several times on a hillside, timing each set-up. Generally one leg is placed up hill and two down.

In both these exercises see that the leveling head plates are brought fairly level by manipulating the legs before trying to level with the screws. Plant one leg in the ground, rest another on the ground, take the third in the hand, and note what results to the leveling head from a radial motion of a leg and what from a circumferential motion.

36. Differential leveling. In this article the curvature of the earth is neglected. To determine the difference in elevation between two points, both of which are visible from a possible position of the level, set up the level in a position such that a rod held on either point will be visible. Turn the telescope toward one point and read a rod held there by a rodman. The rodman will then carry the rod to the other point, and the telescope will be directed toward that point and the rod read. The difference in readings will evidently be the difference of elevation required. Care must be taken to see that the bubble is in the center of its tube when each reading is taken, manipulating the leveling screws as may be necessary, using the pair most nearly parallel to the telescope when it is pointed to the rod. With an average level a rod cannot be read closely at a distance greater than about 300 feet, though in work requiring no great exactness much longer sights may be taken.

If the elevation of one of the points above some base surface is known, the difference of elevation applied to the known elevation gives the elevation of the second point. This opera-

tion is capable of further analysis, thus: The rod reading on the point of known elevation, added to the known elevation, will give the elevation of the line of sight, and is therefore called a *plus* sight. The rod reading on the second point subtracted from the elevation of the line of sight will give the elevation of the second point. This reading is, therefore, called a *minus* sight. From these considerations the following definitions are formulated:

A plus sight, or reading, is any reading taken on a point of known or assumed elevation for the purpose of determining the elevation of the line of sight.

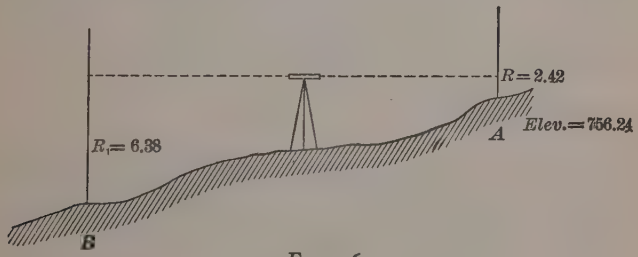


FIG. 36.

A minus sight, or reading, is any reading taken for the purpose of determining the elevation of the point on which the rod is read. Thus in Fig. 36

$$\begin{aligned} \text{Elev. A} &= 756.24 \\ + \text{R} \quad & \underline{\quad 2.42} \\ \text{Line of Sight} &= 758.66 \\ - \text{R}_1 \quad & \underline{\quad 6.38} \\ \text{Elev. B} &= 752.28 \end{aligned}$$

A very bad custom of calling plus sights, "backsights," and minus sights, "foresights," has prevailed in the past. It has been a source of confusion to the beginner and is illogical. It arose probably from the fact that the work in leveling is considered to proceed from the point of known elevation toward the point of unknown elevation, and that, therefore, plus sights are taken in a backward direction, and minus sights in a forward

direction. This is not true always, as will appear later, and hence the nomenclature, "backsight" and "foresight," is ill-chosen. It is true, moreover, that when a minus sight is taken in what may be considered a backward direction, the beginner becomes confused and applies the wrong sign. Hence the terms should be abandoned.

If the points whose difference in altitude is required are so located that rod readings cannot be had on both from one position of the instrument, an intermediate point is chosen that may be used with the first one, and the readings are taken on the first and on the intermediate point. The difference in altitude between the first and intermediate point is thus obtained. The level is then moved to a position between the intermediate and final point, and their difference of altitude is

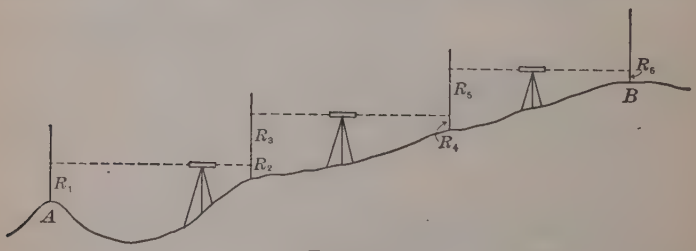


FIG. 37.

determined. The two differences added or subtracted, as the case may be, will give the required difference. It may be necessary to introduce several intermediate points. The work is simply a succession of operations like those of the first case. Figure 37 will make the operation clear.

It is unnecessary to determine the differences of altitude of each set of intermediate points, as the difference between the sum of all the plus sights and the sum of all the minus sights will give the difference of altitude of the first and final point.

If the work is to be well done, the intermediate points that are chosen should be firm, definite points, as the projecting part of a firm rock, the top of a peg firmly driven in the ground, etc.

The form of notes that is kept in differential leveling is as simple as the work. There are three vertical columns, one for the name or number of the point observed, one for the plus

sights, and one for the minus sights. *The readings, both plus and minus,* taken on any point, should appear in their proper columns *opposite the number of that point.*

Exercises

1. A series of rod readings is taken to determine the difference in elevation of points A and B as follows:

Point	$+S$	$-S$
A	4.26	
1	7.37	2.32
2	8.72	1.48
B		0.96

(*a*) Which is the higher point and what is the difference in elevation?

(*b*) Show by a diagram that the algebraic sum of the minus sights and plus sights or the arithmetic difference of their sums gives the difference in elevation of A and B.

The second and third exercises should be performed before reading the article on errors.

2. Determine the difference in level between two points not mutually visible from a common point, but so situated that the work may be done over fairly level ground. Work from the first to the second point and then back, and note the discrepancy.

3. Perform the same operation between points so situated as to require going up and down a fairly long, steep hill, and make the same notes.

4. Determine the angular value of one division of the bubble tube of any leveling instrument, proceeding as follows: Measure a level line — say 100 feet long — set up at one end with one pair of leveling screws approximately in the measured line. After leveling up, bring the telescope to the measured line, and with the leveling screws in the line run the bubble toward one end of the tube, read both ends of it, and average the reading of divisions from the center; while the bubble is in this position read a rod held on the opposite end of the line; then run the bubble to the other end of the tube and read the bubble and rod. The student will determine the rest of the process. A number of determinations should be made for a check and the average taken for greater precision.

37. Some avoidable errors. In engineering work the level is more used than any other surveying instrument, and to

describe the details of its use in the various surveys in which it finds a place would be beyond our purpose. These belong properly to a description of the methods of laying out and executing such engineering works. Some hints as to errors arising through carelessness will be given. A discussion of minute errors involved is necessary only in a treatment of precise leveling.

All leveling, by whatever name it may be called, is differential leveling, and a discussion of the sources of error for this form of leveling is applicable to all forms.

Adjustment. If the instrument is out of adjustment, the line of sight will point up or down when the bubble is in the center of its tube, and distant rod readings will be too great or too small. The method of avoiding errors from this cause is to have distances from instrument to plus rod and minus rod equal because the line of sight will point up or down equally in equal distances, and, hence, both readings will be equally affected and their difference will be unaffected. Distances are made near enough equal by pacing.

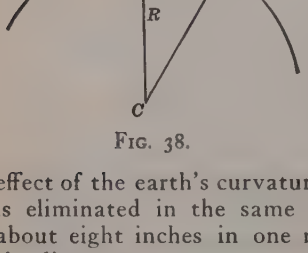


FIG. 38.

Curvature and *refraction.* The effect of the earth's curvature, and to some extent of refraction, is eliminated in the same way. The curvature amounts to about eight inches in one mile and varies with the square of the distance.

The effect of curvature and refraction is shown as follows: In the exaggerated figure (38) let A be a point in the line of sight of a level telescope. Let FD be a normal to the earth's surface, at some distance K — approximately equal to AB, GF, or AD — from the instrument. The level line through A cuts the normal at B, while the horizontal line of sight cuts it at D. DB is then the *correction for curvature* for the distance K. Radius $CA = CB$ being taken equal to GC, we have by geometry (relation between tangent and secant), and a small approximation that the student may discover,

$$DB = \frac{K^2}{2R}.$$

This equation should be compared with the equations of Art. 13, page 23. The effect of curvature is to make the rod read high, and hence to make the point *F* seem low.

Refraction has the opposite effect; that is, the point *E* is seen in the line *AD*. The ray of light from any distant object is bent to a curve that is irregular because of the irregularities in the atmosphere, but which is almost always concave toward the earth's surface, and which is considered for average conditions to be the arc of a circle of diameter about seven times that of the earth. Therefore, the effect of refraction is to reduce the correction for curvature by about one seventh. It is not likely that this value is even closely approximate so near the ground as leveling work is done. The refraction error is thus seen to be uncertain and therefore not wholly avoidable.

The combined effect of curvature and refraction is about 0.001 of a foot in 220 feet, and varies as the square of the distance.

Sidehill work. In leveling up or down hill the uphill sights will always tend to be shorter than the downhill sights, since the extended rod is more than twice the set-up height of the level. Hence, errors of adjustment tend to affect the work unless the sights are made nearly equal by zigzagging.

Turning points. A flat horizontal surface is fairly good for a turning point if the rod is held vertical, but a flat inclined surface is not good, because when the rod is faced in different directions, it will not be likely to rest on the same point. The turning point should be such that the rod will rest on the same spot however it is held. The turning point should be firm and not likely to change elevation.

When extensive differential leveling operations requiring close work are to be carried on (as the careful determination of the altitude of an observatory or other point involving the carrying of levels over many miles and the introduction of many intermediate "turning points"), it is well for the rodman to carry a "point" with him. A very convenient form is shown in Fig. 39. This is made of a triangular piece of boiler plate about three sixteenths of an inch to one quarter of an inch thick and about five or six inches on a side. The three corners are bent down to form, as it were, a three-legged stool, and a round-headed rivet is set in the center. A small hole is drilled in one side, in which to fasten a string or chain. When used, the points are firmly pressed into the ground with the foot, and the rod is

held on the rivet. In some work, notably railroad leveling, this kind of point is not suitable, because it is well to leave every turning point so that it can be found again.

Bubble troubles. The bubble should be in the middle of the tube when the reading is taken or else the line of sight will not be horizontal; therefore, an observation should be made, just before and after the reading, to see that the bubble has not moved. If it has, the instrument must be releveled and the reading taken again. On unstable ground, throwing the weight of the body from one foot to the other may throw the bubble out. Fine work cannot be done on soft ground. A mirror may be attached to a dumpy or Y level, so that the bubble may be observed as the reading is taken. Unequal heating of different parts of the bubble tube will cause the bubble to move toward the warmer part. This may be shown by resting the finger lightly on the tube, to one side of the bubble. Ordinary work must be done in all kinds of weather, and no great precaution can be taken to avoid the effect of the sun on the instrument.

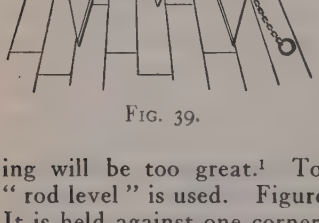


FIG. 39.

Vertical rod. The rod should be held vertical or else the reading will be too great.[1] To accomplish this on nice work a "rod level" is used. Figure 40 shows one form of such a level. It is held against one corner of the rod. Waving the rod back and forth, from and toward the instrument, is recommended in the absence of the rod level. When this is done, the lowest reading observed is used. This serves very well when the reading is several feet; but if it is near the bottom of the rod, and the rod is resting on the middle of its base, a considerable error may be introduced. If the rod is rested only on its front edge, the method is correct.

There are many other hints that could be given to facilitate the work of leveling; but they would occupy too much space,

[1] Although this is the usual statement it is not strictly true. The least reading will be had with the rod *perpendicular* to the line of sight, but because of the earth's curvature a *vertical* rod is not perpendicular to the line of sight except when it is at the instrument. The effect of curvature, which is very small for usual sighting distances, must be ignored to make the statement of the text true. The student should construct a figure and study this point if it is not clear to him.

and the beginner will have to learn details from experience. One point may be mentioned: Never clamp the spindle. It is unnecessary to do this except in adjusting.

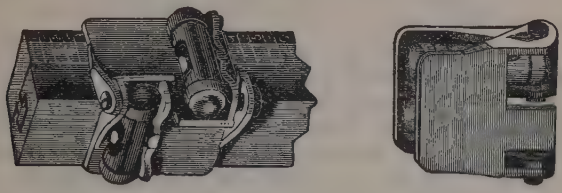


FIG. 40.

EXERCISES

1. If a rod is held 528 feet from a level, what will be the earth's curvature effect on the reading? If held 279 feet away?

2. For theoretic accuracy, is it necessary that all sight distances shall be equal, or only that the plus and minus sight distances for each set-up shall be equal?

3. In leveling up and down hill will the errors of adjustments be eliminated if the sum of all the plus sight distances equals the sum of all the minus sight distances, even though for several instrument settings they have been unequal? Does the same law hold for the effect of the earth's curvature?

4. Why is the lowest reading taken when the rod is waved back and forth? Show by diagram how this may be incorrect when the reading is near the bottom of the rod and the rod is supported on a point back of its front edge.

38. Unavoidable and compensating errors. Some errors are unavoidable, and are usually compensating in character. Such errors may be due to the effect of variations in the atmosphere, failure to read both ends of the bubble exactly and so to have it out of the center of its tube, inability to exactly set a target, etc. After all avoidable errors have been eliminated or allowed for, there will still be some discrepancy between two or more determinations of the difference in level of two points. The degree of precision attained in such a case is discussed by the method of least squares (see page 33). Thus if the difference in level of two points is determined to be 421.656, 421.650, and 421.653, the most probable value is $\frac{1}{3}$ (421.656 + 421.650 + 421.653) = 421.653; the 3 v's are 421.656 − 421.653 = + 0.003, 421.650 − 421.653 = − 0.003 and 0.000; Σv^2 = 0.000009 + 0.000009 + 0.000000, and from Equation 1, page 34, $e = \pm 0.002^{+}$.

This quantity, the probable error of a single determination, is of value only in showing the surveyor what degree of precision he is likely to obtain in a single observation, and hence to tell him when a single observation made under like conditions is probably sufficient.

The probable error of the most probable value — the arithmetic mean — is accepted as a measure of the precision with which a series of observations has been made. The term "probable error" is in some degree misleading. It does not mean that the most probable value is probably in error by the amount of the probable error, nor that it is probably in error by no greater sum than this, but simply means that there is an even chance that the actual error which is not determinable is greater or less than the so-called probable error. In the foregoing example there is an even chance that the error of the mean value 421.653 is more or less than the fraction

$$\frac{\pm\ 0.002}{\sqrt{3}} = \pm\ 0.00115 \text{ (Equation 1, page 34).}$$

The result of the leveling would then be stated as 421.653 ± 0.00115. If another attempt should result in a probable error half that just found, that result would be said to have twice the precision of the result of the example.

In running a long line of differential levels there will be some error for each set-up determination, and since the number of set-ups in different lines will be in some measure proportional to the lengths of the lines, it has become customary to measure the precision of precise leveling by the distance, although a better method would be by the number of set-ups required.

39. Precision to be obtained. In precise leveling, the U. S. Coast and Geodetic Survey requires that the discrepancy (not probable error) between two determinations of the difference of level of two distant points shall not exceed 0.017 of a foot $\times\sqrt{\text{distance in miles}}$, and realizes a much greater degree of precision in its work than this.

The author thinks that in ordinary careful leveling not done with precise levels but with readings taken to 0.001 of a foot, excellent work would be indicated by a difference of 0.01 of a foot to twenty-four settings of the instrument; that is, if a line

of levels is run for twelve settings of average length, and rerun in the opposite direction, the difference in elevation of the starting point, as assumed at the start and as found at the finish, should not be greater than 0.01 of a foot. If it is as much as 0.04 of a foot, the work, if errors of this amount are important, should be rerun. The object of the leveling is a better guide to a decision as to what is good work than any arbitrary limit.

The author's students ran a line of levels from San José, California, to Mount Hamilton, a distance of twenty-seven miles, and were required to repeat the work if the error of any day's run proved to be more than 0.01 of a foot for twelve settings of the instrument in advance. The difference in altitude was about 4200 feet; and at one or two places along the line, differences of as much as 0.1 of a foot were observed between the forward and backward running of a day's work, while the difference between the elevation of the mountain, as determined by the advance work and that determined by the backward work, was 0.023 of a foot. This indicates the compensating character of the errors of observation. The compensation was probably partly due to the fact that the different stretches were run by different men, since it seemed that one man had a tendency to make errors in one direction and another to make them in the opposite direction. The work was done by running a half day ahead, and then rerunning the same stretch backward.

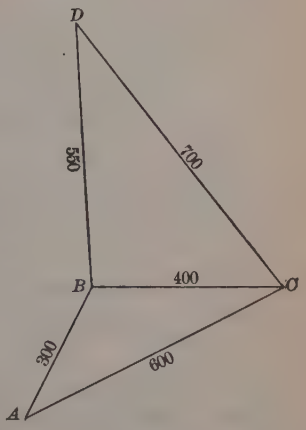


FIG. 41.

40. Adjusting a network of bench marks. In topographical surveys hung on triangles, and in city surveys for the location of a series of bench marks over the city, the several established elevations at triangulation stations and block corners must be consistent, so that no matter where the leveler starts he may always find the same elevation within the limits of error on

arriving at any given station. Moreover, if the stated elevation of a given station is calculated by using the elevations of other surrounding stations, the stated elevation should result. Thus, if in Fig. 41, page 69, the elevation of A is known and is 426.321, and the differences between A and B, B and C, and C and A are found, the algebraic sum of these differences should be zero, and the elevation of A determined from $B-A$, $C-B$, and $A-C$, should be the known or starting elevation.

Thus, suppose the elevation of B is found to be 448.736 and of C, 432.473, then going from A to B to C to A

$$\begin{aligned} B-A &= 448.736 - 426.321 = +\ 22.415 \\ C-B &= 432.473 - 448.736 = -\ 16.263 \\ A-C &= 426.321 - 432.473 = -\ 6.152 \\ &\text{Algebraic Sum} = 0.000 \end{aligned}$$

But suppose in leveling the following differences are found:

$$\begin{aligned} B-A &= +\ 22.400 \\ C-B &= -\ 16.264 \\ A-C &= -\ 6.156 \\ \text{Algebraic Sum} &= -\ 0.020 \end{aligned}$$

The differences are not consistent. They must be adjusted by distributing the error or discrepancy among the several sides in some proportion. The number of set-ups on each side may not be known by the computer. Therefore he must adjust in proportion to the lengths of the sides or the square roots of those lengths on the theory that the number of set-ups, and consequently the number of errors, is proportional to the lengths, and the square root of the number of errors remains uncompensated.

The sum of the sides is 1300 and there will be given to the several sides, adjusting in proportion to lengths,

$$\left.\begin{aligned} \text{to } A\,B,\ \tfrac{300}{1300} \times 0.020 &= +\ .005 - \\ \text{to } B\,C,\ \tfrac{400}{1300} \times 0.020 &= +\ .006 + \\ \text{to } C\,A,\ \tfrac{600}{1300} \times 0.020 &= +\ .009 - \\ &\quad .020 \end{aligned}\right\} \text{ whence } \left\{\begin{aligned} B-A &= +\ 22.405 \\ C-B &= -\ 16.258 \\ A-C &= -\ 6.147 \\ &\quad 0.000 \end{aligned}\right.$$

making the elevation of B and C (A being fixed)

$$\begin{aligned} 426.321 + 22.405 &= 448.726 \quad B \\ \text{and } 448.726 - 16.258 &= 432.468 \quad C \\ \text{or } 426.321 + 6.147 &= 432.468 \quad C \end{aligned}$$

The elevations resulting from the adjustment are not actual elevations but are elevations that are consistent and are, probably, less in error than any others under the assumption made.

If the adjustment is by the square roots of the lengths, those quantities would be obtained and used in connection with their sum just as the individual lengths and their sum are used above.

BC being adjusted, to adjust the triangle BCD the whole error must be distributed between the two sides BD and CD.

In adjusting a large network it is perhaps best to begin with the particular polygon having the largest error, taking them thereafter in the order of size of error, but being careful not to close on an interior polygon all of whose sides are adjusted in connection with surrounding polygons, but are probably inconsistent for the closing polygon.

For extensive work a method of least squares adjustment that adjusts the entire system simultaneously is used, but this requires much labor and has nothing more than a mathematical advantage over the method suggested.

41. Reciprocal leveling. So long as plus and minus sights are at equal distances no error arises in the work because of curvature and refraction, or lack of adjustment of the instrument. It happens not infrequently that sights must be made of unequal lengths, as in crossing a wide river. The following method avoids the errors of curvature, refraction, and adjustment: Establish a peg near each end of the long sight. From a point near one peg read a rod on both pegs. Move the instrument near the other peg and again read on both. The mean of the two differences of elevation obtained will be the correct difference if the same peg be used for plus sight.

Thus, in the exaggerated figures (42 *a* and *b*, page 72), let B be lower than A. Then if c is the error due to curvature from I to B and e is the error of adjustment (plus in the figure), the height of the level line through I above B is $R - e - c$. Similarly,

if e_1 and c_1 be the same errors for the distance IA (really negligible), the height of a level line through I above A is $R_1 - e_1 - c_1$. Likewise, with the instrument near B the heights of the level line through I_1 above B and A respectively are $R_2 - e_2 - c_2$ and $R_3 - e_3 - c_3$. With the instrument near A, the true difference in level between A and B is the difference in the heights of the level line above A and B, or

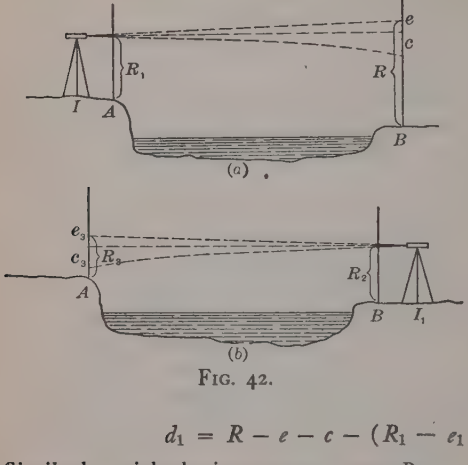


FIG. 42.

$$d_1 = R - e - c - (R_1 - e_1 - c_1).$$

Similarly, with the instrument near B,

$$d_2 = R_2 - e_2 - c_2 - (R_3 - e_3 - c_3).$$

Now since IA and I_1B are practically equal and also IB and I_1A, $e = e_3$, $c = c_3$, $e_1 = e_2$, and $c_1 = c_2$. Therefore, the sum of the two differences or twice the difference in level is

$$d_1 + d_2 = (R - R_1) + (R_2 - R_3);$$

and the difference in level $= \frac{1}{2}\{(R - R_1) + (R_2 - R_3)\}$ or one half the sum of the apparent difference in level as determined from the actual readings.

42. Profile leveling. A profile of a line laid out on the ground is the bounding line of a vertical section that includes the line whose profile is desired. It shows the elevations and depressions along the line. Profile leveling differs from differential leveling in that the elevations of a number of points along the line whose profile is required are obtained from a single setting of the instrument. The principle is the same as that of differential leveling, but the method of keeping the notes and of doing the work is a little different.

The line whose profile is required is first marked out on the ground by stakes or other marks placed at such intervals as may be necessary. These intervals are usually regular, and in railroad surveys are generally one hundred feet. In city streets the interval may be fifty feet. In other surveys the interval may be less or more, according to the nature of the survey. The object usually is to reproduce to scale, on paper, the profile desired. For this purpose profile paper, on which the notes are plotted, is generally used, as will be described later.

Figure 43 shows a map of a line whose profile is desired, which may be assumed to be the center line of a road. It also shows in exaggerated form, and to no scale, the position of the level along the road, both in plan and elevation.

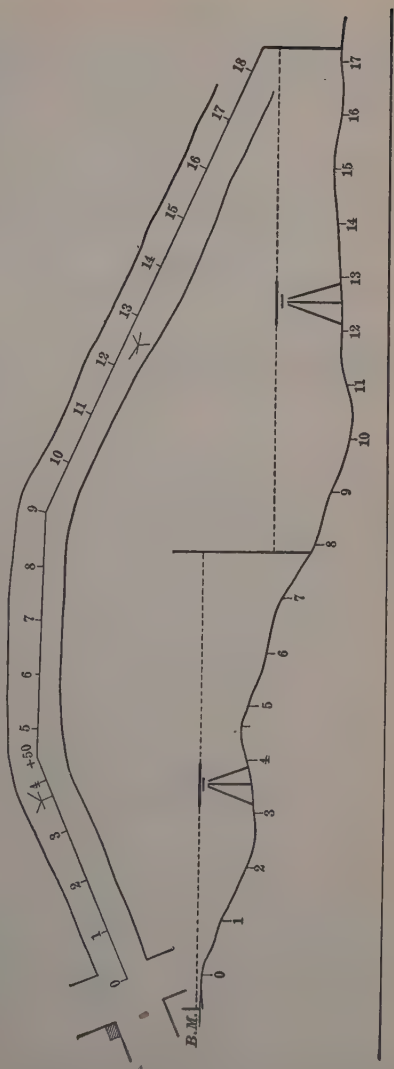


FIG. 43.

The process of leveling is as follows:

Some convenient point is chosen as a bench mark, either because its elevation above some accepted datum is known, or because it is a convenient permanent

point whose elevation may be arbitrarily assumed. A bench mark in leveling is a permanent point of known or assumed elevation from which leveling operations may proceed. In the example given, the B. M. is the corner of the water table of a building, and its elevation is assumed to be 1000.000 feet above some arbitrary datum surface. The elevation of the B. M. should be so assumed as to avoid any minus elevations; that is, the datum surface should be assumed so far below the B. M. that it will be below any point reached in the leveling.

Left Page **Right Page**

LEVELS ALONG GRAFTON ROAD, QUACKENKILL TO CROPSEYVILLE

LEVELER . . . TOUCEDA
ROD . . . . . HIGGINS
March 24, 1895
Weather: Fair, Mild
Work began 6.30 a.m.
Finished 4.30 p.m.

Sta.	+ S.	H. I.	− S.	Elev.	
B. M. No. 1.	0.462	1000.462		1000.000	On the top of the water
0			0.6	999.9	table at the S.E. cor. of
1			2.2	98.3	the brick store of P. Gos-
2			4.3	96.2	ling. Elev. assumed.
+ 50			5.5	95.0	
3			5.2	95.3	
4			4.5	96.0	
+ 60			3.8	96.7	
5			4.3	96.2	
6			6.0	94.5	
7			7.2	93.3	
T. P.	3.602	994.041	10.023	990.439	On stone marked +,
8			4.0	990.0	ten feet right of Sta.,
9			5.3	88.7	7 + 80.
10			7.0	87.0	
+ 20			7.2	86.8	
11			6.6	87.4	
12			6.2	87.8	
13			6.0	88.0	
14			5.8	88.2	
15			5.2	88.8	
16			5.7	88.3	

FIG. 44.

Having selected the B. M., write in the notebook (Fig. 44), under the column marked " Elev.," the elevation assumed, and in the column of stations mark the name of the station whose elevation is given, in this case B. M. No. 1. On the right-hand page write a description of the B. M. so clear and full that another, coming to the place long after, may determine with certainty the exact spot used.

Set up the level at a point convenient to the B. M. and to as much of the line as possible, remembering that it is better to make sights to turning points approximately equal. Having the level set up, read the rod held on the B. M., and record the reading in the column marked " + S.," and add it to the elevation of the B. M. to get the height of the line of sight, which record in the column headed " H. I." (height of instrument). The rod is now held on station 0, and is read, and the reading is recorded in the column of minus sights, opposite station 0. It should be noted that it is usually the elevation of the ground that is wanted, and not that of the top of the stake that marks the station. Therefore, the rod is held on the ground, and not on the stake. The rod is usually read at station points only to the nearest tenth, as it would be folly to try to determine the elevation of rough ground to hundredths. The target is not used for these readings. Points used as B. M.'s are usually read to hundredths or thousandths. Hundredths are generally close enough. The target is used for these readings.

The rod is now carried to station 1 and read, and the reading is recorded in the column headed " − S.," opposite station 1. The leveler, if he is quick, may make the subtraction of the minus sight on one station while the rodman is passing to the next station, and he should practice doing this. He may check his work at noon or night or at any other time when the survey is, for any reason, stopped temporarily. The rod is now read on station 2, and recorded as before. Between station 2 and station 3 there is a decided change in slope, and the rod is held at the point of change, the rodman either measuring the distance from station 2 to the point or estimating it by pacing, according to the importance of exactness in the particular survey. He calls out the " plus " as (in this case) " plus fifty," and the leveler records the distance as shown in the notes, and then reads the rod and records the reading opposite the recorded distance in the − S. column.

It will be noticed that a number of minus sights have been taken with the instrument pointing backward with reference to the direction of progress. It is these sights that are sometimes confused with "backsights" when the terms "backsight" and "foresight" are used.

Thus the work proceeds till the rod is about as far beyond the instrument as it was in the beginning back of it. Not more than three hundred to five hundred feet on either side of the instrument should be used, if fairly good work is to be done. A point is then selected that is tolerably permanent, as a solid stone or a peg driven into the ground. This point need not be on the line that is being leveled, and, indeed, should not be there, if the line is the center of a traveled road.

The rod is held on the peg, which is called a *turning point* or simply "peg," and is read to the same unit as on the B. M., and the reading is recorded in the column of − S., opposite the station T. P. The elevation is then worked out at once, and the level is carried beyond the peg a convenient distance and again set up. A reading is then taken on the rod held on the peg, is recorded in the column + S., and is added to the elevation of the peg to get a new height of instrument, which is noted in the proper column opposite the station from which it was determined. A description of the peg is written on the right-hand page, so that it may be found again, if wanted within a reasonable length of time. The work then proceeds as before. After an interval of, say, half a mile or a mile, a permanent B. M. is established. It may be used as a turning point, or merely determined as is any station, except that the reading will be taken to the unit used for B. M.'s. All points that are likely to be wanted again should be very fully described. Sketches are valuable helps to words in making descriptions. The rodman should keep a separate book in which he records all readings on and elevations of pegs and bench marks.

Other forms of notes are sometimes used by engineers. Some of them have station numbers and elevation in adjacent columns; others have a separate column for minus sights on turning points, and others a separate column for elevations of turning points, etc.

Exercise

Measure a line a mile or more along a hilly road setting stakes at every tape length numbering the starting point 0, the second stake, 1, etc. Run

the level over this line as described for profile leveling, and plot the results on profile paper as described in the next article.

43. Making the profile. The profile is made by plotting the elevations of the points to scale at the proper horizontal distance apart, and connecting the plotted points by a line. Profile paper is generally used for this purpose. One form of ruling for such paper, known as Plate A, is shown in Fig. 45.

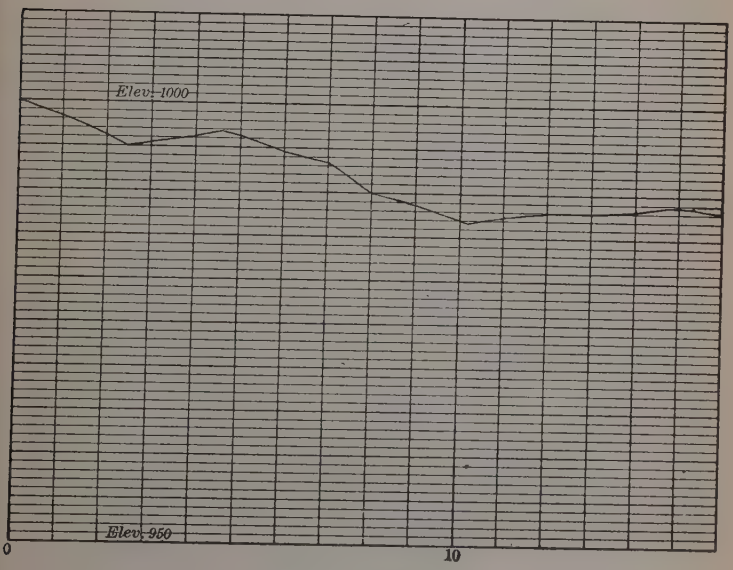


FIG. 45.

There are three common forms, known as Plate A, Plate B, and Plate C. Plate A has four horizontal divisions and twenty vertical divisions to an inch. Plate B has four by thirty divisions per inch; and Plate C five by twenty-five divisions. Profile paper comes in rolls twenty inches wide, or in sheets, and may also be had in half-width rolls.

If profile paper is used, a convenient elevation is assumed for the bottom horizontal line, and a value is assigned to the space between successive lines. For railroad profiles it is

usual to assume one foot as the value of the smallest interval. The paper is so ruled that the five-feet and twenty-five-feet, or fifty-feet lines are accentuated. A smaller value may be assumed for the smallest interval when it is necessary to work to a large scale and to show minute irregularities. It is usual, also, in railroad work to assume the value of the interval between successive vertical lines to be one hundred feet or "*one station*." Each tenth station is then accentuated by the ruling. This scale may be increased also, if necessary. The scales, both horizontal and vertical, having been assumed, it is only necessary to find the position, horizontally and vertically, that will correctly represent any point whose elevation and distance or station from the beginning are known, and to make a small dot at that place. So with the other points. These dots may then be connected with a smooth line with few angles. It may be necessary, occasionally, to make angles, as abrupt changes will show as angles on the small scale drawings; but ordinarily such angles do not occur in nature.

The following method of making the profile, assuming the scale as small as described for railroad work, is rapid. The elevation of the first station is called, and a point is made on the paper; the elevation of the second station is called, the pencil is put down on the paper in the proper place, and a line is drawn back to the first point without making a decided dot at the second station. The line is drawn freehand. The work then proceeds in this way. It seems unnecessary to say that turning points are not plotted, unless they happen to be critical points on line at the ground level — a rare occurrence. The notes in Fig. 44 are shown plotted in Fig. 45.

If profile paper is not used, a base line of assumed elevation is drawn on a piece of paper, the horizontal distances to the points whose elevations have been determined are laid out to scale, and perpendiculars are erected on which the elevations of the points above that assumed for the base line are laid off to vertical scale. The points thus formed are connected by a line.

A 4 H pencil is good for profile work. After the profile is penciled in it is inked either with a fine steel pen, such as a 303 Gillott, or with a right line pen; or better still with a contour pen which is a swiveling ruling pen. Accurate work with the pen requires experience. A light narrow brush shading just

below the profile line improves the appearance, and when used should be put on before the inking is done.

A convenient scale for city streets is:

Horizontal, 1 inch to 100 feet.
Vertical, 1 inch to 10 feet.

Where such a scale is used, it will generally be better to connect the plotted points by ruled lines.

The vertical scale is usually exaggerated, except in profiles made for detail plans of buildings or other works. This is necessary to make the irregularities appeal properly to the eye, since the vertical distances are so small, compared with the horizontal ones, that when a long horizontal distance is laid out to small scale, what would be considered appreciable variations in altitude on the ground, will, if drawn to the same scale as the horizontal distances, appear insignificant on the drawing.

44. Leveling over an area. Sometimes it is required to determine the elevations of a number of points in a comparatively small area, as when a city block is to be graded and the quantity of earth to be removed or supplied is required. In this case, it is customary to divide the block into rectangles or squares, of, say, twenty feet on a side. The elevations of all the corners of these squares or rectangles are determined, and after the grading is done, the same squares are reproduced from reference stakes, previously set outside the work, and the elevations of the corners are again determined. The difference of elevation at any corner, before and after grading, shows the depth of material that has been cut away or filled in. Thus there will be a number of truncated prisms, the lengths of whose edges and the area of whose right sections are known, and hence the volume may be computed.

In such a case as this, after determining the elevation of the line of sight, readings are taken to as many corners as may be seen before the instrument is moved. The work is precisely the same as profile leveling, except that consecutive readings are not necessarily along any one line.

ADJUSTMENTS OF THE LEVEL

45. Focusing the eyepiece. The first adjustment (so called) in any instrument having a telescopic line of sight is the careful focusing of the eyepiece on the wires. If this is not done, there will appear to be a movement of the wire over the image as the eye is moved a little up or down or to the right or left. This is called *parallax*. It is due to the fact that the wire and image are not in the same plane. The eyepiece is best set by turning the telescope toward the sky and moving the eyepiece until the wires appear sharp and black. The setting may be tested as indicated above. This setting should be tested for each person using the level.

46. Adjustments of Y level named. In addition to this focusing of the eyepiece, which can hardly be called an adjustment, the adjustments of the level that the surveyor is usually called upon to make are three in number:

1. The collimation adjustment, which consists in making the line of sight coincide with the line of collimation.

2. The bubble adjustment, which consists in making the bubble axis parallel to the line of collimation and hence to the adjusted line of sight.

3. The Y adjustment, which consists of so adjusting the Y supports that carry the telescope and attached bubble that the latter shall be at right angles to the vertical axis of the instrument, so that the level when leveled in one position may be turned on the spindle 180° and still be level.

The first two are essential to correct work. The third is merely for convenience.

In making any of the adjustments it is practically impossible to manipulate the adjusting screws without disturbing the instrument to some extent. Therefore, an adjustment is rarely made perfectly the first time, and must always be tested and repeated till complete. If an instrument is badly out of adjustment, it will be advisable first to adjust it approximately all round, and then readjust more carefully.

47. The collimation adjustment explained. In Fig. 46 let *o* be the optical center of the objective. It is assumed that the maker has so placed it in its ring that it is practically in the axis of the telescope when drawn clear back as shown. Prob-

ably this is not exactly true, but the error is less than can be determined by the surveyor. Let *D* be the ring that carries the object slide. It is adjusted by the screws *SS*. It is shown much out of the center of the tube. Let w_1 be a wire supposed not to be in the center of its tube, that is, out of adjustment. The outer end of the objective tube slides in the fixed ring *G*. If the instrument is perfectly made, it is evident that, if the objective could be brought to the plane of this ring *G*, the optical center would be in the axis of the telescope tube. When the glass is focused on a very distant object, the objective approximates to the plane of the ring, and hence the optical center is nearly in the axis of the tube, but not necessarily exactly so. In the case presented, it could not be just in the axis.

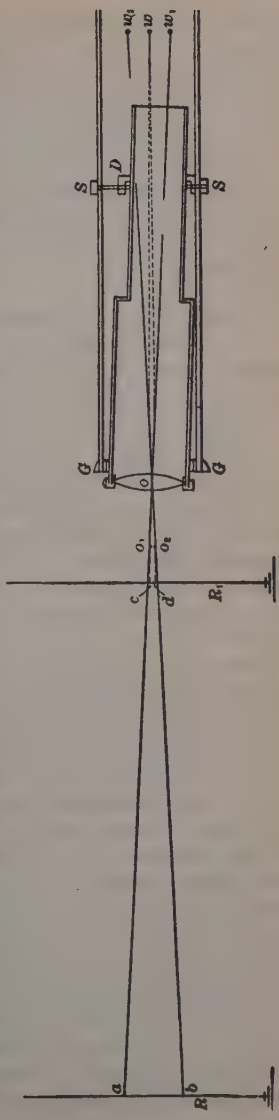


FIG. 46.

If the telescope is directed to a distant object, as a rod *R*, the point *a* of the rod, that will appear to be covered by the intersection of the wires, will be found in the line joining the intersection of the wires with the optical center of the objective. If the telescope is turned upside down, the wire will move to w_2, and the point that will seem to be covered will be the point *b*, found as was *a*. If the optical center is in the axis of the telescope, and the wire is moved to the position *w* in the prolongation of the line joining the point midway between *a* and *b* and the

optical center, it will be in the center of the tube and in adjustment. If the optical center is not exactly in the center of the tube, the position of the wire when moved to read halfway between a and b will not be the center of the tube but will be very near it, depending on the error of the objective. It will be off center about the same amount that the optical center is off. Let it be assumed that the optical center is right, and that therefore the wire is now right. Make the telescope again normal. The intersection of the wires should continue to cover the point midway between a and b.

Let the glass be focused on a near object, as a rod at R_1. Let it be supposed that in throwing the glass out for this purpose o moves to o_1. The reading on the rod will be c, obtained by producing the line wo_1. Invert the telescope, and the reading will be d. If the ring D is now moved, the object slide will swing about GG as a pivot, and o_2 will be moved toward o_1. It is evident that, if this movement is enough to make the wire read halfway between c and d, the objective is centered. If it was right for the distant rod, it may now be assumed to be right for all intermediate distances, since it is supposed that the maker has made the tube true in form. If this is doubted, it may be tested in the same manner for any distance. Since the objective was not exactly centered for the distant rod, the wire was not exactly adjusted, and hence if the test is not made again on the distant rod, a slight error will be discovered which is to be corrected by moving the wire, and the test must again be made on the near rod, which will also probably show some error of objective which must be corrected. By continual approximations the work is completed.

Because the wire is not quite centered the first time, the work may be hastened by adjusting the *slide* for about or quite the whole error instead of half the error. Each wire is adjusted separately, and the objective slide first in one direction and then in the other at right angles thereto. It is well to note that this adjustment of the objective is seldom made because it is seldom necessary. It may be considered to be necessary only when the instrument has been taken apart or has had extraordinarily severe usage. When it is not necessary, the adjustment of the line of sight is a very simple matter. In many makes of instruments the object slide is permanently

adjusted, and hence no provision is made for its adjustment by the engineer.

It must be noted that the image formed by the objective in the plane of the cross wires is inverted, and that an erecting eyepiece inverts the image and the wires so that the top of the vertical wire is seen as the bottom and the right end of the horizontal wire as the left end. For this reason when an erecting instrument is used and it appears that a wire must be moved up or to the right or *vice versa*, it must in fact be moved in the opposite direction. This rule does not apply when twisting the ring to make the horizontal wire at right angles to the axis of revolution. The eyepiece of an inverting instrument does not invert the image or the wires, and hence any necessary movement of the wires is made in the direction apparently necessary when an inverting instrument is used.

48. Collimation adjustment stated. The desirability of having the horizontal wire truly horizontal, and the foregoing explanation, warrant the following statement: To make the first adjustment, set up the instrument and note whether the horizontal wire will cover some distant point as the instrument is turned on its vertical axis. If it will not, loosen two adjacent adjusting screws that hold the wire ring, and turn the ring till the condition described is fulfilled. An effort is made to bring the wire halfway to the point from which it has departed; the screws are set and the test made again. When completed, the horizontal wire is perpendicular to the vertical axis of the instrument. Set the screws and unfasten the clips that hold the telescope in the Y's, and, by means of the leveling screws and the horizontal motion of the instrument about its vertical axis, bring the intersection of the wires to cover a minute distant point. Clamp the horizontal motion, and make the coincidence of the wires and point more perfect by the slow-motion screw and leveling screws. Carefully turn the telescope over in the Y's, not end for end, and note whether the intersection of the wires still covers the point. If not, move the ring that carries the wires, by opposite capstan-headed screws, until the error seems to be one half corrected. This is done by correcting first one wire, then the other. Reset the wires on the point, by the leveling screws and the slow-motion screw, and carefully turn the telescope in the Y's to its former position. If the intersec-

tion of the wires now remains on the point, the adjustment is correct for the distance at which the point is, and this is all that is usually given for this adjustment.

It is correct also for all distances if the object slide is in adjustment. To determine whether this is so, test for the collimation error as just described, but on a very near point. If there appears to be an error, it must be corrected by adjusting the object slide. To do this, remove the ring that covers the screws controlling the slide ring, and adjust with these screws so that rather more than one half the error appears to be corrected. Test to see whether the adjustment is correct, and again test on the distant point, and, if necessary, adjust the wire, and repeat till the adjustments are both correct. If the slide is not adjustable and an error appears to exist, the instrument should be sent to the maker for correction. If now the intersection of the wires appears to be out of the center of the field of view, it is because the eyepiece tube is not properly centered. It may be centered by manipulating the adjusting screws carrying the eyepiece ring. Inverting instruments are permanently adjusted in this respect. Error in this adjustment does not affect the correctness of work done with the instrument.

49. Bubble adjustment. The second adjustment is made in either of two ways. The first is called the *direct method*, the second, the *indirect* or *peg* method. The first depends on the equality of diameter of the rings of the telescope tube that rest in the Y's. Since by the first adjustment the line of sight has been made to coincide with the axis of the tube, it is parallel to the lines joining the points of support in the Y's, if the rings mentioned are equal in diameter. If the attached bubble tube is made parallel to these lines, it will be parallel to the line of sight. To test it, set up the instrument, and, having leveled over both sets of screws, unfasten the clips and level more carefully over one set. Remove the telescope very carefully from the Y's and turn it end for end and replace. If the bubble returns to the center of its tube, the tube is parallel to the lines joining the points of support. If it does not, bring it halfway to the center by the adjusting nuts at the end of the bubble tube. The adjustment is now complete if the instrument has not been moved in making it. Test it by releveling and turning end for end in the Y's, as before.[1]

[1] The principle on which this adjustment depends is stated in Art. 29.

50. The peg method explained. This method is considered better by almost all engineers and is explained as follows: Referring to Fig. 47, if an instrument is set at I_m midway between two pegs P and P_1 and a reading taken on a rod on each peg, the difference in the readings a and b will be the true difference in level, d, of the two pegs. If the level is now moved to I_s, $\frac{1}{10}$ of the distance PP' beyond P (any other proportional distance would do as well), and if the line of sight and bubble were parallel so that the line of sight would be horizontal if the bubble were in the center of its tube, the rod reading on P should be c', but since the line of sight points down, in the case illustrated, the rod reading will be c. Similarly the rod reading

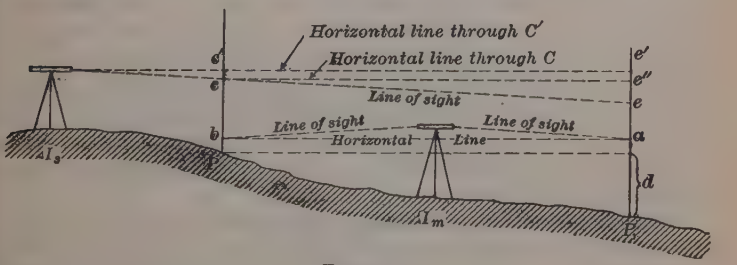


FIG. 47.

at P' should be $e' = c' + d +$ curvature, but is $e = c' + d +$ curvature $- c'c - e\,e''$; or $e = c + d +$ curvature $- e\,e''$.

To find what the reading at P' should be, we apply to the reading c at P the difference in level and the curvature and get the reading e'' which is incorrect by the amount $e'e'' = c'c$, which, because of the similarity of triangles, is $\frac{1}{10}$ of $e\,e''$ (if I_sP were a quarter of the distance PP', $e'e''$ would be one quarter of $e''e$). Therefore, since we have the reading e and the erroneously computed reading e'', if we add to the reading e (subtract if the instrument points up instead of down) $\frac{1}{10}$ of the difference $e\,e''$, we will get the reading e' in a horizontal line with the center of the instrument. If now the line of sight be brought to read e' by the leveling screws, the bubble will move out of the center of its tube and should be brought back to the center by the bubble-adjusting nuts, when the bubble tube will be parallel to the line of sight because both are horizontal. It will

be noticed that there is no inversion here, and hence the apparent error is the real error and not double the real error.

51. The peg method stated. From the foregoing explanation the following statement of the peg method may be made. Set the level midway between two stakes from 200 to 400 feet apart. With the bubble in the center of the tube read a rod on each stake. The difference in readings is the true difference in elevation. Set the instrument beyond one of the stakes a distance, say, $\frac{1}{10}$ that between the stakes and read the rod on the near stake. To this reading apply the difference in level of the two stakes plus the effect of the earth's curvature [1] for a reading on the far stake. Read the rod on the far stake, and to this reading add or subtract, as the case may be, $\frac{1}{10}$ of the difference between the computed and the actual readings for the true reading (this quantity will be added if the actual reading is below the computed reading and subtracted if above). Bring the line of sight to the true reading with the leveling screws, and the bubble to the center of its tube by the bubble-adjusting nuts. Test again for a check, noting that it is not necessary to reset between the two stakes.

Another method of making this adjustment is as follows: Having obtained the true difference in level of the two stakes as in the preceding case, set the instrument between the stakes but near one of them so that the eye end of the telescope in swinging will just clear a rod held on that stake and with one set of leveling screws in line with the stakes. Level up and, looking through the object end of the telescope, mark a reading on the rod with a pencil or pin in the center of the small field of view that will appear. To this reading apply the difference in level of the line stakes, and the curvature correction, for the correct reading on the far stake; set the target at this reading, direct the line of sight at the target by swinging the telescope on its spindle and manipulating the leveling screws, and bring the bubble to the center of its tube by the adjusting screws at its end. Repeat for a check.

52. Lateral adjustment. In many old levels, the telescope is free to turn in the Y's. If this is the case, and the axis of the bubble tube is not in the vertical plane through the line

[1] See Art. 37.

of collimation, and the telescope tube becomes turned a little, the line of collimation will not be horizontal when the bubble is in the center of its tube. To test this, level the instrument and turn the telescope slightly in the Y's. If the bubble remains in the center of its tube, the adjustment is perfect, and nothing need be done. If not, adjust the tube laterally by the lateral adjusting screws at one end of the tube. Most levels have a little pin in one of the clips that fits in a notch cut in a ring on the telescope and prevents the telescope from turning in the Y's. Nevertheless, it is well to know that the adjustment is not much out. After making the adjustment it is well to test the previous bubble adjustment to make sure that it has not been disturbed.

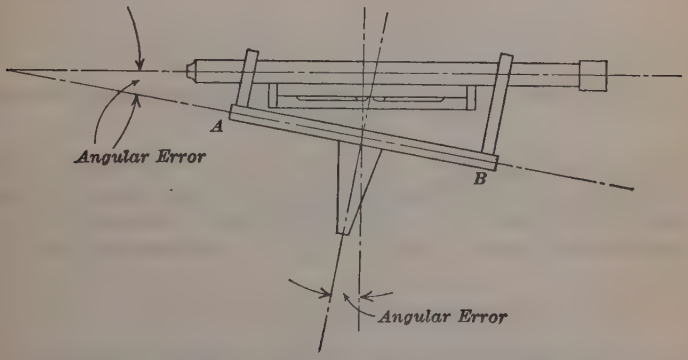


FIG. 48. — First position. Y supports of unequal height; bubble level.

53. Y adjustment. The third adjustment depends on the principle of Art. 29. It is performed as follows: The test consists in leveling the instrument carefully over both sets of screws and more carefully over one set, and then swinging the telescope around 180° on the vertical axis and noting whether the bubble remains in the center of the tube. If not, the axis of the instrument is not vertical, and the lines joining the points of support in the Y's are not perpendicular to it. The conditions and operations will be clear by reference to Figs. 48–51. To correct the error, that is, to make the lines of support perpendicular to the vertical axis of the instrument, move one of the Y's up or down (as may appear necessary by the position as-

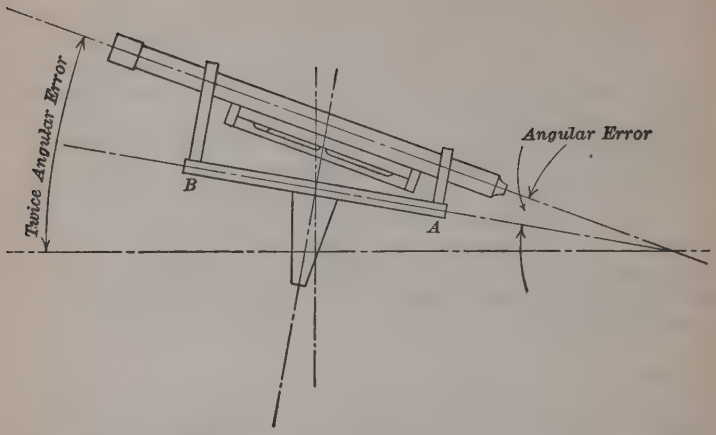


FIG. 49. — Second position. Y supports of unequal height; telescope reversed.

sumed by the bubble) by manipulating the large capstan nuts that hold the Y to the bar, till the bubble has moved *halfway* back to the center of its tube. The lines of support are then perpendicular to the vertical axis of the instrument, provided

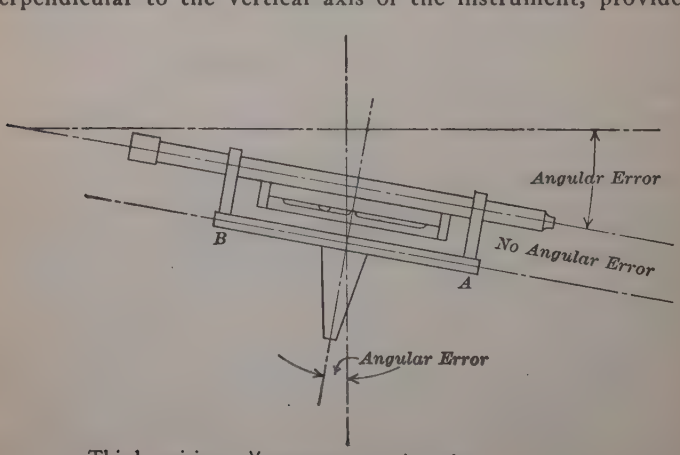


FIG. 50. — Third position. Y supports equal; telescope still reversed but instrument not releveled.

no movement has taken place. To test, relevel and reverse the telescope and make any further correction necessary, and test again until the work is complete.

54. To adjust the dumpy level. Reference to the cut of the dumpy level will show that the axis of the telescope is made by the maker perpendicular to the vertical axis of revolution. It is not supposed that this condition will change. It will also be noticed in the cut that there are no adjusting screws for the object slide. It is permanently adjusted. This is true of all the engineering instruments made by the maker

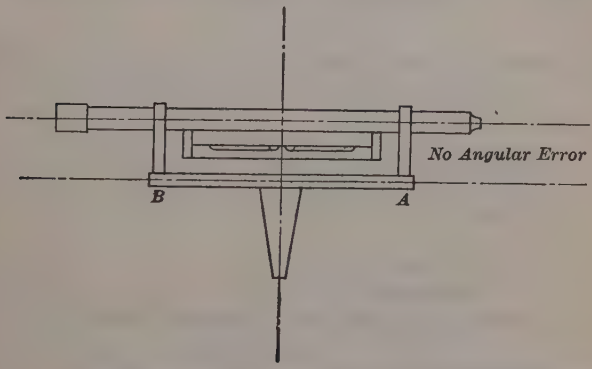


FIG. 51. — Fourth position. Y supports equal; instrument releveled.

of this level. Therefore, since the axis of the telescope is perpendicular to the axis of revolution and the axis of the objective slide is permanently adjusted, if the axis of the bubble tube is made perpendicular to the axis of revolution, and the line of sight is made parallel to the bubble axis, the instrument will be correctly adjusted. In this instrument, then, the order of adjustment is reversed.

First, the bubble tube is adjusted to be perpendicular to the axis of revolution. The test for this condition is made as in the test for the Y adjustment of the Y level. If adjustment is needed, it is made with the nuts at the end of the bubble tube. One half the apparent error is corrected.

Second, the line of sight is made parallel to the axis of the bubble tube. The test for this condition is made by the peg

method, as already described. The adjustment is made by moving the wires instead of the bubble. When the instrument is in perfect adjustment there are three lines that should coincide; viz., the axis of the telescope tube, the optical axis of the objective, and the line of sight.

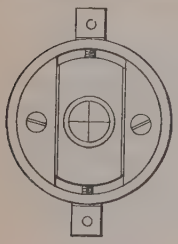


FIG. 52.

The dumpy level, and in fact all levels, should be so made that the vertical wire is not adjustable. A form of ring meeting this requirement is shown in Fig. 52 taken from a valuable English work on Engineers' Surveying Instruments.[1]

55. Summary of level adjustments. The following outline will assist in remembering the adjustments in their proper order.

A. Y *level*

- I. Line of sight coincident with axis of telescope.
  - 1. Parallax.
  - 2. Horizontal wire perpendicular to vertical axis of level.
  - 3. Intersections of wires to axis of telescope — use distant point.
  - 4. Objective slide axis coincident with axis of telescope — use near point.
  - 5. Eyepiece to center.
- II. Bubble axis parallel to line of sight.
  - 1. Vertical adjustment.
    - *a.* Direct method.
    - *b.* Indirect, or peg, method.
  - 2. Lateral adjustment.
- III. Y adjustment — bubble axis perpendicular to vertical axis of level.

B. Dumpy level

- I. Bubble axis perpendicular to vertical axis of level.
- II. Line of sight parallel to bubble axis.
  - 1. Parallax.
  - 2. Horizontal wire perpendicular to vertical axis of level.
  - 3. Line of sight — wires adjusted by peg method.

[1] Stanley's "Surveying and Leveling Instruments."

Exercise

Make complete tests and adjustments of the Y level, and of the dumpy level if available.

56. To adjust any level on a metal base. To adjust in this way, say, a carpenter's level, or a striding level for engineering instruments, place the level on a plane surface, mark its position, and note the position of the bubble in its tube. Reverse the level, end for end, and place it in the same position on the plane surface. If the bubble is as far from the center in the reversed position as it was in the first, the axis is parallel to its base, which means that it is in adjustment. If not, correct one half the difference in readings by whatever screws may be provided for the adjustment of the bubble. If convenient, the bubble may be brought to the center in the first position by tilting the plane surface; then, when the level is reversed, the movement of the bubble from the center will correspond to twice the error of adjustment, which may then be corrected.

MINOR INSTRUMENTS

57. Locke hand level. This is a small instrument for quick approximations where no great degree of precision is required. It is shown in Fig. 53.

Fig. 53.

It consists of a small bubble mounted on a brass tube over a hole in the tube, beneath which, and within the tube, is a prism occupying one half of the tube. A wire is stretched across the hole. There is usually an eyepiece of low magnifying power. The eye, in looking through the tube, will see the wire and the bubble reflected by the prism; and when, if the instrument is in adjustment, the bubble appears to be bisected by the wire, the line of vision past the wire is horizontal. In

use, the leveler and hand level constitute the leveling instrument, and between plus and minus readings the observer must remain stationary. There are other forms of the Locke level than that shown in the figure. The principle is the same in all.

EXERCISE

With a hand level determine the difference in level of two points about a quarter of a mile apart, and compare the result with that obtained by leveling with a Y or dumpy level. Do the same thing for two points a half mile apart. It is desirable that this be done over both level ground and hilly ground.

58. Abney level and clinometer. This is a combination of a hand level and a slope-measuring instrument. Such an instrument is shown in Fig. 54.

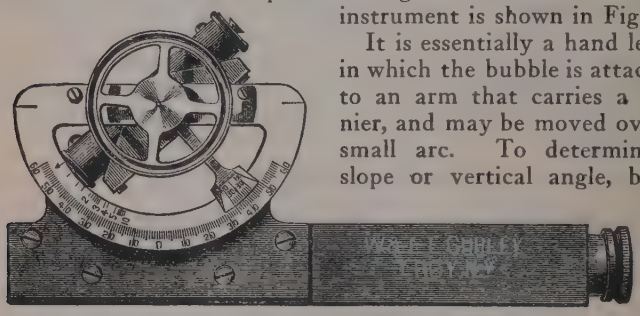


FIG. 54.

It is essentially a hand level, in which the bubble is attached to an arm that carries a vernier, and may be moved over a small arc. To determine a slope or vertical angle, bring the line of sight to the slope, and turn the vernier arm till the bubble is seen on the wire. The bubble axis will then be horizontal, and the angle between it and the line of sight will be indicated by the vernier reading.

EXERCISE

Determine a few slopes with the Abney level and clinometer.

59. Gurley's monocular and binocular hand levels. These are telescopic hand levels by which readings are more definitely determined on a rod at some distance than is possible with the ordinary hand level which is not a telescope. The

Gurley levels are shown in 55. The author considers the monocular hand level the best yet devised.

60. Adjustment of the Locke and Abney levels. For the Locke level, by some means get a horizontal line, as a sheet of water of limited extent; if the latter is not at hand use a leveling instrument, or indeed the hand level itself, held midway between two rods or trees some distance apart. Having ob-

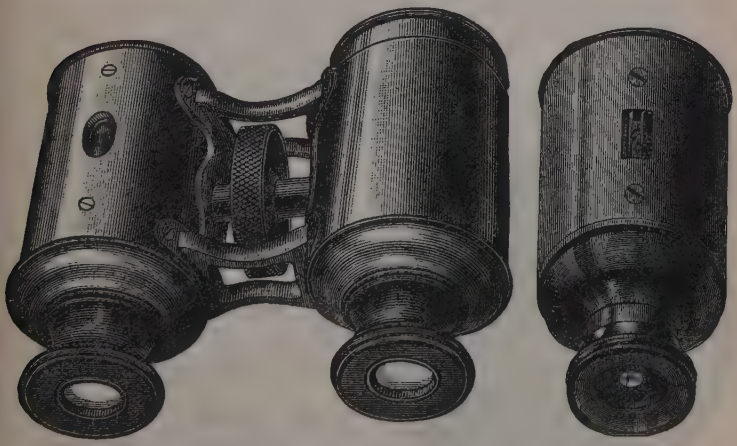


FIG. 55. — Gurley's hand levels.

tained the horizontal line, stand with the hand level at one end of it, and, with the eye and wire in the horizontal line, note whether the bubble appears to be on the wire. If it does not, adjust by moving the frame carrying the wire by means of the little screws at the end of the bubble mounting.

To adjust the Abney level and clinometer, set the vernier to read zero, and test as for the hand level; adjust the bubble by screws provided for that purpose, or adjust the wire by a slide arrangement provided in some instruments.

EXERCISE

Test such Locke and Abney levels as may be available for adjustment, and, if needed, adjust them.

LEVELING WITH THE BAROMETER

61. Barometer described. Very rough leveling work is sometimes done with the aneroid barometer. The barometer is an instrument for measuring the pressure of the atmosphere. The only reliable form is the mercurial barometer. This is not a convenient portable instrument. The aneroid barometer is an attempt at portableness. It is made in various forms, the more common of which is shown in Fig. 56.

It consists essentially of a metallic box, with a thin corrugated top, from which the air is exhausted. With the varying pressure of the atmosphere, the top of the box rises and falls. Its motion is communicated by levers and chains to a pointer moving around a dial, graduated by comparing the movement of the pointer with that of a mercurial barometer. Hence the "inches" on the dial are not true inches nor of uniform length.

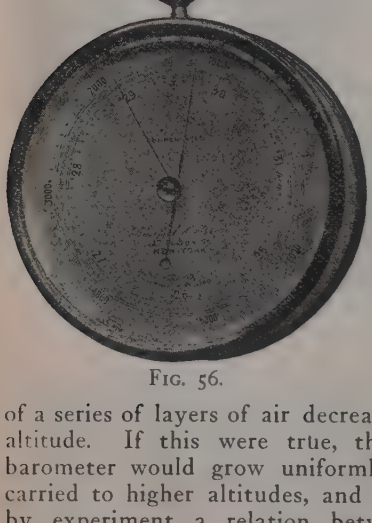


FIG. 56.

62. Theory. The use of the barometer to determine differences of altitude depends on the supposition that the atmosphere is composed of a series of layers of air decreasing uniformly in density with altitude. If this were true, the pressure indicated by the barometer would grow uniformly less as the barometer was carried to higher altitudes, and it would be easy to establish by experiment a relation between the altitude and the barometric reading. The supposition is not entirely correct. The pressure of the atmosphere at any one place is affected by humidity, and very considerably by temperature. Moreover, owing to the movement of the atmosphere, two distant places of equal altitude, having at a given time equal temperatures, will not have equal barometer readings at the same time.

It is difficult therefore to establish a relation between the altitude and the barometer readings. It has been attempted by many persons, with the result that there are many formulas for determining the differences in altitude from the readings of the barometer. These formulas do not differ so much in method of deduction as in the experimental constants used. Perhaps one of the best is that developed by Mr. William Ferrel.[1]

Modified by omitting the terms for correcting for humidity and latitude, which are small in effect, and one or two other still smaller terms, this formula is

$$H = 60521.5\left(\log \frac{B}{B_1}\right)\Big(1 + 0.001017\,(t + t' - 64°)\Big),$$

in which H is the difference in altitude of two points where observations are taken, B and B_1 are the barometer readings at the lower and higher points respectively, and t and t' are the Fahrenheit temperatures at the two points.

63. Barometric tables. It is convenient in tabulating this formula for practical use to consider that $t + t'$ is 100°, and that H is the difference in altitude between two points whose altitudes above a given datum are to be directly determined from the tables. Thus if $t + t' = 100°$, and the given datum is such that the barometer reading is 30 inches, which is approximately average sea level reading, the formula is

$$H = 60521.5(1 + .001017 \times 36)\left(\log \frac{30}{B_1} - \log \frac{30}{B}\right)$$

$$= 62737 \log \frac{30}{B_1} - 62737 \log \frac{30}{B}.$$

Tables may be constructed for the quantity, $A = 62737 \log \frac{30}{B}$ with B as the argument, and such a table would be entered twice, once for each barometer reading. The difference of the tabular values found would be the value of H if $t + t' = 100°$ and there is no correction for humidity. Table III, page 474, is such a table. For other than a mean temperature of 50°, and for average hygrometric conditions, a correction is to be applied

[1] See Appendix, 10, United States Coast and Geodetic Survey Report for 1881.

to the result. This correction is a linear function of the value of H already found, such that

$$H = (A_1 - A_2)(1 + C),$$

in which A_1 and A_2 are the values from Table III, and C is the temperature and humidity correction coefficient. Values of C are given in Table II, page 473.

Examples and Exercises

1. Two barometer readings and corresponding temperatures are at P, 30.76 in. + 70°; at P', 28.42 in. + 60°. What is the difference in elevation?
2. At two other points, Q, 30.26 + 40°; Q', 29.02 in. + 30°. What is the difference in elevation?
3. Measure the height of a neighboring hill with the barometer, remembering to read the temperature as well as the barometer. Check the result with a leveling instrument.
4. Determine the height of some building with the barometer and check with a tape.

64. Practical suggestions. The aneroid barometer should be compared frequently with a mercurial barometer, and the errors of reading at different parts of the scale should be noted. This can be well done only under the bell of an air pump.

The scale of altitudes on some barometers is useless.

Some barometers are marked "compensated," meaning that no temperature corrections need be applied. This is a mistake. The temperature corrections should always be applied.

Always read the barometer when it is horizontal. Carry it in a strong case, and do not let the warmth of the body affect it. Do not use it for altitudes just before, or after, or during a storm. Tap the box gently before reading.

In determining the difference of altitude of two widely separated points, stop once or twice for a half hour or so on the journey between the points, note the reading of the barometer at the beginning and end of the stop, and thus try to determine whether the atmospheric conditions are changing, and, if so, at what rate. A correction can thus be determined for the first reading, on the supposition that the observed change has taken place also at the starting point.

A second barometer may be kept and read at regular intervals at the starting point instead.

64a. Accuracy of the method. If barometer readings are taken simultaneously daily at two widely separated stations for from one to six years, the utmost that can be expected is an error of about one half of one per cent, though occasional results are much closer. Single observations taken on the same day with the same or compared barometers, and at distances of a few hours' walk, will give results with errors of from one per cent to three per cent and more. An error in temperature of five degrees will cause an error in result of one per cent or more. Single observations taken a day or more apart, and at stations a day's journey or more apart, cannot be depended on to give results much closer than the nearest ten per cent, though they will frequently do better than this and sometimes worse.

CHAPTER IV

DETERMINATION OF DIRECTION AND MEASUREMENT OF ANGLES

65. Instruments mentioned. In almost all surveys it is necessary to measure distances and angles.

Any one of several instruments may be used for measuring angles according to the character of the work. The most commonly used are the compass and the transit.

The compass is an instrument for determining directions and, indirectly, angles; while the transit is an instrument for determining angles and, indirectly, directions.

Almost all of the old land surveys of this country have been made with the compass as the angle-measuring instrument; and while this instrument has been, to a great extent, supplanted by the transit, there are still a great many compasses sold annually. The compass is frequently used in preliminary mine surveys and for exploratory surveys, and the compass on the transit instrument is always used in all but city surveys as a check on the angle work of the transit.

A knowledge of the compass and compass surveying is absolutely essential to a clear understanding of old surveys and surveying methods, with which every surveyor must reckon when resurveying lands formerly marked out by and described from compass surveys. For these reasons, the compass and compass surveying should find a place in every treatise on surveying, and will be discussed here. Mention must be made of the sextant, which is an angle-measuring instrument whose use in surveying is confined to certain particular operations, as in exploratory surveying or in the location of soundings taken off shore. A description of this instrument is given on pages 393–396.

Mention should also be made of the solar compass and solar

transit, instruments for determining the true meridian by an observation on the sun. The solar transit is described on pages 257–262.

THE COMPASS

66. Description. The compass consists of a line of sight attached to a graduated circular box, in the center of which a magnetic needle is hung on a pivot.

At any place on the earth's surface, the needle, if allowed to swing freely, will assume a position in what is called the magnetic meridian of the place. If, then, the direction of any line is required, the compass may be placed at one end of the line, and the line of sight may be made to coincide with the line. The needle lying in the magnetic meridian, and the zero of the graduations of the circular needle box being in the line of sight, the angle that the line on the ground makes with the magnetic meridian is read on the graduated circle. If the magnetic meridian coincided everywhere with the true meridian, or even if the angle between them were constant, the compass would be a far more valuable instrument than it is. The compass, however, has its field, and in that field is a very valuable tool. A very good form of compass is shown in Fig. 57, page 100.

It is sometimes set on a tripod head and sometimes on a Jacob's staff, which is a single-pointed staff with a head fitted to receive the compass. The sights *SS* are attached to the main plate of the instrument by the screws seen below the plate. The level vials incased in brass tubes are fastened to the plate by screws passing through the plate from below, by which screws the bubbles may be adjusted parallel to the plate. The needle *N* rests on a steel pivot that is screwed into the plate, and the needle is lifted from the pivot, when not in use, by the ring *R* operated by a lever, which is in turn moved by the screw whose head is seen just below the plate directly in front between *K* and *C*. On the left is a marker *M* used to keep track of the chaining. Next to this is an arc *A* and a vernier, moved by the tangent screw *V*. The arc is attached to the plate, and the vernier to an arm that is fastened to the compass box, which box may be turned thus through a small angle by the tangent screw. This vernier is called a *declination* vernier. Its use will appear later. The compass box is graduated from the north and south points in each direction 90°. When the zero of the vernier coincides

with the zero of the arc *A*, the line of zeros of the compass box is coincident with the line of sight. The spring catch *C* holds the compass on the tripod head, and the clamp *K*, acting on a concealed spring that encircles the tripod head, serves to clamp the instrument so that it will not revolve about the vertical axis. There is a small coil of wire seen on one end of the needle. This counteracts the magnetic force that would otherwise cause the needle to *dip*. This means that the direction of

FIG. 57.

magnetic force or magnetic pull is not horizontal, but is inclined, and if the needle is free to dip, it will lie in the direction of magnetic pull. This *vertical component* of magnetic force that causes the dip varies in different places, and the little coil of wire may be moved nearer to, or farther from, the pivot to allow for this variation.

Since only horizontal angles are measured with the compass and used in land surveying, it is necessary to have the plates horizontal and the sights vertical when working. This is accomplished by means of a ball-and-socket joint attached to the tripod head, and the bubbles on the plate.

The cap on the ball is screwed until the ball can be moved

only by some force, the plate is seized by the hands near the compass box, and tipped till the plate bubbles are centered. If the cap of the ball joint has been set firmly enough, the instrument will stand level while in use. The cap should not be set so firmly as to render likely the springing of the plates in leveling.

The graduations shown on the edge of one of the sight bars are for measuring vertical angles roughly. The eye of the observer held near the bottom peephole in the left-hand sight bar looks past the right-hand sight bar to some point above the level of the instrument, and the finger, a card, or pencil is put on the right-hand sight bar in the line of sight. The position of the card, finger, or pencil on the sight bar gives the reading of the angle of elevation to the object sighted. An angle of depression is found by sighting from the upper peephole of one sight bar past a scale reading from the top down on the other sight bar.

The letters *E* and *W* are on the *left* and *right*, respectively, of the North line of the compass box, instead of on the *right* and *left*, respectively, as might be expected. The reason will appear later.

USE OF THE COMPASS

67. Bearing. The use of the compass is to determine the directions, or, technically, the *bearings* of lines; or to mark out lines whose bearings are given.

The bearing of a line is the horizontal angle between the meridian plane through one end of the line, and the vertical plane including the line, and is measured from the north or south points, 90° either way. Thus, the bearing of a line extending in a direction midway between the north and east, would be N. 45° E. or S. 45° W., according to the end from which the bearing is read. This gives rise to two terms, *forward* bearing and *back* bearing, the forward bearing being the bearing in the direction in which the survey is being run, and the back bearing being that read in the reverse direction. These two bearings (omitting the effect of convergence of the meridians of the two ends of the line, which is an inappreciable amount in an ordinary compass survey) should be numerically equal, but with opposite letters to express the direction. The bearings are never read, east so many degrees south or north, or west so many degrees north or south; but always from the north or south points.

Examples

1. The forward bearings of four lines are respectively N. 1° 30′ E., S. 89° 30′ E., S. 49° 15′ W., N. 88° 45′ W. What are the back bearings?

2. The bearing of a line AB is S. 36° 40′ W., when run from A to B; what is its bearing when run from B to A?

68. To determine the bearing of a line. Set the compass over one end, level the plates, lower the needle, turn the north end of the compass box toward the distant end of the line, and bring the line of sights carefully in the direction of the line. If the declination vernier reads zero, the north end of the needle will show the bearing of the line.

The graduations are usually to half degrees, and in early surveys bearings were read by estimation to quarter degrees. It is now customary to try to read a needle by estimation to the nearest five minutes. Tenths of degrees would be better.

Exercises

1. Set the compass over the southerly end of a line extending approximately northeast; read the bearing of the line and discover why the letters *E* and *W* are on the *left* and *right*, respectively, of the north line of the compass box.

2. Set up a compass at a convenient point and read the bearings to several other points. Set up the compass at several of these points and read the bearings to the point of first set-up. If no compass is at hand, the transit may be used, reading its needle only.

69. To lay out a line of given bearing. Set the compass over a point on the line, level the plates, drop the needle, turn the compass on its vertical axis till the north end of the needle reads the given bearing. The line of the sights is now in the required line, which may be ranged out.

Care must be used to see that the needle moves freely and is not disturbed by turning the compass, or that, if disturbed, it settles again at the correct reading.

Exercises

1. Set the compass over a point and range out lines having the following directions: N. 46° 15′ W., N. 88° 45′ E., S. 36° 30′ E., S. 2° 15′ W., N. 23.2° W., S. 18.6° W., N. 78.8° E., S. 89.9° E.

2. Establish enough other directions to become familiar with compass reading, trying to read to 5 minutes or tenths of degrees.

70. To run a traverse. This is to determine the lengths and bearings of a series of connected lines. These may form the sides of a farm, or inclose a pond, or be the center line of a crooked road. If there are no natural objects to mark the angle points, flags are set at those points by a flagman, the compass is set at each point in succession, and the back bearing of the preceding line and the forward bearing of the line ahead are determined while the lines are being measured by the tapemen. The back bearings are read as a check on the forward bearings and to detect local attraction, as will be explained hereafter. In occasional hasty work the compass will be set only at every second corner, and the back bearing of one line (afterward changed to forward bearing) and the forward bearing of the other line will be determined. Ordinarily this should not be done. If the tapemen have no point ahead to measure toward, they may be kept in line by the man at the compass.

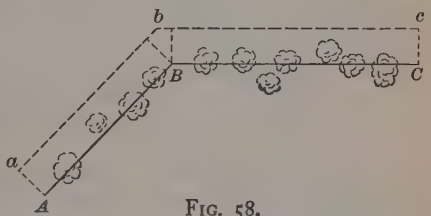


FIG. 58.

Frequently it is necessary to produce a line for a considerable distance through woods or over hills, one end not being visible from the other. If the bearing of the line is known, turn the compass, placed on one end of the line, in the direction of the line, and direct the tapemen as far as they can be seen well. Establish a point by driving a stake in the line at the end of the last measurement, or by merely setting the flag in the ground, to be pulled out when the compass is brought up, set the compass over the established point, get the back bearing for a check, and continue the line.

If a tree or small building obstructs the line, and a few inches on one side or the other are of no consequence, as is the case in most surveys made with the compass, set the compass beyond the obstruction, as nearly on the line as can be estimated, and, turning the sights in the direction of the known bearing, proceed with the line. Other obstacles are discussed on pages

193–200. If the line to be measured is covered by a hedge or high fence, equal offsets are made at a right angle to the line, as indicated in Fig. 58 at *A*, *B*, and *C*, and the direction and length of the offsetted line is determined. In farm surveys, where a corner is anywhere within a two-foot tree or a large post or stone, no great precision is necessary and the perpendicularity of the offset may be estimated by eye. If it is desired to be somewhat careful, a right angle to the line may be laid out with the tape. Sometimes it is possible to measure the line when it is impossible to see along it or to set the compass on one end of it. In the figure the line *abc* is run far enough past the offset at *B* to enable an offset line to be run from *B* to *C*, and the amount of the offset is determined after the convenient line *bc* is determined. A little consideration will always tell one what to do. In compass surveys it is folly to center stakes with tacks.

71. Notes. The notes that are taken may be kept in the following form, if nothing more than a record of the traverse is required:

Station	Bearing	Dist. Chains
etc.	etc.	etc.
D	S. 72° 45′ E.	10.40
C	S. 52° 00′ E.	7.30
B	N. 23° 30′ E.	6.74
A	N. 36° 15′ W.	4.63

The notes read up the page so that the line may appear to be ahead as one walks along with the open notebook. Other notes are required usually, as, distances to points passed, fences crossed, roads and streams crossed, distances right or left to objects near the line, as will be hereafter described. In such cases, a sketch of the work with all the necessary measurements marked on it is the best possible form of notes to take in the field. This may be converted afterwards into the form shown on page 292. The name and date of survey should always be recorded on every page of the notebook used, and the names of those engaged on the work should be entered at least once. The back bearings should also be recorded as read.

Examples of two forms of notes are shown in Figs. 59 and 60. Figure 59 shows a sketch form for the notes of a survey made for John Davis, Oct. 17, 1905, in Rensselaer County, New York, by J. K. Schneider, assisted by Thos. Doran and John Davis. Figure 60 shows another form for the first two lines of the same survey. The middle column of the right hand page represents the line run split in two and separated to allow figures to be

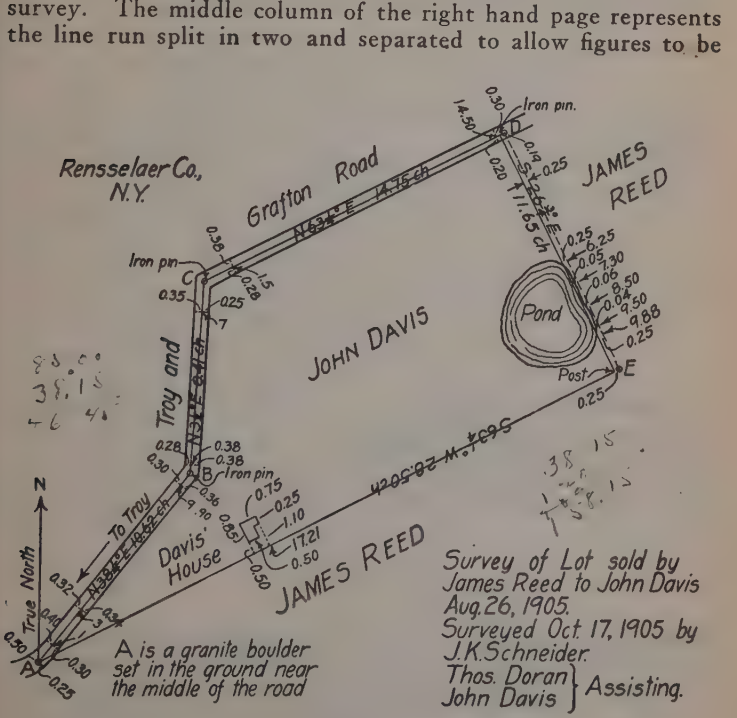


FIG. 59.

plainly inserted. Some surveyors would put only one line on a page to avoid confusion. It may be considered unnecessary to put the offsets in the notes on the left page and also in the sketch on the right-hand page.

72. Angles. To determine the angle formed by two lines at their point of intersection, set the compass over the point of

intersection and read the bearing of each line. From these bearings determine the angle. There will always be two supplementary angles. If the lines are conceived to be run from the end of one to the point of intersection, to the end of the second, and so on, as in running a traverse, the angle between the

Oct. 7, 1905	J.K. Schneider Surveyor Thos. Doran Asst. John Davis Ass't.	Survey for John Davis Lot sold to John Davis by James Reed.			
thence N 3½° E					
10.62 ch. to B	iron pin near center of road			B 10 0.62	
9.90 ch. offset	0.36 R. and 0.30 L. to fence.	0.30		9 9.90	0.36
3 ch. offset	0.34 R and 0.32 L. to fence.	0.32		3	0.34
along road.	1 ch offset 0.30 R. and 0.40 L. to fence.				
S.W. corner hereof and run thence N 38¼° E					
southerly line of this survey marking the		0.40		1	0.30
middle of Troy and Grafton road and on				A	
Set up at A on a granite boulder near		0.50		0	0.25

FIG. 60.

prolongation of line one, and line two, measured in the direction in which the line is conceived to bend, is called the *deflection* angle, or, in a closed survey, the *exterior* angle (Fig. 61).

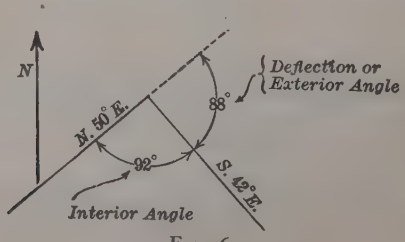


FIG. 61.

EXAMPLES

1. Radiating from a point A, are eight lines of bearings as follows:

AB, N. 53° 45′ W.	AF, S. 86° 45′ E.
AC, N. 36° 42′ W.	AG, S. 24° 36′ E.
AD, N. 18° 34′ E.	AH, S. 20° 30′ W.
AE, N. 34° 28′ E.	AI, S. 36° 20′ W.

Required the angles between the following bracketed lines — always considering the smaller angle:

$\{AB\ AC\}$	$\{AB\ AD\}$	$\{AC\ AF\}$	$\{AE\ AI\}$
$\{AC\ AD\}$	$\{AC\ AE\}$	$\{AD\ AG\}$	$\{AB\ AG\}$
$\{AD\ AE\}$	$\{AD\ AF\}$	$\{AE\ AH\}$	$\{AC\ AH\}$
$\{AE\ AF\}$	$\{AE\ AG\}$	$\{AF\ AI\}$	$\{AD\ AI\}$
$\{AF\ AG\}$	$\{AF\ AH\}$	$\{AB\ AF\}$	$\{AB\ AH\}$
$\{AG\ AH\}$	$\{AG\ AI\}$	$\{AC\ AG\}$	$\{AC\ AI\}$
$\{AH\ AI\}$	$\{AB\ AE\}$	$\{AD\ AH\}$	$\{AB\ AI\}$

2. A traverse is run with the following bearings: N. 42° E., N. 36° E., S. 1° W., N. 50° W. Determine the interior angles. If the traverse is of a closed field, what should be the sum of the interior angles?

3. Let the eight lines of Example 1 be the first eight consecutive courses of a traverse. Determine the deflection angles.

73. Cautions for compass work. It is better to keep the north end of the compass box ahead, and to read the north end of the needle. The north end of the needle is distinguished either by its peculiar shape or by the fact that the coil of wire is on the other end. The north end of the compass box is distinguished by a peculiar figure, usually a conventional *fleur de lis*. If the south end of the box is ahead, read the south end of the

needle. Care should be exercised in the use of the compass, both in directing the sights and in reading the needle, and the beginner should be particularly careful to read from the north or south point according to which is nearer the north end of the needle. Greater differences in results may be expected from different instruments used by the same man, than from the same instrument used by different men, if the men are careful in their work.

If the needle is sluggish in its movement, it may be due to one or both of two causes. The pivot may be blunt, or the needle may have lost some of its magnetic strength. Take the needle off the pivot and examine the pivot. If it is found to be blunt, unscrew it and carefully grind it down on a fine oilstone. If the pivot seems perfect, the trouble is with the needle. Lay the needle down and rub each end from the center out with that end of a bar magnet which attracts the end of the needle that is being rubbed. In passing the magnet back from end to center, raise it some distance above the needle; otherwise the backward movement will tend to counteract the rubbing. The needle may also be remagnetized by placing it in the magnetic field of a strong electro-magnet, as a dynamo. If placed on the magnet in a wrong direction, the south end will, when replaced, point north. To correct this, replace the needle on the magnet in the opposite direction.

Two needles at the same place may not point in the same direction. This will be due probably to the fact that their magnetic axes do not coincide with their geometric axes. This seems to be unavoidable. For this reason the needle should be narrow and deep rather than flat and wide. There is a difference of opinion about this, based on difference of manufacturing processes.

The glass cover of the compass box may become electrified by rubbing, and attract the needle. This is remedied by touching the glass with the moistened finger.

When the compass is being moved, the needle should always be lifted from the pivot. When put away, the needle should be allowed to swing to the magnetic meridian and then lifted. It retains its magnetic strength longer in the meridian.

74. Local attraction. The needle may be drawn from its mean position for a given locality by the attraction of large or

small masses of iron or other magnetic substances. These may be either minerals hidden in the ground or manufactured articles, as agricultural implements in a barn near by, railroad rails, nails in an adjacent house, etc. The chain used in the survey may influence the needle if brought too near, while keys in the pocket of the observer, or steel wire in the rim of a stiff hat, and steel button molds on a coat have been known to give trouble.

Local attraction may be very troublesome, and must be carefully looked out for. It may be discovered and allowed for by simply occupying every corner of a survey, and reading the bearing of each line back as well as forward. It will be evident that at any one station where the compass is set, the local attraction, if any exists, will affect all readings taken from that point by the same amount; and hence, if the bearings of two lines meeting at a point are both determined at that point, the angle of intersection is correctly deduced, even though there is local attraction and neither bearing is correctly determined. If the bearing of a line is determined from each end, and it is found that the two readings are not numerically the same, and it is certain that the bearings are correctly read, there is probably local attraction at one end or the other, or possibly at both ends. Now if another line is set off from one end of the first line, and the bearing of the new line is read at both ends, and the readings are found to agree, the local attraction affected the reading of the first line at the other end. If trouble is encountered on the second line, a third line may be set off from the other end of the first line, the same observations being made as on the second line. If it is feared that local attraction exists, the best method is to read the bearings of all lines at each end, determining the angles between adjacent lines from the bearings taken at their points of intersection. One line may then be assumed to be correctly read, or the true bearing may be determined, and the bearings of the others may be computed from the series of angles.

Example

Find the local attraction in position, amount, and direction, from the following forward and back bearings of the sides of a closed field: *AB*, N. 48° E., S. 47° W.; *BC*, S. 42° E., N. 39° W.; *CD*, S. 40° W., N. 38° E.; *DA*, N. 30° W., S. 30° E.

PLOTTING

75. Plotting a traverse or survey. A traverse is plotted by one of three common methods:

1. With the protractor.
2. By tangent distances or offsets.
3. By latitudes and longitudes. The third method is described in Art. 111.

1. *With the protractor.* A scale being chosen, the survey is examined to see where to lay the meridian and first course of the survey to make the drawing come on the drawing board. This is simple if the latitudes and longitudes of the corners have been determined, as the greatest east and west and north and south distances will be known. Generally the top of the plot should be north, but this is not always possible in a long traverse, as of a road or a railroad survey.

When the initial point and meridian direction are determined and plotted on the paper, the first line is laid out in direction by the use of a protractor, and its length scaled. With parallel ruler or triangles the meridian direction is transferred to the end of the line and the direction of the second course determined with the protractor, its length measured, a meridian drawn through its end and the work continued as before. An improvement consists in laying off the several directions from the original meridian and transferring these directions to their proper places. Thus in Fig. 62 the directions of AB, BC, and CD are laid off at A; AB is drawn and scaled to B; BC is drawn through B parallel to AC' and scaled to C; CD is drawn through C parallel to AD' and scaled to D, etc.

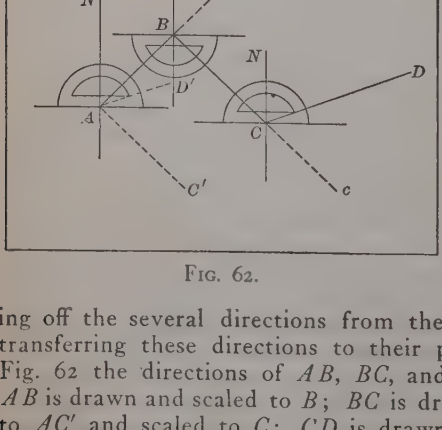


FIG. 62.

Another method of using the protractor is to lay off deflection angles instead of bearings or azimuths. Thus, the line AB

is produced to b and the angle bBC laid off with the protractor; likewise cCD, etc.

The second procedure is recommended as the best for extensive work when some degree of accuracy is required. If a survey has been conducted by deflection angles, the quickest method is to determine the bearings or azimuths of the several lines from the deflection angles and use the second procedure. If the survey is a long traverse, the meridian will be transferred from time to time as the drawing grows.

2. *By tangent offsets.* In this method, deflection angles are usually determined but need not be. Assuming first that bearings or azimuths are used, a distance of 100, or 1000, or 10,000 units is chosen according to the scale of the map and magnitude of the survey, and the natural tangents of the several bearings or azimuths are multiplied by this number by simply moving the decimal point. The chosen base distance is laid off on the meridian both north and south from the initial point, and the several multiplied tangents also laid off from the extremities of the base distances. In Fig. 63, 1000 units are laid off from A to a and e respectively, and 1000 × tan. bearing of $AB = ab$, 1000 × tan. bearing $BC = ec$, and 1000 × tan. bearing $CD = ad$ are measured to b, c, and d, respectively. Lines joining the initial point, and the point thus determined, will give the directions of the several courses and may be transferred to their proper places. If the angle is greater than 45°, it is better to use an east and west line and the cotangent of the angle.

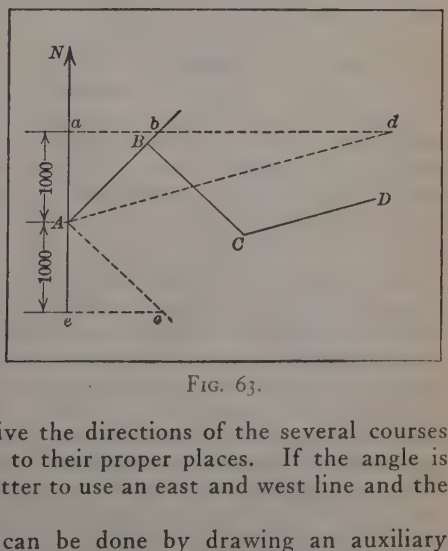


FIG. 63.

Of course the work can be done by drawing an auxiliary meridian or parallel through each corner from which the next course could be laid off. If deflection angles are used, the method will be evident.

Where the scale is large enough this method is probably more precise than the protractor method, unless an expensive vernier protractor is used.

Exercise

Run a traverse around an irregular plot of ground containing several acres, determining simply the length and bearing of each side with the chain or tape and the compass. Read bearings to the nearest $\frac{1}{4}$ degree and use only ordinary care in measuring to the nearest link. Plot the field carefully to a large scale by bearings and distances with a protractor and scale and note whether the plot closes; *i.e.*, whether the final point coincides with the initial point as it should.

MAGNETIC DECLINATION

76. Declination defined. Thus far all bearings have been assumed to be taken with reference to the magnetic meridian. If the magnetic meridian, which is that in which the magnetic needle lies, and the true meridian were coincident at all places on the earth's surface, or if at one place the angle between them were constant, the determination of the true bearing of a line by the compass would be comparatively simple. The needle, however, points to the true north at very few places on the earth's surface, and the angle that the magnetic meridional plane makes at any point with the true meridional plane is called the *magnetic declination.*

77. Variations of declination. The declination is subject at every place to changes, called *variations* of the declination. These are: *secular* variation, *annual* variation, *lunar inequalities*, and *diurnal* variation. Annual variation which is cyclical with the year as the period, must not be confused with annual change of secular variation. There are also irregular variations due to magnetic storms. The annual variation and the lunar variation are both very small and may be neglected entirely. The diurnal variation is nothing at about 10.30 A.M. and 8.00 P.M., and varies between these hours, the north end being farthest east at about 8 o'clock in the morning and farthest west at about 1.30 in the afternoon. The total movement is from five to ten minutes, and the corrections to be applied are as in the Appendix, Table V, page 477. It will be noticed that the

variation changes with the seasons. It also varies slightly with the latitude, and the corrections in the table are about a mean for the United States for the latitudes given.

The secular variation is by far the most important. It is supposed to be periodic in its character, requiring from two hundred to four hundred years to make a complete cycle.[1]

The needle in Paris in 1580 pointed about 9° or 10° east of north, while in 1800 it pointed a little more than 22° west of north. The change seems to have been fairly but not absolutely uniform in rate. The rate of change seems to differ with difference of place, and also with time at the same place. It is almost nothing for a few years when the needle is near its extreme position in either direction.

A line connecting points having the same magnetic declination is called an *isogonic* line, and the line joining points of no declination is an *agonic* line. In Plate VI, at the end of the book, taken from the Coast Survey Chart for 1910 an agonic line is seen extending from near Charleston, S. C., through the lower end of Lake Michigan. All points to the east of this line have west declination, and all points to the west have east declination. This line is moving westward.

Lines of equal annual change also are drawn on this chart, and one line of no annual change appears. To the east of this line, west declinations are increasing and east declinations diminishing, but west of this line east declinations are increasing. The observations of the U. S. Coast and Geodetic Survey seem to show that this line moved about two thirds the distance across the United States from west to east in 20 years (1885–1905). Prior to this period, all isogonic lines in the United States were supposed to be moving westward, but in 1911 all those west of the line of no change were moving eastward. Hence, the end of a cycle has apparently been reached to the west of the United States and is gradually being reached and passed in the United States from west to east. It would appear, therefore, that the agonic line should very soon cease moving west and begin to move east. But this is not certain. The declination for a given place may be determined approximately from this chart, though, from the extreme irregularity of the lines in those portions of the country where observations have been most frequent,

[1] For an extended article on this subject see "United States Coast and Geodetic Survey Report" for 1882. See also the Report for 1908.

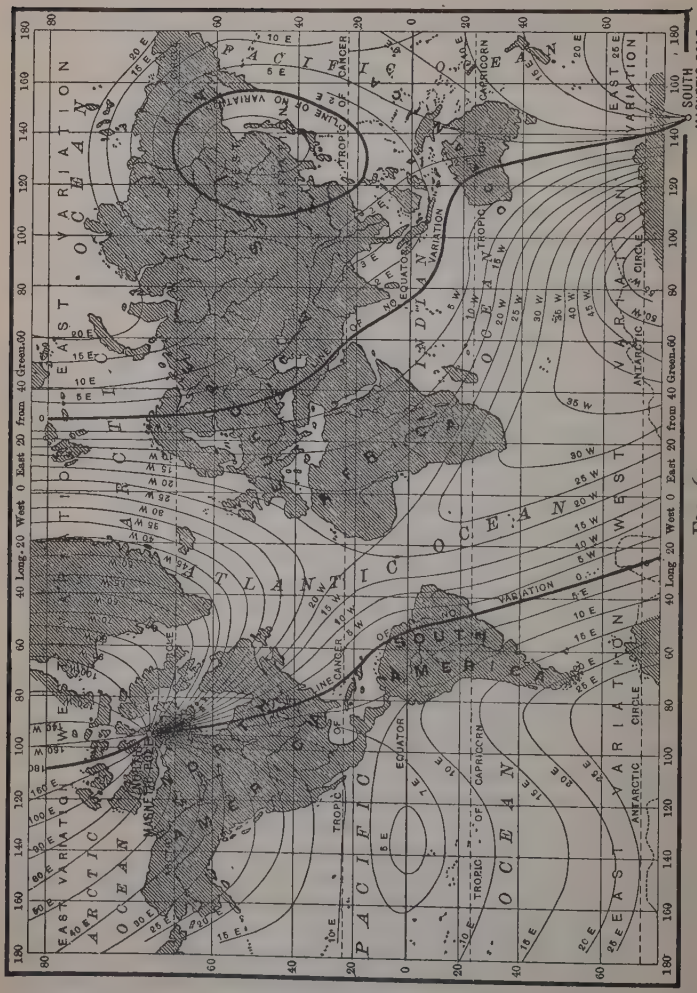

SOUTH MAGNETIC POLE
PACIFIC OCEAN
ATLANTIC OCEAN
INDIAN OCEAN
SOUTH AMERICA
NORTH AMERICA
EAST VARIATION
WEST VARIATION
ARCTIC OCEAN
NORTH MAGNETIC POLE
TROPIC OF CANCER
TROPIC OF CAPRICORN
EQUATOR
ANTARCTIC CIRCLE
LINE OF NO VARIATION
VARIATION
Long. 20 West 0 East 20 from 40 Green. 60

FIG. 64.

as, for instance, in Missouri, it will be seen that too much reliance cannot be placed on the correctness of the lines. Figure 64 is a chart on a very small scale, showing in a general way the declination at all points on the earth.

78. Determination of the declination. Since it is the magnetic meridian that is given by the needle, and since this is not a fixed direction, it should always be required to reduce the magnetic bearings of lines run with the compass to the true bearings. This was not done in the old land surveys made in this country, and the declination was not always noted. The result is that it is difficult to rerun the old lines, as will appear hereafter. From what has been said about secular variation, it will readily be seen that the declination must be known for the place of the survey for the time of the survey. In this country it is sufficient to determine the declination each year for a given place.

The declination is determined by the surveyor by first determining a true meridian (see Chapter IX), and then setting the compass over the southern end of a line in this meridian and reading the bearing of the line. This is the declination, in amount. The direction is, however, reversed, since the bearing of the true meridian referred to the magnetic meridian is read, instead of that of the magnetic meridian referred to the true meridian. The zeros of the declination vernier and its arc should coincide, or the reading will be erroneous.

To read the declination more precisely than can be done on the circle, which is graduated only to half degrees, bring the plane of the sights carefully into the meridian, and then move the vernier by the tangent screw till the needle reads zero, or north. The reading of the vernier will be the declination. Now if the vernier is left set at this reading, a survey may be conducted by the compass and referred to the true meridian, without mental reduction, as the compass will give the true bearing of the lines at once.

The declination should be determined about 10 o'clock in the morning with the compass that is to be used in the piece of work for which the meridian is required. Ten o'clock is suggested because the needle is approximately in its mean position for the day at about this hour.

Examples

1. The magnetic declination at a given place is found to be 10° 30′ E. What will be the bearing of the true north? The true south? The true east? The true west?

2. The magnetic declination is 7° 10′ W. What will be the bearings of the four true cardinal points?

3. A line of an old survey is recorded as N. 18° E. mag. bear. It now reads N. 16° 30′ E. What has been the change in declination in direction and amount?

4. A line of an old survey is recorded as N. 36° 15′ E. mag. bear. It now reads N. 34° 30′ E., and the magnetic declination is now 10° 30′ W. What was the declination at the time of the original survey?

5. A line of an old survey is recorded as N. 36° 15′ E. mag. bear., and the declination is recorded as having been 10° 30′ W. at the time of the survey. The declination is now 12° 00′ W. What magnetic bearing should the line now show?

6. A line of an old survey is recorded as S. 26° 00′ E. mag. bear., declination 10° 30′ W. The declination is now 9° W. What magnetic bearing should be used to retrace the line?

7. The magnetic declination is 10° 30′ E. If the declination vernier is attached to the south side of the compass box, in what direction and by what amount should it be moved so that true bearings may be read by the needle? With sights pointed to the magnetic north, what would the needle read after moving the vernier?

8. A line of an old survey is recorded as N. 30° 15′ E. mag. bear. The declination is now 4° W., and the same line reads N. 30° E. It is desired to set the declination vernier so that the remainder of the survey may be retraced by the recorded bearings. The vernier being attached to the south side of the compass box, what is its movement in amount and direction?

9. Suppose, in the above example, the former bearing had been S. 26° W., and the present bearing S. 26° 30′ W. What would be the movement of the vernier?

10. Is it necessary in the foregoing examples to know the present declination?

COMPASS ADJUSTMENTS

79. Requirements mentioned. To determine the direction of lines properly with the compass, it is necessary

1. That the plate be level;

2. That the sights be vertical, with their plane passing through the zeros of the graduated box when the zeros of the vernier and arc *A* coincide;

3. That the needle be straight;
4. That the pivot be in the center of the circle.

80. Plate bubbles. The instrument is leveled readily if the bubbles are parallel to the plate and the plate perpendicular to the vertical axis. Test the parallelism of the bubbles and plate by the method of Art. 29, and correct, if necessary, by the screws already mentioned. To do this the plate is leveled and reversed on the vertical axis; if either bubble leaves the center of its tube, it is brought halfway back by the bubble-adjusting screws, and the plate is releveled and reversed for a check. The first attempt is not likely to be successful, and the operation is repeated until the adjustment is made. It is assumed that the plate is perpendicular to the vertical axis. If not, it must be made so by the maker.

81. Sights. To test the perpendicularity of the sights, turn the leveled instrument on a suspended plumb line and see whether the line traverses the slits in the standards, turning each end separately to the plumb line. If the sights are found to be *out of plumb*, they may be adjusted by removing them and filing the bottoms till they are found to be correct. With proper care they should not get out, and this is not a common error. Strips of paper may be inserted under one edge, if the surveyor prefers this to filing.

To determine whether the plane of the sights includes the line of the zeros when the vernier is set to read zero, stretch two fine threads through the slits in the sights, and, looking down, see whether the plane of the threads includes the line of zeros of the graduated box. If not, the error should be corrected by the maker. The error may be allowed for, however, by moving the vernier till the zeros are in the plane of the threads, or parallel to it, noting the angle that has been turned, and seeing that thereafter the vernier is set at this angle when working. If the line of sights fails to pass through the center by one tenth of an inch, the error in the direction of a line ten feet long will be about three minutes, and will be less as the line is longer.

82. Needle and pivot. If the needle is straight and the pivot is in the center, the two ends will read 180° apart. If both of these conditions are not fulfilled, the two ends will

read other than 180° apart. If it is observed that one of these faults exists, the instrument should first be tested to determine which one. This is done by turning the plates on the vertical axis and noting whether the difference in the two end readings remains constant: if so, the needle is bent; if not, the pivot is not in the center, and the needle may or may not be bent. If the needle is found to be bent, remove the glass cover from the box by unscrewing it, take the needle from the pivot, and carefully bend it till straight. If the pivot is found to be out of center, remove the needle and carefully bend the top of the pivot over until it is in the center. To determine which way to bend it, turn the box till that position of the box is found that gives the greatest error in end readings of the needle, and bend the pivot at right angles to the position of the needle.[1]

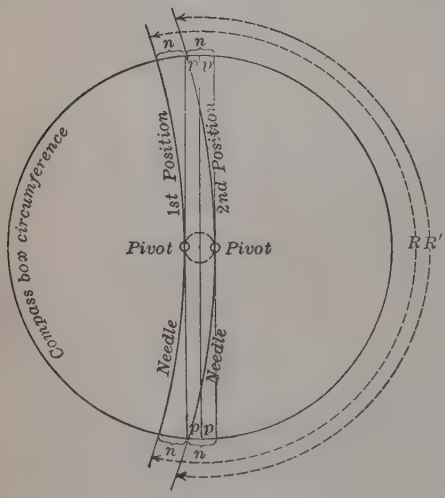


FIG. 65.

If there is reason to think that both errors are involved, turn the instrument as before to get the maximum error of end readings, and read the angle greater than 180° between the ends, calling this R. Now revolve the instrument 180°, in the way described below, and read the angle on the same side of the needle, calling this reading R'. Let p be half of the error due to eccentricity of pivot, and n be half of the error due to bent needle; then, from Fig. 65 it is seen that

$$180° + (2p + 2n) = R; \qquad (1)$$

$$180° - (2p - 2n) = R'. \qquad (2)$$

[1] The student should show the correctness of these statements by a diagram.

Solving for p by subtraction, and for n by addition, there results

$$p = \frac{R - R'}{4};$$

$$n = \frac{R + R' - 360}{4}.$$

Bend the needle so that each end moves over n degrees. Then bend the pivot at right angles to the needle, toward the center, till the needle ends read 180° apart.

The instrument may be revolved 180° by noting an object to which it points when first set and then revolving till the reversed sights point to the same object.

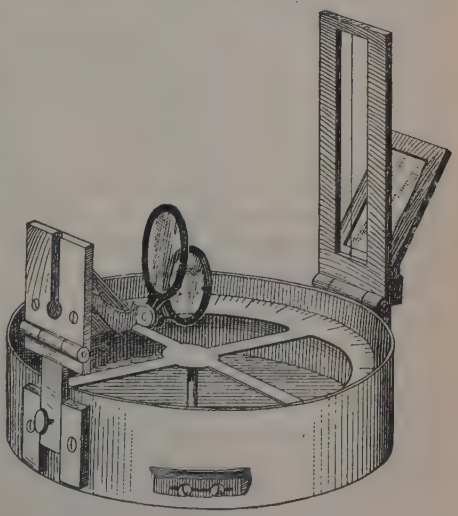


FIG. 66.

83. Special forms of compasses. Some compasses are made with an additional full circle of 360° and a vernier reading to minutes, with suitable clamps and tangent screws; but it may be said that the precision obtained in pointing with the open sights is hardly sufficient to warrant this. If angles are required to be read to single minutes, it is better to use a transit.

The prismatic compass is a very convenient instrument to use on exploratory work. It has folding sights, and, by means of the prism, may be read while being pointed, which is a convenience when the instrument is held in the hand instead of being mounted on a Jacob's staff or tripod. It is usually held in the hand, though it may be used either way. The instrument is shown in Fig. 66.

THE TRANSIT

84. Description. The instrument most used by surveyors and engineers for measuring horizontal angles, and, with certain attachments, for measuring vertical angles and distance, and for leveling, is called a *transit*.

The transit consists of a telescope attached to a vernier which may be moved on a vertical axis around a horizontal graduated circle. There are suitable attachments for controlling the motion of the telescope and the vernier, and for enabling the graduated circle to be made horizontal. The vernier may be clamped to the graduated circle, and they may be revolved together. If this is done, and the telescope is pointed to a given object, and the graduated circle is clamped so that it will not revolve, and if the vernier is then unclamped from the circle, the vernier and telescope may be turned together till the telescope points to a second object. The number of degrees of the circle over which the vernier has passed will be the angle subtended, at the point where the transit is placed, by the two objects seen through the telescope. There are usually two double verniers placed 180° apart.

In the transit shown in Fig. 67, T is the telescope which may be revolved in a vertical plane on a horizontal axis.

The horizontal axis rests in bearings at the top of the standards X, which are rigidly attached to the circular plate that carries the vernier V.

Two level tubes are set on this plate at right angles. When adjusted to be parallel to the plate, these tubes will show whether the plate is level. This plate is made perpendicular to the axis on which it revolves by the maker, and hence, when the plate is level, the axis of revolution is vertical. This axis is conical, and fits inside a conical socket which is the inside of the conical axis of the plate that carries the graduated circle. This latter conical axis revolves in a socket that connects the top and bottom parts of the leveling head. The upper part is leveled by the leveling screws. The lower part of this leveling head screws on the tripod. The clamp C fastens together the vernier plate (sometimes called the *alidade*) and the plate carrying the graduated circle. The latter plate is called the *limb*. When the two plates are clamped together, the vernier plate may still be moved a small amount, relatively to the limb, by means of

the tangent screw *S*. This is for the purpose of setting the vernier at a given reading, or pointing the telescope at a point

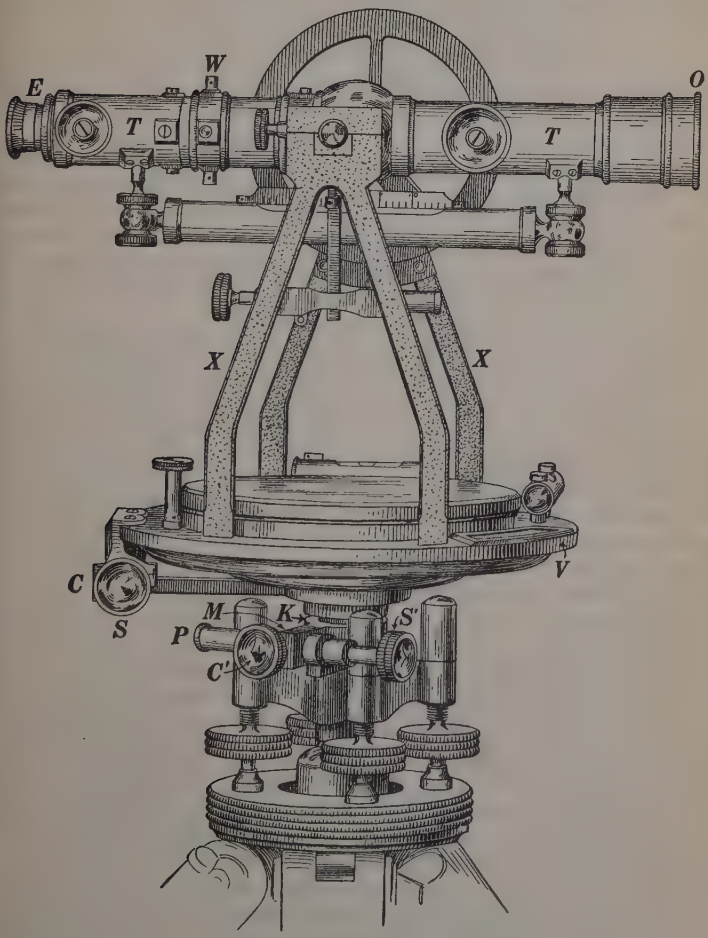


Fig. 67.

more precisely than would ordinarily be possible by simply turning the alidade by the hand. The collar K surrounds the

spindle of the limb, and by the clamp screw C' may be fastened to that spindle. The lug M, which is attached to the collar, being held by the spring in the barrel P and the opposing screw S', in turn fastened to the leveling head, prevents the revolution of the limb, when the collar is clamped to its axis. The limb may, however, be moved a small amount by the tangent screw S' working against the pressure of the spring in the barrel P.

Some instruments, when put away in the box in which they are kept when not in use, are separated into two parts. The upper part of the instrument is separated from the leveling head, and the two parts are placed separately in the box. Others are so made as to be put away in one piece, being merely unscrewed from the tripod and screwed to a board that slides into the box.

There are various forms of transits and various patterns for the different parts, according to the ideas of the different makers. A careful examination of an instrument should be made before putting it to use, that the user may become perfectly familiar with its construction.

The circles are usually so graduated that angles may be read to minutes. Many instruments are graduated to read 30 seconds, some to read 20 seconds, and a few to read 10 seconds. For ordinary land surveying, single minutes are sufficiently precise, while for close city surveying 20 seconds is not too fine.

The author prefers instruments graduated to read to $\frac{1}{100}$ of a degree for ordinary work and to $\frac{1}{200}$ of a degree for fine work, and suggests the gradual abandonment of minutes and seconds in surveying work. While it is true that all angle measurements have been made heretofore on the sexagesimal system, it seems to be time for surveyors to break away from this as they have long ago broken away from the Gunter's chain for engineering work and city surveys, and from the duodecimal system of linear measurement for all engineering surveys, retaining it only in connection with structural work.

There is not the objection to this change that has prevented the general adoption of the metric system in English-speaking countries, nor the adoption of the centesimal system for angle measurements which divides the quadrant into one hundred degrees and the circle into four hundred degrees. The adoption of those measures means changing the unit, while to divide the

degree decimally merely changes the division of the unit from an awkward sexagesimal division to a simple, natural, decimal division. One objection to such a change has been a lack of tables of functions based on a decimal division of the degree; for, although there have been several German tables, none have appeared in the American or English handbooks.[1]

The cost of changing instruments to read in this way when only the vernier must be changed is small, and the cost of a new instrument so graduated is no more than for graduation to minutes or thirty seconds. It is hoped, therefore, that the appearance of decimal tables will open the way for the general adoption of the decimal division of the degree for all angle measurements in surveying.

The telescope is essentially the same as that used in the level, but is shorter and usually of lower magnifying power. It should be inverting, but is usually an erecting glass. The use of the inverting glass is growing. There is a proper relation between the magnifying power of the telescope and the least count of the vernier used; and it is not necessarily the best instrument that has the smallest count vernier, for the reason that the magnifying power of the telescope may be such that a movement of several of the smallest units that can be read by the vernier may give no perceptible lateral motion to the line of sight. The power of the telescope and the least count of the vernier should be so adjusted to each other that a barely perceptible movement of the vernier will cause a barely perceptible movement of the line of sight. The same is true of the magnifying power and the level under the telescope. Methods of testing these relations will suggest themselves.

The line of sight used in angle work is defined by the optical center of the objective and the vertical wire in the telescope, and this line of sight should coincide with the line of collimation, which is the optical axis of the objective, and should be perpendicular to the horizontal axis of the telescope.

The verniers are usually read with small magnifying glasses known as *reading glasses*.

In order that the transit may be set precisely over a point on the ground, a ring or hook is fastened to the center of the lower part of the leveling head from which ring a plumb line

[1] It is expected that this objection will be removed very shortly by the publication of the author's "Field Book for Engineers" which will be based on the decimal division of the degree.

may be suspended. A sectional view of the lower part of the transit in Fig. 67 is shown in Fig. 68. In this transit there is a cap with a small hole in the center of its top shown at P; the cap being removed, the plumb line is passed through the small hole and the end knotted. The cap is then screwed on. It will be seen that the upper leveling plate — in some forms four arms instead of a plate — is attached to the socket in which the vertical axis revolves, and that the nuts in which the leveling screws work are attached to this upper plate. These leveling

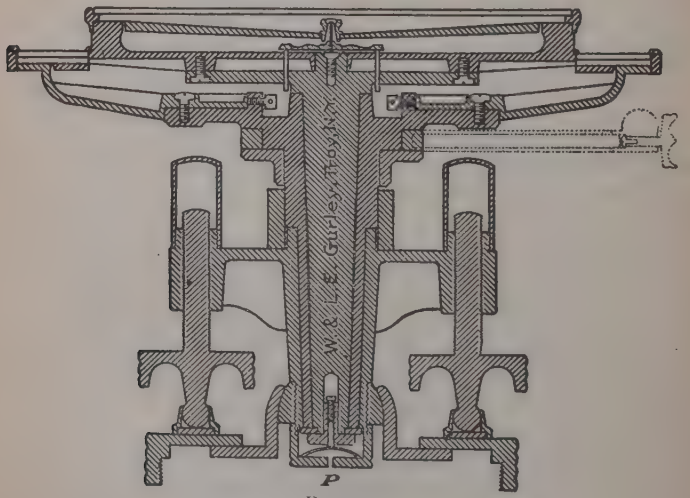


FIG. 68.

screws rest in little cups, that in turn rest on the lower leveling plate. The ball-and-socket joint that permits the leveling of the instrument is separate from the lower leveling plate, and is not quite so large as the hole in the center of that plate. There is an extension plate to this ball-and-socket joint that extends under the lower leveling plate. When the leveling screws are tightened, this extension plate binds against the leveling plate, and the instrument cannot be shifted on the lower plate. If the leveling screws are loosened, the upper part of the instrument may be shifted laterally on the lower plate by an amount de-

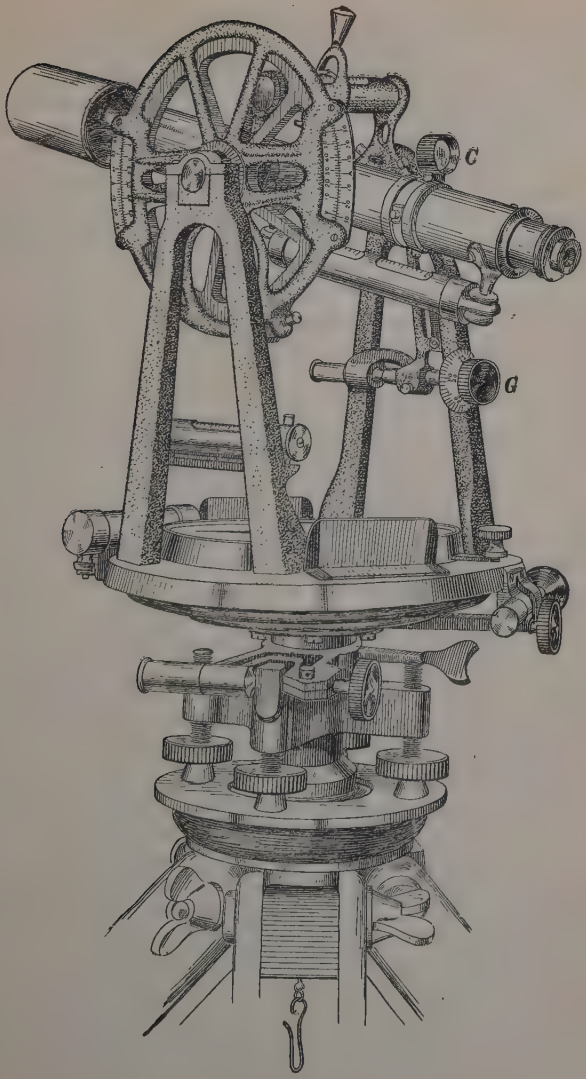


FIG. 69.

pending on the relative diameters of the ball joint and the hole in the center of the lower plate. The device is called a *shifting center* and is found in all modern transits. A sectional view of one make of transit with the various parts named is shown in the frontispiece.

85. The tachymeter. Such a transit as has been described, while still in very general use, is being rapidly displaced by the complete engineer's transit, or tachymeter. This instrument consists of such a transit as has been described, with the following attachments:

1. A level under the telescope, making the transit a good leveling instrument.
2. A vertical circle or arc, by which vertical angles may be read.
3. A gradienter attachment by which very small vertical angles may be set off, as so many feet rise or fall in one hundred.
4. A pair of horizontal wires in the telescope, known as stadia wires.

There are various other attachments for special purposes that will be found described in instrument makers' catalogues, but which it is not necessary to mention here. There are also special forms of transits for use underground. One form of the complete instrument is shown in Fig. 69, page 125.

The level under the telescope and the vertical circle are readily recognized. The gradienter and slow-motion screw, for vertical motion of telescope, is shown at *G*. The telescope is clamped in vertical motion by the clamp *C*. It may then be moved slowly a small amount by the slow-motion screw. The head of this screw is graduated with reference to the pitch of the screw, so that a single revolution of the head corresponds to an angular motion of the telescope of one foot in one hundred, or half a foot in one hundred. In the figure, the head is graduated in the latter way, as is indicated by the double row of figures. The chief use of the gradienter is in setting out grades. Many transits are fitted with the slow-motion screw without the gradienter head. All makers make both the plain and complete transits.

The tachymeter shown in Fig. 69 has a transverse level on the horizontal axis of the telescope to show the horizontality of that axis. The vertical circle has a metal guard carrying two double verniers.

USE OF THE TRANSIT

86. Carrying the transit. While in use, the transit is not removed from the tripod when it is carried from one point to another; but is carried with the tripod on the shoulder of the "transitman." The plumb bob is carried in the pocket without removing the string from the instrument. Before the transit can be properly set up and the angles measured, the various parts must be in proper adjustment. The discussion of the adjustments will, however, be deferred till the use of the adjusted transit has been described.

87. "Setting up" the transit. To "set up" the transit over a point, let the plummet swing free, grasp the uppermost leg of the tripod in one hand, and plant it firmly in the ground at about the right distance from the point, learned by experience. Next grasp the other two legs, one in each hand, and place them symmetrically about the point so that the plumb bob covers the point as nearly as may be. The plates should be observed at the same time and made approximately level. It should be noticed that a sidewise motion of a leg changes the level of the plates without much disturbing the plumb bob, and that a radial motion changes the level of the plates to a less degree, but does disturb the plumb bob. After the transit is set approximately over the point, press the legs firmly into the ground so as to bring the bob more nearly over the point and to make the instrument firmer. Finish centering by the adjustable or shifting center, by loosening two adjacent leveling screws and moving the whole upper part of the instrument till it is centered. If there is no adjustable center, the instrument must be centered by manipulating the legs.

An expert transitman never sets his instrument up near the point to get the leveling head level, moving it bodily afterwards over the point. On level ground he sets the three legs the first time so that the leveling head is fairly level, and only a reasonable pressure on each leg is required to bring the bob almost exactly over the point. On a side hill a little more trouble is experienced, but in the most cramped position the expert transitman sets his instrument but once. On a side hill one leg should always go up hill and two down. The beginner should understand that he has not made a good set-up unless his plates are approximately

level, and he should secure this result even if at first he finds it necessary to move the whole instrument.

When the instrument is finally centered, level the limb by the leveling screws. To do this, place one plate bubble parallel to one set of opposite leveling screws; then the other bubble will be parallel to the other set. Level one bubble at a time, and note that leveling one will, to a slight extent, disturb the other, which must then be leveled again. If the lower plate is not fairly level, "leveling up" will disturb the centering of the plumb bob, which must then be corrected. Continue till both bubbles are in the centers of the tubes. In using the leveling screws care must be taken not to bind them. If one set begins to bind, the other set should be loosened. The screws should always be set so that there is no lost motion, but also so that they are not tight. If this precaution is not observed, the screws soon wear so that they are unreliable. Opposite screws are turned at the same time, both thumbs in or out as the case may be. The beginner should practice until unconsciously he turns the screws the right way to move the bubble in the required direction.

Set the vernier that is to be used at the proper reading (this will often be zero), and the instrument is set up. To set the vernier, loosen the clamp that holds the limb and vernier together and move the two with the hands until the proper reading is almost obtained, as seen by the unassisted eye; fasten the clamp firmly but not hard, and finish the setting with the slow motion screw, using a reading glass. Some keen eyes hardly need a reading glass for minute reading instruments, but it is better to use one. In completing the setting with the slow-motion screw care should be taken not to move the vernier a little too far and then set it back with a reverse motion of the screw. Should this accidentally occur, it should be set back by a turn or so of the screw and again brought forward to avoid the effect of lost motion.

It is necessary to become expert in setting the transit, as much time may be lost by awkward work in setting up.

Exercises

1. Set up a transit over a point on fairly level ground noticing the effects on the leveling head and plumb bob of different motions of the leg—*i.e.*, radial and circumferential — and then set up several times over the same

point, changing the positions of the legs each time and noting the time required for the complete operation.

2. Set up over several points on a fairly steep hillside, and note the time required.

88. To produce a straight line with the transit. Set the instrument over one extremity of the line, and with the limb free to turn, turn the telescope on the other extremity or on a point in the line. Clamp all motions except the vertical motion of the telescope, and, with either of the horizontal tangent screws, but preferably with the lower screw, bring the line of sight to bisect exactly the distant point. Transit the telescope, and set a point beyond it at any desired place in the line of sight. This point will, if the instrument is in adjustment, be in the straight line produced. If it is feared that the transit is slightly out of adjustment, unclamp the limb, turn the instrument till the line of sight again cuts the first point, clamp and set precisely, and then transit and set a new point beside the first one set; the point midway between the two set will be a point in the continuation of the straight line. This is called *double centering* and is the same operation as the test of the adjustment of the vertical wire, to be hereafter explained. The method of ranging out a number of points in a straight line in the same direction will be apparent without explanation.

Exercises

1. Set up a transit where a clear sight of at least a quarter of a mile may be had, preferably on a cool day in a grassy field. Using an ordinary surveyor's range pole and with all horizontal motions clamped, set a stake in the line of sight at a distance of 100 feet and center it for a tack. Without changing the circles set and center stakes at distances of 500, 1000, and 1500 feet. Without changing the circles again, set the flag in line for a tack at the 100 feet stake, the 500, 1000, and 1500 feet stakes, and note the variations from the first settings. The stakes should be driven flush with the ground so that they are not seen from the instrument. They may be quite short but should be fairly wide across the line of sight so that no new ones need be driven at the distant points. Perform the operation several times and determine the probable error of a single setting at each distance. Perform the same operation on a hot day in the same field and then on a paved city street.

2. Set up over a point A in a fairly level stretch. Set a point B from 200 to 400 feet distant and, with this as a backsight, prolong the line BA to a

point C some 200 to 400 feet distant by transiting the telescope. Double center and check the work, finding the true prolongation if the first is found to be in error.

3. Having determined the true line, set the transit successively $\frac{1}{8}$, $\frac{1}{2}$, and 1 inch to one side of a point and with B for a backsight prolong the line AB to points C', C'', C''', opposite C, double centering for exactness. Note the errors at C and compute the angular errors of prolongation, first the angles CBC', CBC'', CBC''', second the angles CAC', CAC'', CAC'''. Note that these last angles are the angles that would be made in the prolongation of the line AC if correctly prolonged from one of the erroneously set points at C.

89. To measure an angle with the transit. Before using the transit, the limb or circle and vernier should be examined to familiarize the user with the method of graduation and the least count of the vernier. Set the instrument over the vertex of the angle, with the vernier at zero. With the lower motion, *i.e.*, the limb clamped by the lower clamp and hence called lower motion, bring the line of sight to bear approximately on one of the distant points, clamp, and with the lower tangent screw make an exact bisection. Loosen the alidade and bring the line of sight approximately to the second point, clamp, and complete the bisection with the upper tangent screw. Read the vernier. The reading will be the angle sought. This assumes the numbering on the limb to extend both ways from 0° to 360°. It is, of course, unnecessary to set the vernier at zero. When the instrument has been set on the first point, a reading may be taken and noted, and subtracted from the reading found on pointing the telescope at the second point. In this case the vernier may pass the 360° point, and it will then be necessary to add 360° to the final reading before subtracting the first.

Exercises

1. (*a*) Set up over a well-defined point and measure the angles between several other permanent and well-defined points as church spires, flagpoles, lightning rods, corners of buildings, etc., beginning the series at the leftmost point. Let the sum of all the angles be less than 360°; *i.e.*, do not close the circle. Measure each angle three times as follows: with the vernier at zero set the line of sight on the left point using the lower motion, clamp, and tangent screw; loosen the alidade, turn to the right point, and set carefully with the upper clamp and slow-motion screw; read the vernier and record the angle; loosen the lower clamp and with lower motion, clamp, and

tangent screw, set again on the left point; loosen the alidade and set on the right point *without reading the angle;* loosen below, and again set on the left point; loosen above, and again set on the right point and read the vernier. The final reading should be about three times the first if the work has been correctly done. If $\frac{1}{3}$ the final reading equals the first angle within less than the least count of the vernier, the work may be regarded as satisfactory. If not, it should be repeated. Proceed with each angle similarly, and finally read the whole angle that includes all the smaller angles in the same way, and compare the value found with the sum of the several smaller angles, in each case using $\frac{1}{3}$ of the third reading for the value of the angle. The plates should be kept level but should not be releveled during one measurement of an angle.

(*b*) Invert the telescope and read the same series of angles beginning at the rightmost point and reading from right to left. Compare all of the results with the corresponding results of the first measurements and, if they differ at all, take the means of the two values of each angle and compare the sum of the means of the smaller angles with the mean of the two values of the whole angle. The means will be nearest to the true values.

(*c*) Repeat the examples 1 (*a*) and 1 (*b*) twice to get three complete records, and compare the several means of the three complete records with the corresponding mean values of each complete record.

2. (*a*) Proceed as in 1 (*a*) using the same *number* of angles, but close the circle making the sum of the angles 360°, and compare the difference between the sum of the angles and 360° with the corresponding difference obtained in 1 (*a*).

(*b*) Proceed as in 1 (*b*) modified as in 2 (*a*), and compare the results.

(*c*) Proceed as in 1 (*c*) modified as in 2 (*a*), and compare the results.

3. (*a*) With a minute reading instrument set over any point at random, carefully set the vernier at zero, and with the lower motion bring the line of sight to bisect a minute distant point; with the upper motion set off an angle of 28° 33′ or other convenient fractional degree but whole minute angle and carefully set and center a stake about 200 or 300 feet away. Loosen both motions, again set the vernier at zero, set the line of sight on the distant point with the lower motion, and again set off the angle previously measured and set and center a point beside the first one set; perform this operation 3 or 5 times and determine the probable error of a single measurement. This exercise should give some idea of the precision with which an angle may be laid out.

(*b*) Perform the same operation with 30″ and 20″ reading instruments if they are available. Compare the several precisions.

90. Errors in angle measurement. Errors in angle measurement may be due to:

1. Mistakes in reading;
2. Errors of adjustment of instrument;

3. Errors in setting the vernier or the line of sight due to personal peculiarity of the surveyor who always sets a mark a little to the left or right, or who is unable to distinguish fine differences because of lack of sharpness of vision;

4. Atmospheric disturbances, the air "dances" (heated air currents), or is hazy or smoky;

5. The effect of the sun on the instrument, producing unequal expansion of the parts;

6. Coarseness of graduations. (No matter how finely an instrument is graduated, it is unlikely that any angle at random will be exactly a whole number of the least count divisions of the vernier and hence the angle will be read too large or too small.)

Mistakes are as likely to be one way as another and can be eliminated or discovered only by great care and repeating measurements. Errors of adjustment are constant or cumulative and can be eliminated by reading angles in opposite directions with telescope direct and inverted. "Personal equation" errors are likely to be small and to be partly or wholly eliminated by opposite direction readings. Atmospheric disturbances cause accidental errors that are likely to be partially compensating. Inability to read the angle exactly, because it is not a whole number of the least count division of the vernier, causes errors as likely to be too small as too large and there will be a tendency to read the division nearest to the truth. In fine work this error is partly eliminated by measuring an angle a number of times and dividing the final reading by the number of measurements. Precision in sighting depends on magnifying power of the telescope, atmosphere, and pitch of the slow motion screws of the alidade and limb.

If an angle is measured several times, and read each time, all of the readings should agree to the least count of the vernier and any reading will be apparently as good as the mean which will be the same as any reading; but if several measurements be made as described in exercise 1 (*a*) of the preceding article, the final reading divided by the number of measurements will generally give a value differing from any single reading by less than the least count of the vernier, and probably more precise than any single reading.

Assuming that mistakes and errors of adjustment are eliminated, any given angle measurement is likely to be correct to the nearest 30″ with a minute reading instrument — that is, it is

not likely to be in error by more than 30″ — to the nearest 15″ with a 30″ reading instrument and to the nearest 10″ with a 20″ reading instrument, to the nearest 200th degree (= 18″) with a 100th degree reading instrument and to the nearest 400th degree (= 09″) with a 200th degree reading instrument.

In the exercises of the preceding article a single triple measurement of an angle in one direction does not eliminate errors of adjustment, hence it could not be used as a complete determination of an angle in a least squares discussion of the precision of the mean of several measurements. The values to be used as complete determinations would be the mean values found in 1 (*b*) after reading in opposite directions with telescope direct and inverted.

Considering only accidental and compensating errors, the probable error of a single complete determination of any angle determined n times is $e_1 = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$. In this equation v is the difference between any single complete determination and the mean of all the complete determinations. The probable error of the mean of n complete determinations of a single angle is

$$e = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}} = \frac{e_1}{\sqrt{n}}$$

and the probable error of the sum of m angles independently determined is $E_m = \sqrt{e_1^2 + e_2^2 + e_3^2 + \cdots + e_m^2} = \sqrt{\Sigma e^2}$. If the sum of the several angles must by the nature of the case be some fixed quantity — as when measuring several angles at a single point and closing the circle, the sum must be 360° — the error is the discrepancy between the sum of the measured angles and the fixed quantity that it should equal. In such a case, the discrepancy is divided equally among the several angles, if each was determined but once, since there is no reason to suppose one was measured more correctly than another. If each was fully determined more than once and the several probable errors computed, the error would be distributed in proportion to the square roots of the probable errors of the angles. Thus, if the total error is S seconds, the several probable errors, e_1, e_2, e_3, etc., the correction to angle 1 is $\frac{S\sqrt{e_1}}{\Sigma\sqrt{e}}$, to angle 2, $\frac{S\sqrt{e_2}}{\Sigma\sqrt{e}}$, etc.

When only a part of the circle is completed and the sum of the several angles is itself measured as a whole angle, there is no reason for distributing any error since the precise value of the sum is not known, but to make the work appear consistent:

1. The mean value of the measured whole angle may be assumed to be correct and the discrepancy between this and the sum of the several angles distributed equally between the angles;

2. The discrepancy may be distributed among all the angles including the measured whole angle, the sign of the correction to the whole angle being opposite to those of the several angles;

3. If each angle has been completely determined more than once and its probable error computed, the discrepancy may be distributed among all the angles, including the whole angle, in proportion to the square roots of the respective probable errors. If there is but a single complete determination of each angle, there is no reason to suppose one has been better measured than another and the discrepancy should be equally divided among *all* the angles.

Exercises and Examples

1. Tabulate the errors in position of points distant 100, 200, 500, and 1000 feet due to angular errors of 0° 01′, 00′ 30″, 00′ 20″, 00′ 10″, respectively.

2. In Fig. 70 the lines AC and CD are determined and assumed to be correct and each 400 feet long. B is determined by measuring the angles at A and D. If B must be so determined that the precision of determination of sides AB and BD shall be $\frac{1}{10000}$, what is the maximum error that may be made in the angles at A and D? (If both angles are in error, the discrepancy will be slightly more than if only one is in error; this extra effect may be neglected.) On reasonably level or uniformly sloping ground is it likely that the location of B will be better determined by linear or angular measurements?

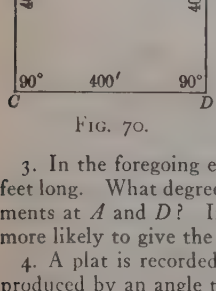


Fig. 70.

3. In the foregoing example suppose that the sides AC and BD are 1200 feet long. What degree of precision must be reached in the angular measurements at A and D? In this case would linear or angular measurements be more likely to give the required precision?

4. A plat is recorded in part as in Fig. 71. If D is to be located on AE produced by an angle turned at C: (*a*) How much out of place on the line AE will D be if an error of 0° 01′ is made at C? (*b*) What is the largest

error permissible in the angle at C if ED is to be determined within 1 in 10000? (*c*) Why is this different from the precision required in the preceding examples?

5. By least squares discussion determine the probable error of a single complete determination of each angle of those measured for Exercise 1 of Art. 89; of the mean of the three determinations of each angle; of the sum of the mean values for the three determinations, and compare the last with the probable error of the mean of the three measurements of the whole angle.

91. Azimuth. The *azimuth* of a line is the horizontal angle the line makes with a reference line, as a meridian. It differs from bearing, in that it is measured continuously from 0° to 360°. If the azimuth of a point is mentioned, there is implied another point and a meridian through it from which the azimuth is measured.

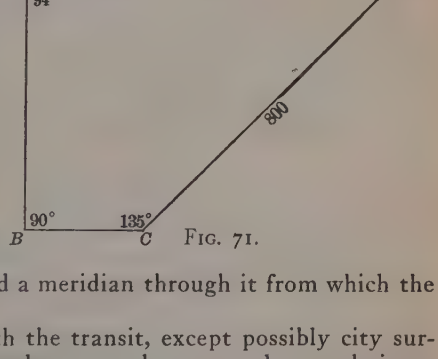


FIG. 71.

In making surveys with the transit, except possibly city surveys and others where angles are to be repeated several times, it is better to use azimuths than bearings. It is necessary, however, in writing descriptions of property, to reduce the azimuths to bearings simply to conform to long-established usage with which lawyers are familiar.

It is customary to reckon azimuths from the south point around by way of the west, north, east, south, 360°. This is the practice of astronomers and geodesists. It is believed to be more convenient for the surveyor to begin with zero at the north point and read to the right 360°. The reason for this will appear in Chapter V.

As in bearings, there is "forward" azimuth and "back" azimuth, referring, as in bearings, to the direction of progress of the survey. The "back" azimuth of a line will always differ from its "forward" azimuth by 180°. To find it conveniently, 180° is added when the forward azimuth is less than 180° and subtracted when the forward azimuth is more than 180°.

Examples

1. A traverse is run with the following azimuths, zero azimuth being north: 42°, 144°, 181°, 230°. What are the bearings?

2. Another is run with the following azimuths: 306° 15′, 18° 34′, 93° 15′, 200° 30′. What are the bearings?

3. What are the azimuths of the following lines of Example 1, Art. 71: *AC*, *AE*, *AG*, *AI*?

4. Determine the exterior or deflection angles in Example 2.

92. Traversing. The method of traversing with the transit is as follows: In Fig. 72, let it be required to determine the lengths and azimuths of the courses of the crooked road, *A*, *B*, *C*, *D*, etc. It will be assumed that the magnetic meridian *SN* is the meridian of the survey. Set the transit over a tack in a stake driven at *A*, and, with the vernier set at zero, turn the

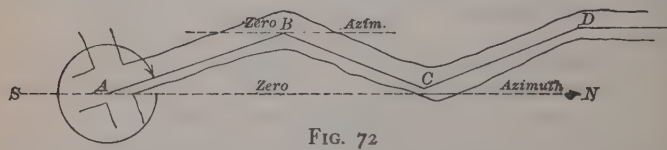


Fig. 72

telescope by the lower motion in the direction *AN* as defined by the needle, and clamp the lower motion. For the purpose of using this line again, if necessary, set a stake in the line *AN*, some distance toward *N*, and put a tack in it in the exact line. This is necessary because the needle would not give the direction twice alike to the nearest minute, which is desired in transit work. Unclamp the alidade and turn the telescope toward a point in the first turn of the road, as *B*. Clamp the alidade and set a stake at *B*. In most transit work all stakes marking stations to be occupied by the transit are " centered " by setting a tack in the precise line. Read and record the vernier and the needle. They should agree within the limit of precision of the needle reading. The angle read is that which has been turned to the right, and in the figure is greater than 270°. The mistake of reading the wrong circle must be avoided. The needle reading may be recorded as bearing, or may be mentally reduced to azimuth and so recorded. If the compass box is graduated continuously 360°, the needle will give azimuths at once.

Measure *AB*, and, while this is being done, take the transit to *B* and set up over the tack that has been put in the stake. With the same vernier used at *A* (let it be called vernier *A*) bring the instrument to read the azimuth *AB* minus 180°, which is the "back azimuth" of *AB*. Point the telescope to *A* by using the lower motion, clamp, and make the exact pointing with the lower tangent screw. By so doing the instrument is *oriented;* that is, the zeros of the limb are made parallel to their positions at *A*. (The telescope has not been transited, and, if the alidade is now unclamped and the telescope turned in azimuth just 180°, the line of sight will be in *AB* prolonged, the vernier will read the azimuth of *AB*, and the zeros of the limb will be seen to lie in the meridian of the survey. This intermediate step is not usually taken separately in practice, but is introduced here for the clearer understanding of the method.)

With the alidade unclamped, turn the telescope in the direction of the next point, as *C*. Clamp the alidade, set a stake and tack at *C*, and read and record the vernier and needle. The reading of the vernier will be the azimuth of *BC* referred to *AN*, and the needle should read the same or the corresponding bearing. Measure *BC*, take the transit to *C*, set the vernier at the back azimuth of *BC*, and proceed as at *B*. Continue the work thus until the traverse is complete.

Many engineers measure the deflection angles instead of the azimuths, and in railroad surveying this is still quite common. The better way is to use azimuths from which deflection angles may be had if desired. In running by azimuths, the telescope is not transited, hence the effect of error of collimation adjustment is eliminated. If deflection angles are used, every setting should be double centered.

In making a traverse of a closed field, the interior angles may be measured, thus avoiding the errors likely to result from transiting the telescope, as in measuring deflection angles, and making it easy to double all angles for check.

93. Notes. The form for the field notes of the survey just outlined is the same as for compass traverse (see page 104), except that azimuths instead of bearings are recorded, and one or two extra columns are used in which to record the needle readings and calculated bearings as a future evidence of the correctness

of the work. The notes for the survey of the preceding paragraph are shown in Fig. 73.

The best method for traversing a road or other long line not the boundary of a field, lot, or farm, is to adopt the stationing of railroad surveys. In these surveys each chain or tape length is a station, the tape length or the stake marking its end being indiscriminately called a station, the context usually showing which is meant. In traversing, the stake may be called the station, the initial point numbered zero, the end of the first tape

Sta.	Azimuth Back	Azimuth For'd	Distance Feet	Bearing Mag.	Bearing Calcul'd
		&c.	&c.	&c.	
D	146-17	358-20		N1°45'W	N1°40'W
C	211-26	326-17	763.8	N33°45'W	N33°43'W
B	158-42	31-26	686.7	N31°30'E	N31°26'E
A	0-00	338.42	742.4	N21°15'W	N21°18'W

Note.- The calculated bearing is determined by changing the Azimuth to Bearing.

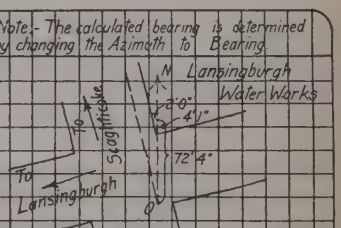
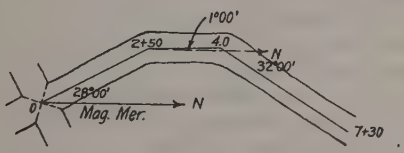

N
Lansingburgh
Water Works
4°0'
4½'
To
Scaghticoke
72°4'
To
Lansingburgh
0
Sta. 0 is 10ft to the E of the center of the Scaghticoke road as defined by the fences and in the S. line of the Lansingburgh - Brunswick road, and is a 2"x2" stake driven flush with the ground. Zero Azimuth is Magnetic North and is determined by a stake set just inside the fence of the Lansingburgh W.W. property as shown.

FIG. 73.

length or 100 feet, one, etc. If the length of the first course is 250 feet, the stake marking its end, which is to be used as a transit point, is numbered 2 + 50, the next measurement on the second course is 50 feet to station 3, etc. The number of any station indicates the distance in stations of one tape length from the beginning, and if, as is usual, the tape is 100 feet, the number of any station is the distance from the beginning in hundreds of feet. The stake set for a transit point will be a hub driven flush with the ground, centered with a tack or not as the precision of the survey requires, and marked by a guard stake, set far enough to one side to be out of the way of traffic, or marked by surveyor's crayon on the roadside fence. The

guard stake should be marked with the number of the station, reading from the top down facing the hub.

If side distances are required to locate objects, as the fence lines of the road or the shore line of a pond or stream in another case, they will be taken at regular intervals along the traverse, or at irregular intervals as may be necessary to get the required information.



Sta.	Azimuth Back	Azimuth For'd	Needle Read	Needle Calculated
6	& c			
5				
4	179°00'	32° 00'	N32°05'E	N32°00'E
3				
+ 50	152°00'	359°00'	NO°50'W	N1°00'W
2				
1				
0	0°00'	332°00'	N28°00'W	N28°00'W

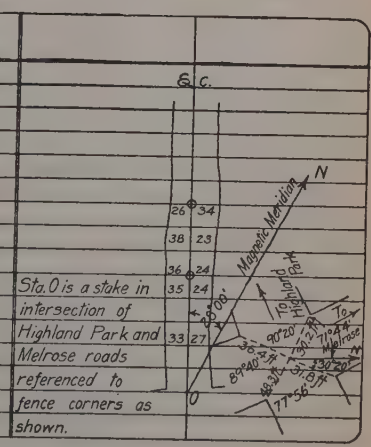


FIG. 74.

The notes may be kept as shown in Fig. 74 for the survey indicated in the upper part of the figure. The red line usually found in the center of the right-hand page of field notebooks may be regarded as the line being run, and for convenience in orienting one's self, the notes will run up the page from the bottom. The sketch on the right-hand page should be full and complete. It will appear distorted because the line being run has angles, while the red line representing it has none. Instead

of angling the line, the features sketched will be angled out of place. A ruled space to a station or tape length may be adopted for a rough scale in sketching. The stations occupied by the transit are marked by centered circles; the azimuths and back azimuths are both recorded for possible future references. The needle reading check is recorded. Both the actual needle reading and the reading calculated from the azimuth are recorded. The notes of the side measurements are best put on the sketch.

If the traverse is run by deflection angles instead of azimuths, then these angles appear in place of the azimuths with a letter *L*, or *R*, to denote which way the angle is turned from the preceding course produced. No angle would appear at Sta. 0, but a note that the azimuth or bearing of the first course is so many degrees from Mag. North as shown on the sketch. The first two columns would be as shown in Fig. 75.

Sta.	Deflection Angle
7	
6	
5	
4	33°00'R.
3	
+50 2	27°00'R.
1	
0	Azimuth 332°00' from Mag. North.

Fig. 75.

If the traverse is of the boundaries of a farm or field, it is not usually called a traverse, but a survey. In old surveys made roughly with chain and compass, the distance along any given course to various objects crossed, as streams, fence lines, roads, summits of hills, was noted, and in such work no form of notes is so good as that shown at page 293 combined with a sketch of the line being run and the objects noted. If one is making a complete map of a farm, such measurements and notes are still desirable, but if one is making the survey simply to enable him to write a description of the property, or to verify a former description or locate corners for fencing, such measurements and notes are unnecessary, and only the lengths and azimuths or bearings of the several sides need be given, together with a complete description of every corner of the field, with such references as will enable a stranger to identify it when coming with the notes years later. More on this subject will be found under General Surveying Methods.

94. Transit vs. compass. Any land survey may be made with the transit, the object being, of course, the same as if made with the compass. The essential difference is that the work is

done with a greater degree of precision with the transit, and azimuths are used instead of bearings. One difference that should also be noted is this: the use of the compass makes every line independent of every other line, so far as direction is concerned, since the direction of each line is determined independently by the needle. With the transit this is not so. An error in one line is carried through the remainder of the survey; and, being an angular error, the error of position of the final point of the survey is much greater than it would be if there were an error in the direction of but one line of the survey. It may be said, however, that with the needle check always applied, there is very little probability of an error of this kind, and the use of the transit is advised in all work of importance. Where speed and roughly approximate results are the chief requisites, the compass is the better instrument to use, unless the measurements may be made with the stadia, in which case the transit still remains the better tool. The two may be combined. The directions may be determined with the compass attached to the transit, and the distances read with the stadia, thus securing a maximum economy of time. In this case the stakes would not be centered with tacks, one or two inches making little error in such work, since the compass may not be read to less than five minutes or one twelfth of a degree, and this by estimation. Five minutes of arc means 0.15 of a foot in one hundred feet. Moreover, the distances, if read by the stadia, may not be determined closer than the nearest half foot or possibly the nearest foot.

Exercises

1. Run a traverse around an irregular plot of ground with sides of various lengths by azimuth, using the direction of the first side as zero azimuth, occupy the initial point, and redetermine the azimuth of the initial side from the back azimuth of the final side. Ignoring the last determination of the first side, figure the exterior deflection angles and see if their sum is what it should be. What should it be? (Does this operation furnish any check on the work?) If there are reëntrant sides, angles measured in the direction of the survey (clockwise or counterclockwise) should be called *plus* and those in the opposite direction *minus*.

2. Run the same field, omitting the measurements of the sides, by deflection angles, transiting the telescope at each set-up to produce a line or turn an angle, finally occupying the initial point and measuring the deflection to the initial side. Sum the angles and compare with what they should sum. Does this operation furnish a check on the work?

95. Needle checks on azimuths. In making surveys, if the compass box is not provided with a declination plate or vernier, on which the declination may be set off so that the needle will read north when pointing in the true meridian, it is often convenient to make the magnetic meridian of the place the meridian of the survey, to facilitate checking by the needle the angles measured with the transit. It is inconvenient to do this even then, because the compass is graduated from two points 90° each way, while for azimuths the transit is graduated continuously through 360°. This difficulty is obviated by making a similar graduation on the compass box. If one has a transit that is not graduated in this way, he may graduate a paper ring and paste it on the glass cover of the box. The graduations need be only to degrees. It will be found that such a check on angle measurements is very valuable. If the ring has been properly set, and if the magnetic meridian has been chosen as the meridian of the survey, the reading of the compass should always agree practically with that of the transit. If, instead of the magnetic meridian, some arbitrary meridian has been assumed as the meridian of the survey, the paper circle should be placed so that the needle will read zero when the telescope is pointed in the direction of the assumed zero azimuth, the vernier of the transit reading zero. The needle check is good to the nearest five minutes except where there is local attraction.

Some transits have the graduations of the limb numbered as are those of the compass box. This is an old method. It is better to provide continuous numbering on the compass box.

ADJUSTMENT OF THE TRANSIT

96. Requirements. The transit is used for measuring horizontal angles; that is, angles subtended at a given point by the vertical planes through it and the two other observed points. If these two other points are not in the same horizontal plane, the vertical motion of the telescope must be used to bring the line of sight down from a higher point to the horizontal plane of the instrument, and the line of sight of a lower point up to the same plane. Now, the line of sight is the line in the instrument that is directed to the distant point, and the line that is revolved down or up, as the case may be, to the horizontal plane of the instrument, which is the horizontal plane through the

center of the horizontal axis. It is evident that this line must revolve in a vertical plane, to properly project the distant points into the horizontal plane of the instrument. It will also be evident that the vertical axis of the transit must be truly vertical in order that the line of sight, when projected into the horizontal plane of the instrument and then turned in azimuth, may move in a horizontal plane. If the axis of revolution is not vertical, the lateral motion of the instrument will be in an inclined plane. In order that these necessary conditions may obtain, certain adjustments of the instrument must be made properly.

The adjustments in their proper order necessary to bring about these required conditions are:

1. The adjustment of the plate bubbles to be perpendicular to the vertical axis, and hence presumably parallel to the plates, the usual statement of this adjustment being to make the plate bubbles parallel to the plates.

2. The adjustment of the line of sight to be perpendicular to the horizontal axis of revolution, and hence presumably in the axis of the telescope tube, and coincident with the line of collimation. It will then revolve in a plane (not necessarily vertical) when the telescope is swung on the horizontal axis.

3. The adjustment of the horizontal axis to make it perpendicular to the vertical axis, and hence truly horizontal when the adjusted plate bubbles have been centered in their tubes.

These are all the adjustments of the plain transit. If there is a bubble on the telescope and a vertical circle, the following are added:

4. To make the axis of the bubble under the telescope parallel to the line of sight so that leveling can be done.

5. To make the vertical circle read zero when the adjusted telescope bubble is in the center of its tube, so that vertical angles may be read correctly.

Every adjustment consists of two parts: the test to determine the error, if any, and the rectification of the error found.

97. The plate bubbles. If the plate bubbles are perpendicular to the axis of revolution, that axis will be vertical when both bubbles are in the centers of their respective tubes. The test is made and the adjustment is performed as described for the compass.

Both bubbles are brought to the center of their tubes by

manipulating the leveling screws, and the instrument is turned 180° in azimuth. If the bubbles leave the centers, they are brought halfway back by the adjusting screws and the rest of the way by releveling. The operation is repeated until the bubbles will stay in the centers of their tubes when the transit is revolved on its vertical axis. The principle upon which this test and adjustment rests is stated in Art. 29.

If one bubble gets broken, the other may be used for both, by first leveling with the bubble in one position and then in another, 90° from the first, exactly as in setting up the level, and by repeating the operation till the axis is vertical. But two operations would be necessary were it not for the fact that the second leveling will, to some extent, disturb the first.

The adjustment being made, all points of the instrument will revolve in horizontal planes if the bubbles are both in the centers of their tubes. This adjustment should be tested every day, although probably it will be found correct for a considerable number of days in succession. If the transit is out of adjustment all around, it is better to make each adjustment in order approximately and then carefully repeat them all.

98. To make the line of sight revolve in a plane when the telescope is turned on its horizontal axis. If the line of sight is not perpendicular to the axis of revolution of the telescope, it will describe, in revolving, the surface of a cone whose axis is the horizontal axis of revolution of the telescope; and if the line of sight is perpendicular to the axis and the axis is not horizontal, the line of sight will describe, on being revolved, an inclined instead of a vertical plane. The student should make a mental picture of these conditions.

The intersection of the wires should be in the line of motion of the optical center of the objective; for, if not, then the line of sight will not be fixed in its position in the telescope tube, and, if made perpendicular to the axis of revolution for one distance, it would not be so for some other distance that would require a shifting of the object slide for new focusing. The axis of the object slide should move coincident with the axis of the telescope tube in order that the line of sight may be made exactly perpendicular to the horizontal axis of revolution. Theoretically, its line of motion should pass through the horizontal axis. In the instrument shown in Fig. 69 the objective

is permanently adjusted in motion by the maker. In some other instruments, however, this is not the case.

The two wires are adjusted separately, and it is not unusual to omit the adjustment of the horizontal wire. This should not be omitted if the instrument is to be used for leveling or for measuring vertical angles. The first adjustment of the vertical wire is to make it perpendicular to the horizontal axis.

1. *To make the vertical wire perpendicular to the horizontal axis.* Carefully focus the eyepiece by turning the telescope toward the sky and moving the eyepiece in or out until the wires appear black and distinct; turn the telescope so that the bottom, or top, of the vertical wire cuts a minute point not far distant; revolve the telescope on the horizontal axis and note whether the vertical wire remains on the point; if it does, no adjustment is necessary; if not, loosen two adjacent capstan-headed screws that hold the ring carrying the wire, and move the ring around by the screws till by trial the adjustment is found to be complete. The holes by which the adjusting screws pass through the telescope tube are larger than the screws to allow this motion.

2. *To make the line of sight coincident with the geometric axis of the telescope.* Construct a pair of Y standards of wood fastened to a firm block of wood and with such a distance between them that the telescope tube, after being removed from its standards, may rest in the Y's near its ends. Notches cut in the opposite sides of a box, as a starch box, will answer very well. The Y's of a Y level may sometimes be used. Having made the vertical wire vertical as described in the last adjustment, remove the telescope with its horizontal axis from the standards, and rest it in the Y supports. Now test and adjust the wires (and the object slide if it is adjustable and needs it) in precisely the same manner as described in the first adjustment of the level. If the geometric axis of the telescope has been made exactly perpendicular to the horizontal axis, and the two are coincident where they cross, the line of sight is now perpendicular to the horizontal axis. After replacing the telescope in its standards, the vertical wire is tested and, if necessary, corrected as explained in 3. For the horizontal wire no further test is necessary. With erecting instruments having permanently adjusted eyepieces the horizontal wire may be centered with sufficient accuracy by bringing it to the center of the field of view, after which the vertical wire may be tested and adjusted as in 3

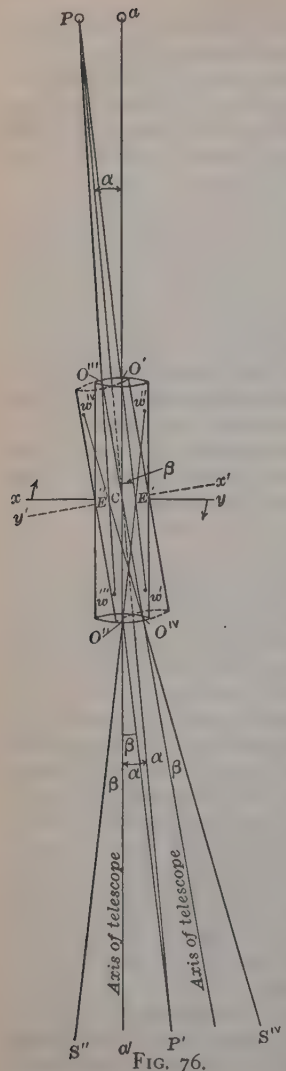


FIG. 76.

and the use of Y's may be omitted. With other than permanently adjusted erecting eyepieces, the method by the Y's is the only correct method yet devised for the adjustment of the horizontal wire.

3. Referring to Fig. 76, C is a point over which the transit is set up, O' is the center of the objective of the telescope, a part of the barrel of which is shown in outline by the heavy full lines, w' is the position of the vertical wire off the axis of the telescope tube, $w'O'$ is the line of sight which makes the angle $w'O'C = \beta$, the angular error of adjustment with O' focused for the distance to P, with the axis of the tube, and points to a distant point P. The axis of the tube extended fails to coincide with the line CP by the angle α, less than β by the angle $w'PC$, a practically negligible quantity depending for its value on the distance to P. The horizontal axis of the telescope is shown by the line xy. If the telescope is now transited on this axis, w' will go to w'', O' will go to O'', and the line of sight will pass through a point S'', the line $w''S''$ making an angle of $\alpha + \beta$ with the extension of the line PC as shown in CP'. The sum of α and β is practically but not exactly equal to 2β, twice the real angular error of adjustment, and if one could locate the point P', it would now be necessary to move the wire in the tube so that the line of sight would fall about midway between P' and S'' when the instrument would be adjusted for collimation, and if the line

of sight were now turned again on P by transiting the telescope and moving the alidade through an angle α, and the telescope again transited, the line of sight should pass through P'. If the transit had been first set at P and the two points C and P' set in line by a foresight, then P' would be located and the preceding method of testing and adjusting would be the proper procedure.

But it is not usual to set three points thus in line and, if the transit is badly out of adjustment, it cannot be done except by double centering as heretofore described but not explained. As this process of double centering also furnishes a test of the adjustment, it will be explained.

When the transit has been set at C, the line of sight brought to P, the telescope transited and the point S'' in the new direction of the line of sight noted, the transit is turned in azimuth until the line of sight again falls on P. Referring to Fig. 76 let it be imagined that the turning has been clockwise. It will not be necessary to turn 180° in azimuth, but 180° less 2 α and the new position of the telescope is shown by the heavy dotted lines, the ends of the horizontal axis being $y'x'$. The line of sight will be w''' O''', cutting P. And again the axis of the telescope does not fall in the line CP by an angle $PCO''' = \alpha$, but on the opposite side of PC. If the telescope is now transited, w''' will go to w^{IV}, O''' to O^{IV} and the line of sight will pass through S^{IV} as far from P' as S'' is, and will make an angle of $\alpha + \beta = 2\beta$ nearly, with the line PCP'. It is now known that P' lies midway between S'' and S^{IV} and that the error of adjustment will be rectified if the wire is moved enough so that the line of sight passes through a point practically $\frac{1}{4}$ of the distance from S^{IV} to S''. It will be exactly $\frac{1}{4}$ if CP' equals CP and $O''S''$ equals $O''P'$. This may be seen by considering the triangle $CP'O''$, whose sides and, therefore, angles, are respectively equal to those of the triangle $CO'P$, so that the angle $P'O''a' = \alpha + O''P'C = \beta$. Hence, it is usual to describe the test and adjustment of the line of sight as follows:

Set up the instrument on a fairly level stretch of ground (not necessarily over any particular point); set a centered stake 200 or 300 feet away or select a minute point some distance away and turn the line of sight on it by either one of the horizontal motions, the other being clamped; transit the telescope and set a point some distance away, calling it #1; turn in azimuth

till the first point sighted comes in the line of sight, clamp and set carefully, transit the telescope, and if point #1 is not covered by the line of sight, set another point, #2, in the line of sight, measure $\frac{1}{4}$ the distance from #2 to #1, and adjust the vertical wire by the adjusting screws that hold the wire ring until the line of sight covers the quarter point; transit the telescope, and by either horizontal motion, set on the first point sighted, transit again, and the line of sight should fall midway between #1 and #2. If it does not, repeat the whole operation or adjust the wire in the last position of the telescope so as to cover a point halfway between the point sighted and the middle point, and repeat the test and adjustment until complete.

In moving the adjusting screws it should be remembered that in an inverting telescope the wire is to go the way it seems that it should, in an erecting telescope the opposite way. One capstan screw must be loosened and the opposite one tightened, which means that with the adjusting pins in the tops of both screws both pins should go forward or back at the same time by equal amounts. It is difficult to do this exactly right, and at the last both screws should be tightened but not forced hard.

Usually it will be found necessary to perform the operation more than once. All bisections should be made with the greatest possible precision, using the clamps and slow-motion screws of all motions except the vertical motion of the telescope.

Probably the horizontal wire will not be appreciably disturbed by this adjustment, but may be tested as before. If it is found necessary to correct it, the vertical wire must be tested again as above. If the instrument is to be used for leveling or for reading vertical angles, it is just as necessary as in the level that the horizontal wire be adjusted properly so that the line of sight shall be correct for all distances.

99. To make the horizontal axis perpendicular to the vertical axis. This is done that the line of sight may revolve in a *vertical* plane when the telescope is plunged or transited. The adjustment is made in any one of several ways. Two will be given.

1. Hang a long plumb line from some tall and firm support, as a second story window sill. Having set up the transit a short distance, say twenty feet, from the line, turn the line of sight as defined by the intersection of the wires on a point in the line near its upper end, and clamp azimuth motion. Swing

the telescope vertically, noting whether the intersection of the wires remains on the plumb line. If not, raise or lower one end of the axis of revolution of the telescope till the intersection of the wires will follow the plumb line.

2. Set up the instrument near some tall building or other high object, setting the intersection of the wires on some well-defined point near the top, and clamp the azimuth motion. Plunge the telescope downward and set a point on the ground near the building or object. Reverse in azimuth, transit the telescope, and set again on the point near the top and clamp in azimuth. Plunge the telescope, and see whether the intersection of the wires falls on the same point as before, near the bottom. If it does, the axis is horizontal and needs no adjustment. If not, adjust the axis by raising or lowering one end till the wire will cut the same point below when plunged from the upper point with telescope direct and reversed. The lower point that is vertically under the upper point will be nearly or exactly midway between the first two points set. The student may draw a figure and explain why it would not be exactly midway ordinarily. The difference is not likely to be large enough to be measurable. It is better not to attempt to adjust in either elevated or depressed position through some particular part of the apparent error. The axis should be adjusted slightly and the adjustment again tested. When completed, the instrument as a transit is adjusted.

100. Level under the telescope. To use the transit as a level, the level under the telescope must be adjusted by the peg method as described for the leveling instrument in Chapter III, adjusting the bubble, not the wire.

101. Vertical circle. If the transit has a vertical circle, this should be adjusted so as to read 0° when the bubble under the telescope is in the center of its tube, after the last-mentioned adjustment for leveling has been made. For this purpose the vernier of the vertical circle is adjustable. To make the adjustment, loosen the small screws that fasten the vernier to the standard and slide the vernier along, as is indicated by the position of the zero of the circle, till the zeros of scale and vernier coincide, the bubble being in the center of its tube.

It may be preferred to read the vernier when the bubble is in the center and use this reading as an error to be applied to

all vertical angles measured. Care must be taken to apply the error with the right sign to plus and minus angles.

102. Eyepiece. After the wires have been adjusted they may not appear in the center of the field of view. This is because the eyepiece is not properly centered. There need be no inaccuracy in work done with the transit if this is not corrected, but better seeing will result if it is corrected. This may be done by moving the ring through which the eyepiece slides, just as the wire ring was moved, there being a set of screws for this purpose next back of the wire screws. In the inverting instrument shown in Fig. 69 and some erecting instruments, this adjustment is permanently made.

103. Errors of graduation and eccentricity. Errors of graduation may exist, but such as are likely to occur in modern machine graduated instruments cannot be detected by ordinary means. The center of the graduated circle may not lie in the axis of rotation, and the line joining the zeros of the verniers may not pass through the center of the graduated circle. If the latter condition exists, the verniers will not read 180° apart, except possibly at some one point in case the first condition also obtains. The second condition simply means that the verniers are not 180° apart, and no error will result from this cause if the same vernier is read for both pointings for the measurement of an angle. If the second condition exists and the first does not, the angular distance between verniers will be the same for all parts of the circle; while, if the first condition exists, the angular distance for different parts of the circle will not be constant. These two facts furnish methods of testing for eccentricity that will be evident to the student who is familiar with the discussion of eccentricity in the compass. The surveyor will not attempt to correct the instrument for eccentricity, but will return it to the maker for such correction.

104. Summary of transit adjustments. The following outline will assist the student in remembering the adjustments in their proper order:

I. Axis of plate bubbles perpendicular to vertical axis of instrument.

II. Line of sight perpendicular to and presumably coincident with horizontal axis of telescope.
   1. Parallax.
   2. Vertical wire perpendicular to horizontal axis of telescope.
   3. Intersection of wires to axis of telescope.
   4. Objective slide axis coincident with axis of telescope.
   5. Line of sight perpendicular to horizontal axis of telescope.
   6. Eyepiece to center.

III. Horizontal axis of telescope truly horizontal.

IV. Axis of telescope bubble parallel to line of sight.

V. Error of vernier of vertical circle.

CHAPTER V

LAND SURVEY COMPUTATIONS

105. General considerations and definitions. It will be evident that, if the lengths and bearings of all the sides of a closed field have been determined, these lengths may be reproduced to scale on paper, with the proper bearings referred to a meridian line drawn on the paper. It will be evident further that, if these lines and bearings have been determined correctly and have been laid off consecutively, joining end to end, they should form a closed figure similar to the field on the ground. A survey of a closed field rarely " closes " on the drawing board, owing to the facts that distances cannot be measured exactly; that the various sides, offering different obstacles, will not be measured with the same proportionate error; that if a compass is used, the bearings are ordinarily determined only to the nearest quarter degree, and that even if a transit is used, these bearings are not determined exactly. If the drawing is done with the aid of a protractor, the survey may close on the drawing, even though there is some error, because the errors are too small to show with the scale used. It is necessary, therefore, to resort to computation to determine just what the error of closure is.

To balance a survey is to determine the error of closure and, if it is not greater than is allowable, to distribute a quantity equal to the error, but of opposite sign, among the several sides, changing the notes accordingly, so that the survey shall " balance " or close. This is necessary because a description of a piece of property appearing in a deed ought to be such that it will indicate a possible closed tract, and such descriptions are, or should be, made from surveyors' notes. Moreover, the area cannot be found for a field that is not closed — it includes the surface of the earth. Before describing the method of balancing it will be necessary to explain a few terms.

The *latitude of a point* is its perpendicular distance from an assumed base parallel.

The *longitude of a point* is its perpendicular distance from an assumed base meridian.

The *latitude of a line* is the latitude of its middle point.

The *longitude of a line* is the longitude of its middle point.

The perpendicular distance of the final end of a given line from a latitude parallel through its initial end is the *latitude difference of the line*.

The perpendicular distance of the final end of a given line from the meridian through its initial end is the *longitude difference*.

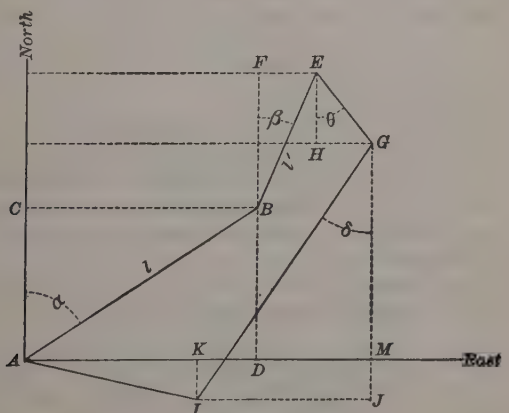


FIG. 77.

In Fig. 77 the latitude difference of the line AB is DB; the longitude difference is CB. If α is the bearing of the line,

$$DB = AC = l \cos \alpha,$$
$$CB = l \sin \alpha.$$

Therefore, the latitude difference is the length of the line multiplied by the cosine of the bearing; and the longitude difference is the length of the line multiplied by the sine of the bearing. This rule should be memorized, for it is used in many kinds of work.

The latitude and longitude differences of the line BE are respectively:

$$BF = l' \cos \beta,$$
$$FE = l' \sin \beta.$$

The latitude and longitude of the point B, referred to the meridian and parallel through A, are respectively AC and CB.

The latitude and longitude of the point E, referred to the same meridian and parallel, are respectively $AC + BF$ and $CB + FE$.

Latitude differences measured north are considered positive, and those measured south negative. Longitude differences measured east are considered positive, and those measured west negative. Measuring north, east, etc., refers to the direction of progress of a survey. If the line AB is run from A to B, it has north latitude difference and east longitude difference, while if it were run from B to A, the survey going around the field from A to I to G, etc., it would have south latitude difference and west longitude difference.

Latitude difference and longitude difference were formerly and still to some extent are called *latitude* and *departure* respectively. If one-word terms are preferred to the terms "latitude difference" and "longitude difference," the best to use are northing, southing, easting, and westing, there being, however, four terms instead of two.

North latitudes are likewise positive, and south latitudes negative. East longitudes are positive, and west longitudes negative.

The latitude and longitude differences of the line EG are respectively $-EH$ and $+HG$. The latitude and longitude of the point G are respectively:

$$\text{Lat.} = AC + BF - EH = MG,$$
$$\text{Long.} = CB + FE + HG = AM.$$

Similarly, the latitude and longitude of the point I are respectively:

$$\text{Lat.} = AC + BF - EH - GJ = -MJ = -KI,$$
$$\text{Long.} = CB + FE + HG - JI = +AK.$$

From the above discussion, the method of determining the latitude and longitude of any point in a traverse will readily be seen.

The latitude of any point in a traverse is the latitude of the initial point of the traverse plus the algebraic sum of the latitude differences of all the courses of the traverse to the given point.

The student may formulate a similar statement for the longitude of a point in a traverse.

In the figure, the latitude and longitude of the initial point are zero. In some cases this may not be so.

Since in a closed traverse the initial and final points are coincident, their latitudes and longitudes must be the same, and hence, the algebraic sum of the latitude differences and the algebraic sum of the longitude differences should each equal zero.

106. Error of closure. If the latitude and longitude differences of the courses are computed, and the algebraic sum of each equals zero, the field closes; otherwise it does not.

If the field does not close, and if it is drawn on paper by laying off from a meridian and parallel the latitudes and longitudes of the several corners in order, it will be found that the last point, which should be coincident with the first, is some distance from it. This distance is known as the error of closure, and is the hypotenuse of a right triangle, whose two sides are the error in latitude and the error in longitude. Its length, therefore, is the square root of the sum of the squares of the errors of the latitude and longitude. Thus, in Fig. 78, page 156, the notes of a survey are plotted in the lines *AB*, *BC*, *CD*, and *DE*, and the field fails to close by *EA*. The error in latitude is *AF* and in longitude *FE*. If the error of closure is *very* small, or nothing at all, it indicates the probability that the angles of the field have been correctly determined. If the angles are correct, the bearings are either correct or all wrong by equal or compensating errors. It also indicates that the lengths of the sides have been measured correctly so far as their relative lengths are concerned, though, perhaps, their true lengths have not been determined. If a compass has been used, the correctness of the bearings depends on the correctness with which the declination has been determined and the degree to which local attraction, if any, has been eliminated; and the determination of the true length of the lines depends on the length of the chain or tape and the precision with which it has been handled.

Usually it is assumed, however, that the constants of the compass and chain have been correctly determined, and that, therefore, the error of closure is a true measure of the precision of the survey. The precision of the survey is indicated usually by saying that the error of closure is one in so many obtained

by dividing the error of closure by the perimeter of the field and reducing the fraction by dividing both terms by the numerator. The surveyor should know whether the error of closure is such as to be the result of justifiable or unjustifiable lack of precision in the work, or gross error. It may be said in this connection that, in ordinary farm surveying, an error of one in five hundred, obtained by dividing the errors of closure by the

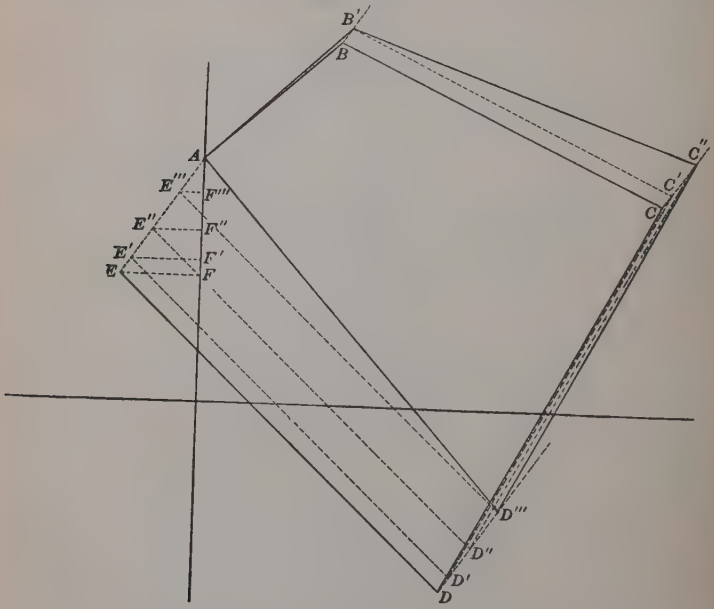


FIG. 78.

total perimeter of the field, is tolerable, and that the precision should be, for good farm work, as high as one in two thousand, or better.

With a transit and a tape and considerable care in the chaining, it is not at all impracticable to secure a precision of 1 in 12,000 to 1 in 15,000, even in country work over rolling ground. But such precision is not necessary in farm work except where the farm joins a municipality and is being surveyed for subdivision.

BALANCING THE SURVEY

107. Fundamental hypotheses. If a compass is used, the method of correcting the notes is based on the supposition that any errors are due as much to inaccuracy of angle measurement as to a lack of precision in measuring lines. This does not mean that the error of closure due to any one line is as much owing to erroneous linear measurement as to erroneous determination of bearing; but that it is probable that in a line in which an error of bearing would tend to produce the observed error of closure, such an error has been made, and little or no error in measurement of length; while in a line in which an error of length would tend to produce the observed error of closure, such an error in length has been made with little or no error in bearing. This means that a line, whose bearing is about normal to the line which represents the error of closure, would be corrected in bearing and not in length; while a line that is parallel to the direction of the line representing the error of closure would be corrected in length and not in bearing. Lines whose bearings are intermediate between those mentioned would be corrected in both bearing and length, the greater portion of the error being given to bearing or length in proportion to the effect that each would have on the error of closure; thus, if the bearing of the line were nearly normal to the line representing the error of closure, it would be corrected more for bearing than for length, and if nearly parallel to that line, it would be corrected mostly in length.

The plausibility of the supposition that errors of closure are due as much to errors in bearing as to errors in length is clear when it is remembered that, if bearings are not read with greater precision than to the nearest quarter degree, the possible error may vary from nothing to seven and one half minutes. The error resulting from an error in bearing is proportional to the length of the line on which the error is made, and an error of seven minutes gives an erroneous position for the end of a line of one foot in five hundred, or as much as would be allowed in measurement of lines.

It is assumed that errors in length will be proportional to the lengths of the lines measured, though it is more probable that they are proportional to the difficulties involved in the measurement *and* to the lengths.

108. The method explained. If the assumptions are all true, it will follow that the error of closure should be distributed among the several sides of the field in proportion to their several lengths. The demonstration of this is as follows:

In Fig. 78 let $ABCDE$ be the plot of the notes, EA being the error of closure, known as the closing line. The line AF is the error in latitude, and FE is the error in longitude. Let the length of $AB = a$, $BC = b$, $CD = c$, $DE = d$, and let the sum of these lengths be s. Divide EA into four parts,

$$EE' = \frac{a}{s}\overline{EA},\ E'E'' = \frac{b}{s}\overline{EA},\ E''E''' = \frac{c}{s}\overline{EA},$$

$$E'''A = \frac{d}{s}\overline{EA}.$$

Lay off the lines BB', CC', DD', equal and parallel to EE'. Also lay off $C'C''$ and $D'D''$, equal and parallel to $E'E''$. Also lay off $D''D'''$ equal and parallel to $E''E'''$. Connect $AB'C''D'''A$. This will be the correctly closed field, and to each side will have been given such a portion of the entire error of closure as that side is of the whole perimeter. From the similarity of the triangles AEF, $AE'F'$, $AE''F''$, etc., it will be seen that the same proportion of the error of latitude and the error of longitude has been given to each side, whence the following:

RULE: *Correct the latitude difference and longitude difference of each course by an amount determined by the proportion; the required correction in latitude* (*or longitude*) *is to the total error in latitude* (*or longitude*) *as the length of the course in question is to the entire perimeter of the field.*[1]

It will be observed that the courses nearly parallel to EA have been corrected mostly in length, while those nearly normal to EA have been corrected mostly in bearing.

From the corrected latitude and longitude differences new bearings and lengths are computed.

109. The practice. This method is not followed strictly in practice. It is, as has been seen, based on certain assumptions as to the probable mode of occurrence of the errors. In a

[1] Least squares demonstration of this rule will be found in Wright's "Adjustment of Observations." See also Trans. Am. Soc. C. E., Vol. XLV, p. 453. 1901.

given case it may be that the surveyor knows, or is reasonably certain, that a greater portion of the error is due to the difficulty encountered in measuring one side, and in such a case he would give the greater portion of the error to that side. In determining that this is true, he must first ascertain the direction of the closing line to see whether the line supposed to be difficult to measure is parallel or nearly parallel to the closing line, so that an error in measurement would be responsible for the resulting closing line. If the supposedly difficult line is not parallel to the closing line, the error is probably not in that line but in another that is nearly parallel, or in the bearing of a line that is nearly at right angles to the closing line. If the error seems to demand it, the survey should be examined in the field. The method may be called balancing by judgment.

In the absence of mistakes, it is probable that the measured lines are recorded all too long. Hence those lines nearly parallel to the closing line and whose direction of measurement would produce the observed error should be shortened, while those lines which are nearly at right angles to the error of closure should be changed mostly in bearing. Thus in Fig. 78 line *AB* should not be shortened because this would increase the error, and it should not be lengthened because that would be proper ordinarily only on the assumption of a mistake which would demand remeasurement, but line *CD* should be shortened a good deal, while lines *BC* and *BA* may both be changed in bearing. These changes are brought about by properly correcting the latitude and longitude differences of the courses to be changed. In this the surveyor uses his judgment.

When the lines are all of equal difficulty of measurement, or when one who knows nothing of the field work balances the survey, the first method should be used. In case the person making the survey also balances it (as should always be the case, when possible), the second method should be followed.

The original notes of the survey should not be destroyed. The new latitude and longitude differences and lengths and bearings of courses may be written in the notebook in red ink over the originals. The originals may be useful in the future in determining how the corrections were made. It is not common to correct the observed bearings and lengths to correspond to the corrected latitude and longitude differences; but it is well to do this, and it must be done if a consistent description of the field is to be written.

110. When a transit is used. In case there is no reason to believe that there is any error in bearings, and therefore the entire error of closure is due to lack of uniformity in measurement, the survey should be balanced in such a manner that only the lengths shall be corrected. No satisfactory rule has yet been devised to accomplish this, and it must usually be done by trial, observing which lines are nearly parallel to the error of closure and changing the lengths of these lines until the field closes.

If the field is rectangular and one side may be taken parallel to the meridian of the survey, or assumed so for the purpose, the following rule accomplishes the desired result:

RULE: *The correction in latitude (or longitude) difference for any course is to the total error in latitude (or longitude) as the latitude (or longitude) difference of the course is to the arithmetical sum of the latitude (or longitude) differences.*

This rule may be used in balancing surveys of the approximately rectangular areas of the U. S. public land subdivisions, city blocks, etc.

Some surveyors think the rule applies to any survey for which a transit is used. It is doubtful if this is true, and in close work in which a transit is used and care is taken to make the precision of linear measurement about equal to that of angle measurement, as is usual in city work, the rules of Art. 108 or that of Art. 109 should be used.[1] Linear errors of 1 in 10,000 correspond to angle errors of about 20″, 1 in 5000 to about 40″, and 1 in 3400 to 1′.

111. Plotting a traverse by latitudes and longitudes. This is the best method of all for plotting a traverse or survey. If the survey is not an extended one, as is a railroad survey, but can be put all on one piece of drawing paper on an ordinary drawing board, a single reference meridian and parallel are drawn through an initial point. From the meridian and parallel, the latitudes and longitudes of the corners of the survey are laid out by scale and the points thus found are connected with straight lines representing the courses. If balanced latitudes and longitudes are used, the several sides of the plot should scale the corrected lengths of the courses, which fact gives a check on the work.

[1] See Trans. Am. Soc. C. E., Vol. XLV, p. 453. 1901.

If the survey is a long one, squares of, say, 1000 feet on a side, may be laid out very carefully along the general line of the survey determined a little ahead as the work advances, the measurements of latitudes and longitudes of corners or points of the survey being taken from the nearest 1000-foot line, so that every measurement will be within a single scale length. If the line extends many miles east and west, the direction of the meridian will change, but not unless changed in the survey. It should not be changed on the drawing oftener than about once in 10 miles in any but high latitudes. (See Convergence of Meridian, Art. 211.)

AREAS

112. Double longitudes. When the survey has been balanced, the area may be computed. This is usually done by the use of a formula involving the products of the latitude difference and double longitude of each side of the survey. The double longitude of a line is simply twice its longitude, or the sum of the longitudes of its ends. The latitude differences are taken from the table of corrected latitude differences that has been prepared in balancing the survey. A convenient method for determining the double longitudes must be found.

If the most westerly corner of the field is chosen for the initial point through which the reference meridian is supposed to be passed, all of the D.L.'s will be positive, the necessity of considering their signs in the subsequent work of finding areas is avoided, and the work thus simplified. In Fig. 79 let NS be the reference meridian, the longitude differences of the various sides being as shown, with the signs prefixed to each. It is evident that the double longitudes of the first and last courses will be numerically equal to their respective longitude differences, but the sign of the D.L. of the final course will be opposite to that of its longitude difference. The D.L. of any other course, as BC, is the sum of the longitudes of its extremities.

$$\text{D.L. of } BC = bB + cC = d_1 + d_1 + d_2.$$

In words, the D.L. of BC is the D.L. of AB plus the longitude difference of AB, plus the longitude difference of BC.

Again, the D.L. of CD is

$$cC + dD = d_1 + d_2 + [d_1 + d_2 + (- d_3)]$$
$$= [d_1 + d_1 + d_2] + d_2 + (- d_3).$$

The bracket is the D.L. of BC; hence
The D.L. of CD is the D.L. of BC plus the longitude difference of BC plus the longitude difference of CD.

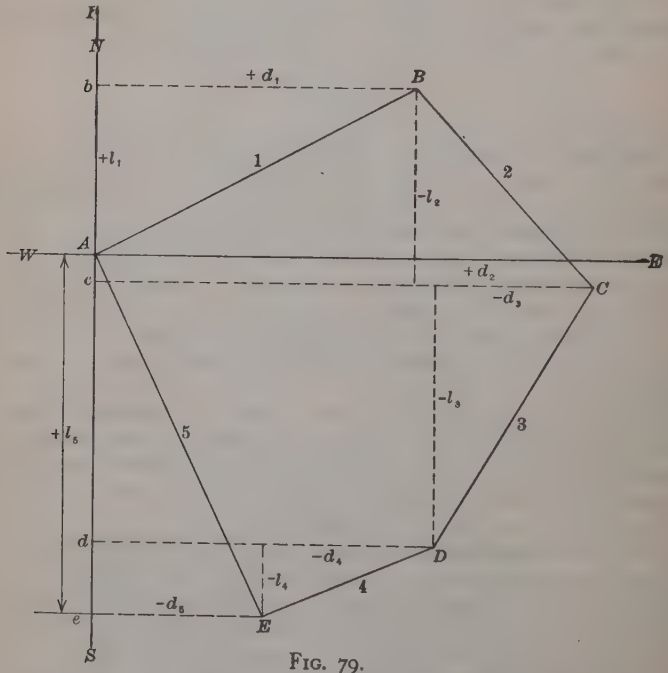


Fig. 79.

It is customary to take the reference meridian through the most westerly corner of the field, but this is not necessary; it may be taken through any corner, and is frequently taken through the first, or beginning, corner of the survey. Thus, in Fig. 80, page 164,

$$\text{D.L. of } BC = bB + cC = d_1 + d_1 + (- d_2).$$

In words, the D.L. of BC is the D.L. of AB plus the longitude difference of AB plus the longitude difference of BC.

Again, the D.L. of CD is

$$\begin{aligned} cC + (-dD) &= [d_1 + (-d_2)] + [d_1 + (-d_2) + (-d_3)] \\ &= [d_1 + d_1 + (-d_2)] + (-d_2) + (-d_3), \end{aligned}$$

or the D.L. of CD equals the D.L. of BC plus the longitude difference of BC plus the longitude difference of CD.

From the result of this investigation may be formulated the

RULE: *The double longitude of any course is equal to the double longitude of the preceding course, plus the longitude difference of that course, plus the longitude difference of the course itself.*

In applying this rule, due attention must be paid to the signs of the quantities. This rule applies to the first course as well as to the others, if the preceding course is considered to be zero.

To determine the D.L. of the various courses, select one course as the first. Its longitude difference is its D.L. To this add its longitude difference and the longitude difference of the second course, and obtain the D.L. of the second course; to this add the longitude difference of the second course and the longitude difference of the third course, and obtain the D.L. of the third course, etc. The correctness of the work is proved if the D.L. of the final course is found numerically equal to its longitude difference.

113. To find the area of the field. The area in Fig. 79 evidently equals the sum of the trapezoids $bBCc$, $cCDd$, and $dDEe$, less the triangles AbB and AeE.

Beginning with the triangle AbB and taking the triangles and trapezoids in order around the figure to the right and using the signs of the double longitudes (all plus) and latitude differences, the following equation results:

$$\begin{aligned} 2 \times \text{Area } ABCDEA = {} & l_1 \times bB - l_2(bB + cC) - l_3(cC + dD) \\ & - l_4(dD + eE) + l_5 \times eE \end{aligned}$$

$$\text{or, Area } ABCDEA = \tfrac{1}{2}\begin{cases} l_1 \times \text{D.L.}_1 - l_2 \times \text{D.L.}_2 - l_3 \times \text{D.L.}_3 \\ - l_4 \times \text{D.L.}_4 + l_5 \times \text{D.L.}_5 \end{cases}$$

which is equivalent to saying:

The area of a closed field is

One half of the algebraic sum of the products of the several latitude differences into the corresponding double longitudes.

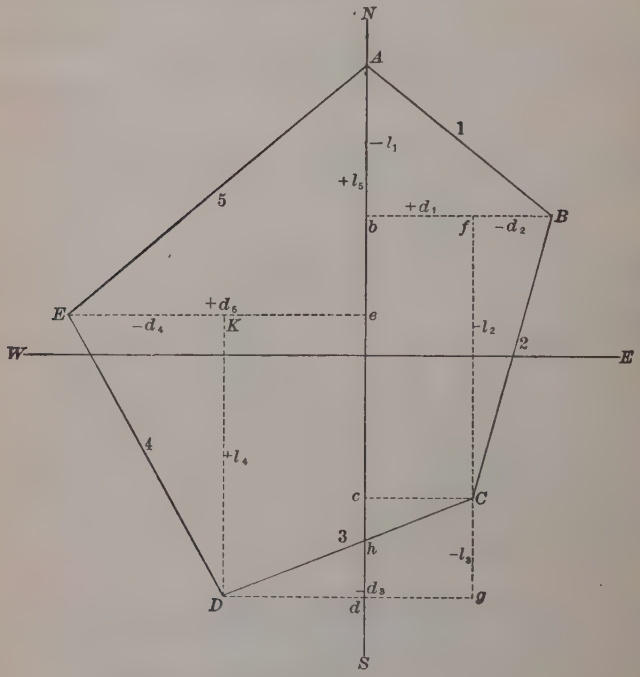


FIG. 80.

Whence the

RULE: *To find the area of a closed survey multiply the latitude difference of each course by the double longitude of the course, and note the signs of the products. Divide the algebraic sum of these products by two.*

The student may try to show the correctness of the rule, using Fig. 80.

The sign of the result is of no consequence, and depends on the position assumed for the reference meridian and the direction of the survey around the field, whether clockwise or counterclockwise. Thus, if the survey had proceeded from A to E to D, etc., courses AE and BA would have had negative latitude differences and the others all positive, so that the area would appear positive.

MODEL EXAMPLES

114. Logarithms. When formerly, in land surveying, bearings were read only to quarter degrees, there were published for convenience what were known as traverse tables. These were nothing more nor less than tables of natural sines and cosines of the angles from 0° to 90° for every quarter-degree multiplied by 1, 2, 3, 4, etc., to 10, and in some tables to 100. The use of such a table made it unnecessary to multiply the sines and cosines of the bearings to get the latitude and longitude differences, since for each digit in the number expressing the length of the course the differences could be read from the table and brought to the right amount by moving the decimal point. The several quantities were then added.

With modern methods of work the compass is read to the nearest five minutes, and when a transit is used the angles are determined to minutes. Such a table as has been described then becomes useless. The proper table to use is one of logarithmic sines and cosines. A single computation involving not more than two or three figures can be performed more quickly perhaps without the use of logarithms, but any series of computations or a single computation involving five or more figures can be more quickly performed by logarithms. This does not mean that one unaccustomed to the use of logarithms can work with them as fast as without, but a very little practice with them will in any case substantiate the above claim. The student should familiarize himself with a good set of logarithmic tables.[1] The question will arise whether four-place, five-place, six-place, or seven-place tables should be used. The decision must be based on the size of the quantities involved in the computation and the precision required in the result. A four-place table will give

[1] The tables of Bremiker or Vega are recommended for work requiring great precision. Gauss or Crockett or the tables of this book are recommended for five-place tables.

results correct to three significant figures and almost correct to four significant figures, probably within one or two units in the extreme end of the table. A five-place table will give results correct to four significant figures and within one or two units in the end of the table, to five significant figures, and so on.

Angles read to the nearest minute may be a half minute in error; read to the nearest 20 seconds they may be in error by 10 seconds. In a right triangle having 1000 feet for base and acute angle of 34° 30′ measured with a minute reading instrument, the results in calculating the opposite side with different tables will be studied (Figure 81).

1. Let it be supposed that the 1000 feet is measured with a precision of 1 in 50,000; that is, the 1000 feet is correct within 0.02 of a foot either side of 1000 feet or the true distance lies between 999.98 and 1000.02. This is a very high degree of precision for ordinary surveying, probably five times the precision obtained in excellent railroad surveying and in general town surveying outside of large cities. The angle may lie between 34° 29.5′ and 34° 30.5′.

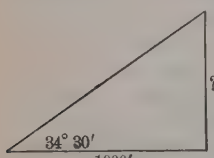


FIG. 81.

Let a five-place table of natural tangents be used. The last place is uncertain and may be a half unit less or more than the tabular value.

First assume the angle to be exact. The tangent of 34° 30′ is 0.68728 and may be anywhere between 0.687275 and 0.687285; therefore, if the base is measured exactly right, the opposite side may be anywhere between 687.275 and 687.285. The result is obtained to the nearest hundredth of a foot, which is the limit of possible measurement and is closer than a precision of 1 in 50,000, which is as close as the work warrants. But let it be supposed that the angle is measured only to the nearest minute. The value of the tangent may then be anywhere between that for 34° 29′.5, and that for 34° 30′.5, with a possibly double indefiniteness in the last figure due to interpolation between two uncertain quantities. These tangents are respectively 0.687065± and the opposite side of the triangle may lie anywhere between 687.06+ and 687.49+, showing a large discrepancy in the fourth place, a much larger discrepancy than the character of the linear work warrants.

The discrepancy would be greater for a larger angle and less for a small one. A four-place table would give 687.3 for 34° 30′ with a possible range between 34° 29′.5 and 34° 30′.5 of 687.1− to 687.5+, or just as good a result for the work in hand as is obtained with a five-place table.

This discussion shows two things, namely:

1. It is useless to make linear measurements with a precision of more than 1 in 3500 if angles are to be read to the nearest minute only.

2. It is useless to use tables of more than four places for angles read to the nearest minute only.

It is difficult for many persons to bring themselves to use the smaller tables because they seem to see a greater precision in the use of tables giving results that are true to five and six significant figures, and fail to realize that the field work on which the computations depend does not warrant any such degree of precision, which is therefore only a seeming precision that is misleading and does not exist in fact. It is true that linear measurements can in general be made with greater precision than the angle work gives and, hence, it is the angle work that fixes the precision and the tables to be used. The following rules may be formulated:

For angles read to the nearest minute use four-place tables.
For angles read to less than 0° 00′ 30″ use five-place tables.

For work in general requiring certainty in the third significant figure use four-place tables, in the fourth significant figure five-place tables, and in the fifth significant figure six-place tables.

But it must be remembered that no ordinary surveying work is done precisely enough to warrant results certain to more than four significant figures, and that five-place tables are as extensive as are warranted by any land, topographic, railroad, or other surveys except the most refined city, bridge, and geodetic surveys.

Computation labor is increased about 50 per cent by using five-place tables instead of four-place tables, and about one third by using six-place tables instead of five-place tables.

115. Example stated. The notes of the courses of a survey are as follows:

N. 69° E.	437.0 ft.
S. 19° E.	236.0 ft.
S. 27° W.	244.0 ft.
N. 71° W.	324.0 ft.
N. 19° W.	183.5 ft.
	1424.5 ft.

It is required to balance the survey and determine the area of the field. This example will be worked out in detail as a model for the student. He is advised to note carefully the systematic arrangement of the work, as by such system much time is saved. It is a case from practice.

116. Balancing. Finding the most westerly corner by inspection or a rough sketch, using it as the beginning corner, and letting L represent latitude differences and M longitude differences, the computation is arranged as shown on page 169.

Explanation: We first write the logarithm of the length of the course, and above it the logarithmic cosine of the bearing, and below it the logarithmic sine. The logarithm of L is then obtained by adding up, and the logarithm of M by adding down. The L's and M's are then taken out and placed in their respective columns with their signs, and each column added algebraically, giving the result -4.9 in L and $+7.9$ in M. The error of closure is then found. In the example given it is 1 in 151, entirely too large, and the field should be rerun. The errors in L and M are now distributed among the courses in proportion to the lengths of the sides. It is not necessary to be exact about this, and it is done by inspection. Thus $\frac{437}{1424.5}$ is a little less than one third, hence the first corrections are 1.5 in L and 2.5 in M. The other fractions are treated in the same way, the second being somewhat less than one sixth, etc. If the corrections thus determined do not sum up exactly to the respective total corrections, some one or more of them is slightly altered to make the sum correct; thus, in the above example, the corrections first written for the fourth L and the fifth M were 1.2 and 1.1 respectively, and these were changed to 1.1 and 1.0 to make the sums equal 4.9 and 7.9 respectively. The balanced L's and M's are now written in the column of L's and M's under the old L's and over the old M's. The lengths and bearings

COMPUTATIONS FOR THE HARRINGTON SURVEY

QUANTITY	LOGS	L	M	dL's AND dM's	QUANTITY	LOGS
L	2.19481	+156.6		$+\frac{437}{1424.5}\times 4.9=1.5$	435.2	2.63872
cos 69°	9.55433	+158.1			sin 68° 42′	9.96927
437.0	2.64048				405.5	2.60799
sin 69°	9.97015		+405.5		158.1	2.19893
M	2.61063		+408.0	$-\quad\times 7.9=2.5$	tan 68°42′	0.40906
L	2.34858	−223.1		$+\frac{236}{1424.5}\times 4.9=0.8$		
cos 19°	9.97567	−222.3				
236.0	2.37291					
sin 19°	9.51264		+ 75.5			
M	1.88555		+ 76.8	$-\quad\times 7.9=1.3$		
L	2.33727	−217.4		$+\frac{244}{1424.5}\times 4.9=0.8$		
cos 27°	9.94988	−216.6				
244.0	2.38739					
sin 27°	9.65705		−112.1			
M	2.04444		−110.8	$-\quad\times 7.9=1.3$		
L	2.02319	+105.3		$+\frac{324}{1424.5}\times 4.9=1.1$		
cos 71°	9.51264	+106.6				
324.0	2.51055					
sin 71°	9.97567		−308.2			
M	2.48622		−306.4	$-\quad\times 7.9=1.8$		
L	2.23931	+173.5		$+\frac{183.5}{1424.5}\times 4.9=0.7$		
cos 19°	9.97567	+174.2				
183.5	2.26364					
sin 19°	9.51264		− 60.7			
M	1.77628		− 59.7	$-\quad\times 7.9=1.0$		
		−440.5	+484.8			
		+435.6	−476.9			
		−4.9	+7.9			

Error of closure $= \sqrt{4.9^2+7.9^2} = 9.4 \pm$ and $\frac{9.4}{1424.5} = \frac{1}{151}$ —.

are now inconsistent with the balanced L's and M's, and should be corrected to be consistent. This is done in this example for

the first course only. The tangent of the bearing is $\frac{M}{L}$, hence, write the log M and subtract the log L and get log tan. Above log M write log sin and, subtracting up, get log length.

This completes the balancing.

117. Areas by latitude differences and double longitudes. From the balanced M's the double longitudes are computed, and from the balanced L's and D's, as we may call the double longitudes, the double areas are computed. The work is systematized as in the following table, in which the first column shows the computation of the D's, the second column gives the sign of the corresponding L's opposite their logs in the third column, which shows the log computation of the double areas found and placed in the fourth column.

		Logs	Double Areas
M of the First course is its D =	405.5	2.60799	
		+ L 2.19893	
Of the First course + M =	+ 405.5	4.80692	+ 64110
Of the Second course + M =	+ 75.5		
Second course D =	+ 886.5	2.94768	
		− L 2.34694	
+ M =	+ 75.5	5.29462	− 197070
+ M =	− 112.1		
Third course D =	+ 849.9	2.92937	
		− L 2.33566	
+ M =	− 112.1	5.26503	− 184090
+ M =	− 308.2		
Fourth course D =	+ 429.6	2.63306	
		+ L 2.02776	
+ M =	− 308.2	4.66082	+ 45795
+ M =	− 60.7		
Fifth course D =	+ 60.7	1.78319	
		+ L 2.24105	
		4.02424	+ 10574
			−381160
			+120479
			2)260681
			43560)130340.5
			2.992 + acres

It is believed that the work is self-explanatory. The work is as it was originally done but there was no excuse for using five-place tables here or in the balancing, and even with the five-place tables there are only four certain figures in the final square foot area that is divided by 43,560. The student should work the whole example with four-place logarithms and compare the results.

The results of the foregoing computations are usually tabulated in the form as shown on page 172.

Examples

Find the error of closure, balance the survey, plot, and compute the areas of the following fields, using four-place tables for the first four and both four-place and five-place for the fifth, and study the two results.

	Sta.	Bearing	Chains	
1.	A	S. 51° 10′ E.	5.05	
	B	S. 58° 10′ W.	4.63	
	C	N. 29° 15′ W.	4.16	
	D	N. 45° 30′ E.	2.87	
				Area, 1.682 ± *A*.
2.	A	N. 84° 00′ W.	9.04	
	B	S. 21° 15′ W.	12.34	
	C	N. 72° 15′ E.	12.92	
	D	N. 9° 30′ E.	6.68	
				Area, 9.264 ± *A*.
3.	1	S. 7° 25′ W.	4.35	
	2	S. 62° 05′ W.	6.94	
	3	N. 2° 35′ E.	4.01	
	4	N. 34° 25′ E.	3.64	
	5	N. 83° 00′ E.	4.51	
				Area, 3.124 ± *A*.
4.	1	S. 62° 15′ E.	5.12	
	2	S. 71° 15′ E.	4.66	
	3	S. 5° 30′ W.	12.00	
	4	S. 80° 15′ W.	12.46	
	5	N. 2° 15′ E.	9.46	
	6	N. 25° 45′ E.	9.40	
				Area, 17.68 ± *A*.

	Sta.	Azimuth	Feet	
5.	1	107° 15′	1082.4	
	2	196° 15′	1590.6	
	3	246° 05′	1293.6	
	4	13° 55′	2442.0	
				Area, 48.36 ± *A*.

Station	Course	Distance Feet	Lat. Diff. +	Lat. Diff. −	Long. Diff. +	Long. Diff. −	Balanced Lat. Diff.	Balanced Long. Diff.	Double Longitudes	Double Areas +	Double Areas −
A	N. 69° E.	437.0	156.6		408.0		+ 158.1	+ 405.5	+ 405.5	64110	
B	S. 19° E.	236.0		223.1	76.8		− 222.3	+ 75.5	+ 886.5		197070
C	S. 27° W.	244.0		217.4		110.8	− 216.6	− 112.1	+ 849.9		184090
D	N. 71° W.	324.0	105.5			306.4	+ 106.6	− 308.2	+ 429.6	45795	
E	N. 19° W.	183.5	173.5			59.7	+ 174.2	− 60.7	+ 60.7	10574	
		1424.5	435.6	440.5	484.8	476.9	0.0	0.0		120479	381160

Error of closure $= \sqrt{4.9^2 + 7.9^2} = 9.4 \pm$ or $\frac{9.4}{1424.5} = \frac{1}{151}$ —

2)260681

43560)130340.5

2.992+ acres.

If the field is balanced by weighting the sides, a column of weights and one of weighted lengths are inserted between the third and fourth columns of the table. If the distances are in chains, the result is square chains, and is divided by 10 instead of 43560 to reduce to acres.

In the fifth example, the distances are apparently more precisely (not necessarily more accurately) measured than the angles. The angles measured to the nearest minute call for four-place tables. The distances read to the nearest tenth of a foot are apparently read to 1 in from 10,824 to 24,420. Probably no such degree of precision obtains, and since the angles govern the precision in any case, it is sufficiently precise probably to use only the nearest foot figures of the distances in balancing. It *may* be that the angles are right to within a few seconds, and it will do violence to some minds to omit the last figure of a determined quantity. Nothing worse than a loss of time will result if five-place tables are used, so that he who prefers may use five-place tables and feel the satisfaction that *perhaps* his result is better than if he had used the four-place tables.

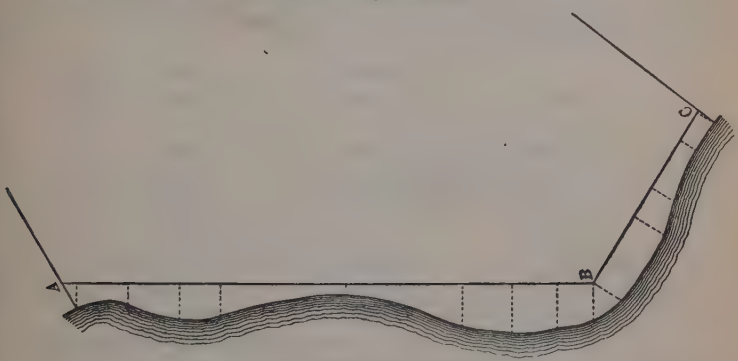


FIG. 82.

118. Irregular areas by offsets. Frequently it occurs that one side of a field that is to be surveyed is bounded by a stream or the shore of a lake, and that the bounding line is quite irregular and not easily run out.

In Fig. 82 the irregular shore line is a boundary. To determine the area, two auxiliary courses, *AB* and *BC*, are run and used with the remaining sides to compute the area on their left. Left and right refer to the direction in which the lines are run. The additional area between these lines and the shore is obtained by measuring offsets normal to these lines. These offsets are measured to the points of change of direction of the shore, and their lengths and distances from *A*, *B*, or *C*, are noted. The area is then computed by considering the portions between the shore and base line and two adjacent offsets to be trapezoids or triangles. When the curve of the shore is such that the offsets

may be taken at regular intervals along a base line, the area is found by applying the following:

RULE: *To the half sum of the initial and final offsets add the sum of all the intermediate offsets, and multiply the result by the common distance between offsets.*

If the offsets are taken at irregular distances, the area may be found as described in Art. 131.

EXAMPLE

The following offsets were taken on the sides of a line indicated, at points 100 feet part. Required the area between extreme boundaries.

L.	DIST.	R.
137.4	0	55.1
93.5	100	76.5
49.3	200	83.3
78.2	300	79.9
102.0	400	69.7
100.5	500	59.5
96.6	600	83.3
144.2	700	70.5
111.9	800	54.4
71.4	900	101.2

Area, 153,585 square feet.

SUPPLYING OMISSIONS

119. Necessity for. At times it is impossible to measure the length of a line; at other times it is impracticable to determine its bearing directly; and sometimes it is impossible to do either. Moreover, it may happen that the length of one line and the bearing of another have not been determined. If any one of these conditions prevails, it becomes necessary to supply the omission in the notes. To do this leaves the work without a check, as all the errors are thrown into the quantity that is supplied; hence, no omission that can in any way be supplied in the field should be permitted.

The cases that may occur are the following:

I. The length of one side may be wanting.

II. The bearing of one side may be wanting.

III. The bearing and length of one side may be wanting.

IV. The bearing of one side and the length of another may be wanting.

V. The lengths of two sides may be wanting.

VI. The bearings of two sides may be wanting.

120. General discussion. It should be clear from what has preceded, that the algebraic sums of the latitude and longitude differences of the known sides are respectively equal to the latitude and longitude differences necessary to close the field (but are of opposite signs), or to the latitude and longitude differences of the wanting sides plus the errors in latitude and longitude of the known sides. Since there is no means of determining these errors, the sums are taken as the latitude and longitude differences of the defective sides.

It is possible, therefore, to write two independent equations as follows:

1. The algebraic sum of the products of the lengths of the defective courses by the cosines of their respective bearings equals the algebraic sum of the latitude differences of the known sides, but has the opposite sign.

2. The algebraic sum of the products of the lengths of the defective courses by the sines of their respective bearings equals the algebraic sum of the longitude differences of the known sides, but has the opposite sign.

Having but two equations, there can be but two unknown quantities, and these may be as in Art. 119.

If l_1 and l_2 and θ_1 and θ_2 are the lengths and bearings of the defective sides, and if Σ is taken as a sign indicating the algebraic sum of any series of quantities, as in this case the latitude and longitude differences, and L and M represent latitude and longitude differences respectively, the above two equations may be written as follows:

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 = -\Sigma L, \quad (1)$$
$$l_1 \sin \theta_1 + l_2 \sin \theta_2 = -\Sigma M. \quad (2)$$

These are, of course, equivalent to the following general equations, which should be true for a closed field:

$$\Sigma L = 0,$$
$$\Sigma M = 0.$$

121. Cases I, II, and III. These are solved most readily by applying equations (1) and (2). If but one quantity is wanting, as the length or bearing of a side, either equation (1) or equation (2) will solve, one of the left-hand terms becoming part of the right member. Two values may be obtained by using both equations. In this case adopt the mean and balance the field in the usual way.

If both the length and bearing of one side are wanting, one of the left-hand terms in each equation becomes part of the right member, and equation (2) may be divided by equation (1), giving

$$\tan \theta = \frac{-\Sigma M}{-\Sigma L}.$$

Having θ, the length is found by substituting in either equation.

The signs of the sums of the known differences will indicate the direction of the defective side. Thus if ΣM is minus and ΣL is plus, the closing side must have a plus M and a minus L, and hence must run east and south.

122. Model example. In the example of Art. 115, let the bearing and length of the second course be wanting. Adding the original L's and M's, we get

Course	L	M
1	+ 156.6	+ 408.0
2		
3	− 217.4	− 110.8
4	+ 105.5	− 306.4
5	+ 173.5	− 59.7
	+ 435.6	− 476.9
	− 217.4	+ 408.0
	+ 218.2	− 68.9

There is then to be added south latitude and east longitude in order to close the field, and hence the line to be supplied runs southeast.

The tangent of its bearing is

		Logs	
$\tan\theta = \frac{68.9}{218.2}$		1.83822	
		2.33885	
Log tan θ		9.49937	$\theta = 17^\circ\ 31.5'$.
Log sin θ		9.47874	
Log length		2.35948	length = 228.8.

The whole error of closure, which was large, being thrown into this course, its bearing and length have been materially altered.

EXAMPLE

In Example 1, Art. 116, let it be supposed that the second course is wanting. Supply it.

123. Cases IV, V, and VI. The author thinks the remaining cases are better solved by special methods as follows:

Case IV. 1. *The two imperfect sides are adjacent.* With the algebraic sums of the known latitude and longitude differences, compute the length and bearing of a closing side. This side with the two defective sides will form a triangle in which two sides and one angle are known. The triangle may then be solved, giving the required quantities.

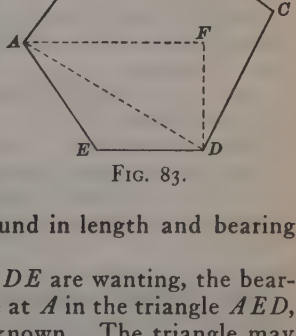


FIG. 83.

Thus in Fig. 83 if AE and ED are defective, the algebraic sum of known L's will be minus FD and the M's plus AF. The closing side, which must run northwest, is DA found in length and bearing from the right triangle AFD.

If the length of EA and bearing of DE are wanting, the bearings of EA and DA will give the angle at A in the triangle AED, and the lengths of ED and DA are known. The triangle may be solved for the angle at E, then the angle D becomes known and EA may be found. Solution is by the sine formula. The bearing of DE may be had from either angle D or E, and the bearing of the adjacent side.

2. *The defective sides are not adjacent.* Imagine all of the courses between the two defective ones on one side shifted along one of the defective sides parallel to themselves so that they join the other known courses between the defective ones. Then by shifting, the defective side being used as a guide, a closed field can be formed with the two defective sides adjacent, and the procedure can be as in the preceding case. Thus, in Fig. 84 the defective sides are DE and AB. If the side AE is imagined shifted parallel to itself to the position BF, and AB to EF, a closing line DF may be computed, and this will form a triangle with ED and EF ($= AB$).

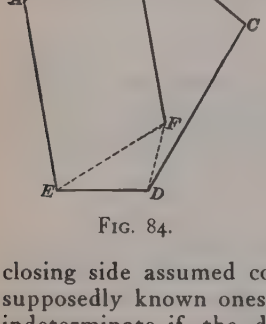


FIG. 84.

The side FB will have the same latitude and longitude difference as EA.

Case V. Treat in the same way as Case IV. In the triangle that results there will be known one side and the three angles.

If the three angles determined from the bearings do not sum 180°, the error should be distributed equally to the three angles, the bearing of the closing side assumed correct, new bearings computed for the supposedly known ones and the triangle solved. This case is indeterminate if the defective sides are parallel, unless the area is known.

Case VI is solved in the same way, and in this case the three sides are known.

Cases IV and VI are not determinate unless enough is known of the field to show which of two angles that correspond to a given sine or cosine is the correct one to use. This will be evident from the following algebraic discussion of Cases IV, V, VI.

124. Algebraic solution. Let the sum of the known latitude differences be denoted by L and the sum of the longitude differences by M. Then for these cases may be written,

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 = -L, \quad (3)$$
$$l_1 \sin \theta_1 + l_2 \sin \theta_2 = -M. \quad (4)$$

In Case IV let there be wanting l_1 and θ_2. Solving (3) for cosine θ_2 and (4) for sine θ_2, squaring and adding and then solving for l_1, there results

$$l_1 = \pm\sqrt{l_2^2 - M^2 - L^2 + (L\cos\theta_1 + M\sin\theta_1)^2} - (L\cos\theta_1 + M\sin\theta_1).$$

It is evident that there are here two values for l_1. If the proper value may be determined by a knowledge of the field, θ_2 may be found by substitution in either (3) or (4).

For Case V there will be wanting in (3) and (4) l_1 and l_2. Solve each for l_1, equate the resulting expressions, solve for l_2, and derive

$$l_2 = \frac{M\cos\theta_1 - L\sin\theta_1}{\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2} = \frac{M\cos\theta_1 - L\sin\theta_1}{\sin(\theta_1 - \theta_2)},$$

from which it is seen that the value is indeterminate when $\theta_1 = \theta_2$, or the defective sides are parallel.

For Case VI there will be wanting θ_1 and θ_2. Solve equation (3) for cos θ_1 and equation (2) for sin θ_1. Square both resulting equations and add, getting

$$L\cos\theta_2 + M\sin\theta_2 = \frac{l_1^2 - L^2 - M^2 - l_2^2}{2\,l_2}.$$

Let the right-hand member be represented by P. Then

$$M\sqrt{1 - \cos^2\theta_2} = P - L\cos\theta_2;$$

square, and solve for cos θ_2, deriving

$$\cos\theta_2 = \frac{PL \pm M\sqrt{M^2 + L^2 - P^2}}{M^2 + L^2},$$

from which it is evident that there are two values, the correct one to be determined by a knowledge of the survey. In the algebraic solutions, careful attention must be paid to the signs of the trigonometric functions. Since no bearing is greater than 90°, the signs, according to usual trigonometric conceptions, would all be positive; but in equations (3) and (4) those signs must be used that will produce the proper sign for the latitude

and longitude differences; thus, for a S. E. bearing, the cosine would be negative and the sine positive.

125. Model example. Let the lengths of sides 2 and 4 of the model example of Art. 115 be wanting. Adding the original L's and M's, we get

Course	L	M
1	+ 156.6	+ 408.0
2		
3	− 217.4	− 110.8
4		
5	+ 173.5	− 59.7
	+ 330.1	+ 408.0
	− 217.4	− 170.5
	+ 112.7	+ 237.5

It is seen that southwest bearing must be that of the closing line. Its bearing and length are obtained as in the last example.

	Logs	
$\tan\theta = \frac{237.5}{112.7}$	2.37566 2.05192	
Log tan θ	0.32374	$\theta = 64° 37'$ S. W.
Log sin θ	9.95591	
Log length	2.41975	length = 262.9.

We now have a triangle composed of this closing side and the two wanting sides. This triangle[1] is formed by shifting one of the wanting sides parallel to itself. In this triangle there are known the bearing and length of the closing side just found and the bearings of the two other sides, and, hence, there are known all the angles and one side.

If the apexes are lettered A, B, C, and the sides opposite the apexes a, b, c, respectively, and if a is the closing side just found, b side 2, and c side 4, we have from the known bearings

[1] Let the student draw the triangle.

Angle $A = 52° 00'$
Angle $B = 44° 23'$
Angle $C = 83° 37'$
$180° 00'$

whence $b = \frac{a \sin B}{\sin A} = \frac{262.9 \sin 44° 23'}{\sin 52° 00'}$

Log 262.9	2.41975
Log sin 44° 23′	9.84476
	2.26451
Log sin 52° 00′	9.89653
Log b	2.36798
	$b = 233.3$

Solve for course 4. The other problems are similarly solved.

Examples

1. In Example 2, Art. 117, let it be supposed that the bearing of the second course and the length of the third course are wanting. Supply them.
2. In the third example, let it be supposed that the bearings of the second and fourth courses are wanting. Supply them.
3. In the fifth example, let it be assumed that the lengths of the second and fourth courses are wanting. Supply them.

COÖRDINATES

126. Definitions. It is becoming common to call the latitude and longitude of a point the *coördinates* of the point.

The base parallel and meridian are known as the *coördinate axes*, and their intersection as the *origin of coördinates*. The coördinates of the origin are evidently both zero. As with latitudes and longitudes, ordinates measured east, or to the right from the reference meridian, are considered positive; those measured west, or left, are negative; those measured north, or up from the base parallel, are positive; and those south, or down, are negative.

It is not uncommon to speak of the latitude ordinates as *ordinates*, and the longitude ordinates as *abscissas*.

In the city of New York, and in some other cities, many corners to public property, and many or all private property corners, are located by coördinates, the well-established line of some important thoroughfare being taken as a reference meridian, the origin and meridian being marked by monuments. The true meridian through a well-defined point would perhaps do as well, except that, if the subdivision of the city is on the

rectangular plan, and not exactly "with" the cardinal points, a great deal of computation would be avoided by making the axes parallel to the street lines.

127. Elementary problems. A large amount of surveying work is much facilitated by the use of coördinates, particularly by the use of the methods of the two following problems:

1. Given the coördinates of two points, to find the bearing and length of the line joining them.[1]

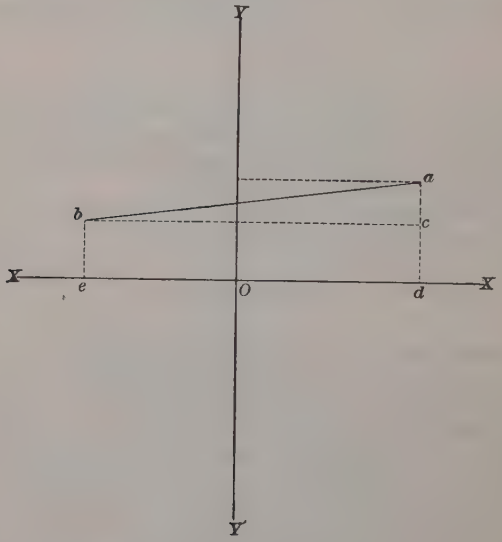


FIG. 85.

In Fig. 85 the coördinates of a and b are given. The angle cab is the bearing of ab, but the angle cba will be found first because the smaller angles can be found with greater precision.

$$\tan cba = \frac{ac}{bc}.$$

$ac = ad - cd$ = difference of ordinates.
$bc = Od - (-Oe)$ = difference of abscissas.

[1] The same as supplying a wanting course, Art. 117.

$$\text{Length of } ab = \frac{bc}{\cos cba}.$$

To solve this problem, therefore,

log tan smaller angle = log smaller difference of ordinates *minus* log larger difference of ordinates.

(The angle thus obtained is the bearing when the latitude difference is the greater, and is the complement of the bearing when the longitude difference is the greater.)

log length = log greater difference of ordinates *minus* log cos of angle found.

The letters to affix to the bearing may be determined by inspection of the coördinates of the points.

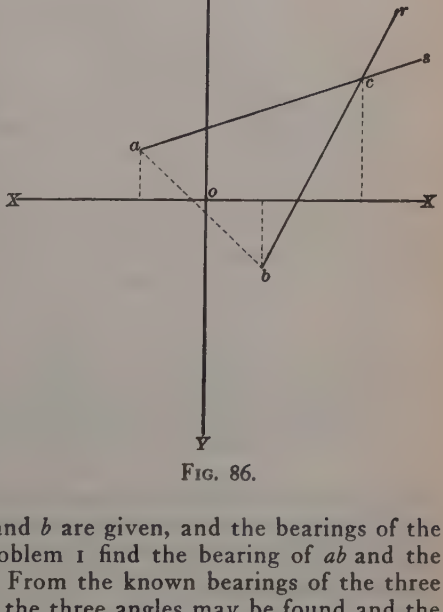


Fig. 86.

2. Given the bearings of two lines and the coördinates of a point on each, to find the location, *i.e.*, the coördinates, of the point of intersection of the lines,[1] Fig. 86.

The coördinates of *a* and *b* are given, and the bearings of the lines *as* and *br*. By Problem 1 find the bearing of *ab* and the *logarithm* of its length. From the known bearings of the three sides of the triangle *abc*, the three angles may be found and the triangle solved for either *ac* or *bc*. From the determined length of either *bc* or *ac* and its bearing, its latitude and longitude differences may be found and applied to the coördinates of *a* or *b*, according to the line chosen.

It is not necessary to find the length of either side unless the

[1] This problem is the same as supplying two wanting lengths.

conditions of the particular case require it; only the logarithm need be found.

This problem may be solved as an example in analytical geometry by those who have studied that subject.

128. To find the area. Areas are frequently more readily computed by the method of *coördinates*.

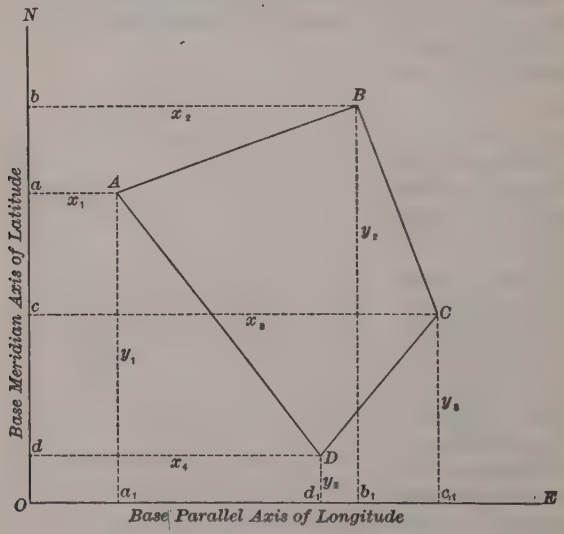


FIG. 87.

It will be seen in Fig. 87 that the area of the closed field $ABCD$ is the sum of the two areas, $bBCc$ and $cCDd$, less the sum of the two areas, $bBAa$ and $aADd$. Expressed in an equation this becomes

$$A = (y_2 - y_3)\frac{x_2 + x_3}{2} + (y_3 - y_4)\frac{x_3 + x_4}{2} - (y_2 - y_1)\frac{x_1 + x_2}{2} - (y_1 - y_4)\frac{x_1 + x_4}{2}$$

$$= \tfrac{1}{2}\{y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1 - (y_1x_4 + y_2x_1 + y_3x_2 + y_4x_3)\}. \quad (1)$$

This equation expressed in words as a rule is:

RULE: *To determine the area of a closed field when the coördinates of its corners are known: Number the corners consecutively around the field. Multiply each ordinate by the following abscissa and sum the products. Multiply each ordinate by the preceding abscissa and sum the products. One half the difference of the two sums, subtracting the second from the first, is the area of the field.*

Equation (1) may be written,

$$A = \tfrac{1}{2}\{y_1(x_2 - x_4) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_1 - x_3)\}. \quad (2)$$

This equation expressed in words as a rule is:

RULE: *To determine the area of a closed field when the coördinates of its corners are known: Number the corners consecutively around the field. Multiply each ordinate by the difference between the following and preceding abscissas, always subtracting the preceding from the following. One half the sum of the products is the area required.*

The second rule ordinarily involves less work than the first, but there are certain cases in which the first is used to better advantage. It will be evident that the coördinates of a field are the latitudes and longitudes of the corners.

129. To make the coördinates all positive. To lessen the danger of making errors in signs it will be better to arrange the axes so that the field shall lie wholly within the northeast quadrant. All the coördinates will then be positive. This may be done as follows: Determine the latitudes and longitudes of the corners of the field in the usual way with reference to the meridian and parallel through one corner, preferably the most westerly corner. If the most westerly corner is chosen, the longitudes will all be positive without further arrangement. To make the latitude ordinates all positive, add to each latitude a quantity equal to the greatest southern latitude. This will move the reference parallel to the most southern point. If considered more convenient, any round number, as 100, greater than the most southern latitude, may be added. Ordinarily an inspection will be sufficient to enable the computer to assume proper coördinates for the first corner beforehand.

130. Model example. Let the area of the field of the model example of Art. 115 be required by coördinates. The work is the same up to and including the determination of the balanced L's and M's. An inspection of these demonstrates that the first corner is the most westerly corner, and that the fourth corner is the most southerly, and that it is 283.9 feet south of the first. Therefore, if it is desired to make all coördinates positive, the reference meridian will be passed through the first corner, and the origin of coördinates will be taken on this meridian 300 feet south of the first corner. The coördinates of the corners and the area of the field are then found as in the table, in which the y's are the latitude ordinates and the x's are the longitude ordinates.

Corner No.	Y	Logs	Diff. X's	X	Double Areas. Sq. Feet
1	300.0	2.47712		0.0	
		2.53757	−344.8=2−5		
		5.01469			+103440
	+158.2			405.5	
2	458.1	2.66096		405.5	
		2.68214	+481.0=3−1		
		5.34310			+220340
	−222.3			+ 75.5	
3	235.8	2.37254		481.0	
		1.56348	− 36.6=4−2		
		3.93602			− .8630
	−216.6			−112.1	
4	19.2	1.28330		368.9	
		2.62356	−420.3=5−3		
		3.90686			− 8070
	+106.6			308.2	
5	125.8	2.09968		60.7	
		2.56691	−368.9=1−4		
		4.66659			− 46408
	+174.2			−60.7	
	300.0			0.0	+323780
					− 63108
					2)260672
				43560)130336	
					2.992 + acres.

Rather more work is required by this method than by the method of double longitudes. This is not the case when the corners have been determined by random lines rather than by a continuous traverse around the field.

When the coördinates of the corners are alone known, the method just given is by far the quickest, since it would be necessary to compute the L's and M's for each side before the D's could be obtained or the areas, just as it was necessary here to compute the coördinates from the L's and M's.

EXAMPLE

The latitudes and longitudes of a series of points are as follows, measured in chains:

A	Latitude	+ 13.63
	Longitude	+ 10.20
B	Latitude	+ 1.26
	Longitude	+ 5.10
C	Latitude	− 3.30
	Longitude	+ 7.20
D	Latitude	− 3.30
	Longitude	− 6.60
E	Latitude	+ 7.10
	Longitude	− 10.20

Find the area included by the broken line joining the points. (Shift the coördinate axes so that the coördinates are all positive.)

131. Elongated areas by offsets. A special case, in which the application of this method of coördinates is used advantageously, is the determination of elongated, irregular areas, the measurements for which consist of offsets at unequal intervals along a straight line. In this case, the line from which the offsets are measured is assumed as the reference parallel. Thus, in Fig. 88 it is required to determine the area between the line AH, the irregular line *abcdefgh*, and the two end offsets. The corners of the closed field are A*abcdefgh*H and their coördinates are as shown in the figure. x_1 and y_1 and x_2 and y_{10} are zero. The equation that would be written, following the second rule of Art. 128 would be as follows:

$$\begin{aligned} A = \tfrac{1}{2}\{ & y_1(x_2 - x_{10}) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_5 - x_3) \\ & + y_5(x_6 - x_4) + y_6(x_7 - x_5) + y_7(x_8 - x_6) \\ & + y_8(x_9 - x_7) + y_9(x_{10} - x_8) + y_{10}(x_1 - x_9)\}. \end{aligned}$$

y_1 and y_{10} being zero, the first and last terms will disappear; x_1 and x_2 being zero, the second term becomes y_2x_3, and the third term y_3x_4.

What is perhaps a less confusing system of writing these quantities is to write each ordinate and its corresponding abscissa in the form of a fraction, connecting each ordinate with the abscissas whose difference is to be taken as a multiplier. Thus, calling the x's ordinates and the y's abscissas (it is immaterial which is ordinate or abscissa),

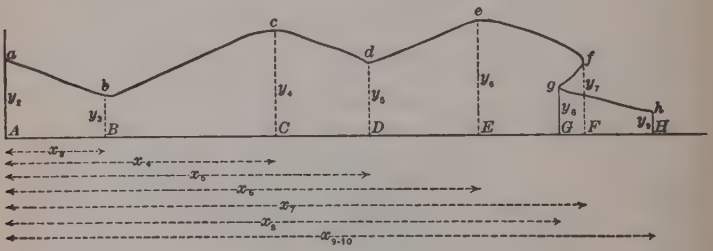


FIG. 88.

$$\frac{x_1}{y_1}\times\frac{x_2}{y_2}\times\frac{x_3}{y_3}\times\frac{x_4}{y_4}\times\frac{x_5}{y_5}\times\frac{x_6}{y_6}\times\frac{x_7}{y_7}\times\frac{x_8}{y_8}\times\frac{x_9}{y_9}\times\frac{x_{10}}{y_{10}}\times\frac{x_1}{y_1}$$

The downward lines to the right show the following or positive abscissas, and the downward lines to the left show the preceding or negative abscissas. The lines may be omitted as soon as the student becomes thoroughly familiar with the work. This arrangement applies to any closed field and not alone to the elongated strip last described, though it is particularly applicable to that case since the quantities may be arranged for computation as they are taken in the field. Applications of the coördinate method of surveying will be found on pages 433–439.

EXAMPLE

1. The following offsets were taken on one side of a base line, to determine the area between it and an irregular boundary line. The distances are all from zero. The result should be reported to the nearest square foot with a ± sign following.

Distance. Feet	Offset. Feet
0	20.7
202	31.5
358	42.6
825	53.2
984	36.1
1223	40.7

Area, 49,698.25 sq. ft.

132. Zero azimuth. It is thought that the reason for suggesting that zero azimuth shall be the north point, will now be clear. Since north latitude and east longitude are considered positive, and south latitude and west longitude, negative, a system of azimuths should be so arranged that the signs of the trigonometric functions of any azimuth shall agree with the signs of the corresponding latitude and longitude differences. Figure 89 shows the arrangement of signs of coördinates, and the correspondence of the signs of the trigonometric functions. The quadrants are not numbered as in trigonometrical figures, but this is immaterial; second quadrant trigonometric signs agree with second quadrant azimuth signs, etc.

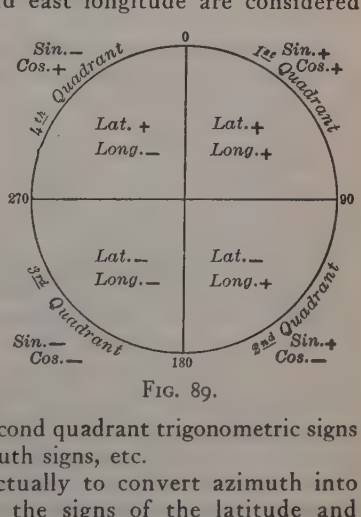


Fig. 89.

Thus there is no necessity actually to convert azimuth into bearing in order to determine the signs of the latitude and longitude differences, nor to carry in mind any other than the ordinary scheme of signs given in any work on Trigonometry. The signs of the coördinates could, of course, be changed to suit a south zero azimuth, but the custom among all people to look to the north as the orienting point, and the long use of the signs given for coördinates, seems to make it better for the surveyor to use the north as zero azimuth. Were it merely a matter of changing technical terms used only by the surveyor, such as changing "departure" to "longitude difference," the case would be different.

Example

Find the area of the field shown by the notes by coördinates, without converting the azimuths to bearings.

Sta.	Azimuth	Chains
1	128° 50′	5.05
2	238° 10′	4.63
3	330° 45′	4.16
4	45° 30′	2.87

DIVIDING LAND

133. Occurrence of the problem. It becomes necessary sometimes to divide a field into two or more parts of equal or known areas. This occurs when one man, as John Jones, sells to another, as Paul Smith, x acres to be laid off in the northeast corner of Jones's field. It occurs also in the division of inherited lands among the heirs, and in the determination of lands sold for taxes. When the taxes are not paid on a given piece of land, the land is sold to the lowest bidder. This means that the land is put up at auction for the taxes and expenses of sale, and that the person who agrees to take the least part of the whole piece, and pay therefor the taxes and expense of sale, is given a title to that portion of the land that he agrees to take. This title is redeemable by the original owner within a certain time, specified by law, after the expiration of which time, if the title has not been redeemed, it becomes vested in the purchaser forever. In any case that may arise the original tract will be fully known, either by previous surveys or by surveys made at the time and for the purpose of the subdivision. It will be known also in what way the land is to be divided, and the problem then becomes simply one of Geometry or Trigonometry.

134. Solution of the problem. The method of solving two common problems will be given, and others may readily be devised, on which the student may test his ingenuity.

1. It is required to lay off from a given field A acres by a line beginning at a given point in a given side. Plot the field to scale. Let Fig. 90 represent the field so plotted. Let m be the given point. Imagine the line mD to have been run to the

corner nearest the probable ending point of the required line mg. The point g on the line DE will first be determined as follows: Consider $mBCD$ as a closed field with length and bearing, or azimuth, of one side, mD, wanting. Find bearing and length of mD, and area of $MBCD$, which call a. Then

$$\text{Area } mDg = A - a.$$

In the triangle mDg, the angle at D is known, and the side mD. The area of the triangle is

$$A - a = \tfrac{1}{2}\, pk \sin D;$$

whence

$$k = \frac{A - a}{\tfrac{1}{2}\, p \sin D}.$$

The triangle may then be solved further, giving the bearing and length of mg.

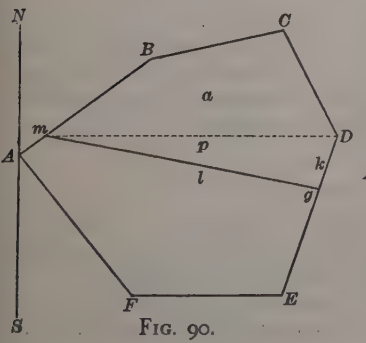


FIG. 90.

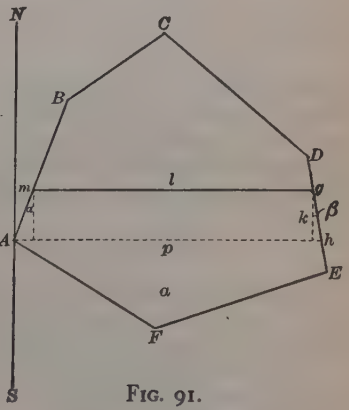


FIG. 91.

2. It is required to lay off from a given field A acres by a line extending in a given direction.

Let Fig. 91 represent the given plotted field. Select a corner of the field, as A, such that from this corner a line may be run in the given direction, cutting off, as nearly as may be, the required area A acres. Let mg be the required line, the area $AFEgm$ being equal to A. If Am were known, the line mg could be run. Since the length l should be computed for a check, and since it is somewhat simpler to determine l first

rather than Am, l will be first determined. Imagine the line Ah run in the given direction. Consider $AFEh$ a closed field with two wanting lengths; viz., Ah and Eh. Determine these and the area $AFEh$, which call a.

The area $Amgh = A - a$. Then

$$\frac{l+p}{2}k = A - a. \quad (1) \qquad l = p - k(\tan \alpha + \tan \beta). \quad (2)$$

α and β are known from the bearings;

whence $$k = \frac{p - l}{\tan \alpha + \tan \beta}. \quad (3)$$

Substituting in (1), $$\frac{l+p}{2} \times \frac{p - l}{\tan \alpha + \tan \beta} = A - a;$$

whence $$l = \pm \sqrt{p^2 - 2(A - a)(\tan \alpha + \tan \beta)}. \quad (4)$$

l being known, k is determined from (1), or (3), and Am, or hg, from the small right triangles. In the field, find the point m, and run l on the given bearing to its intersection with DE at g. See that the length agrees with the computed length, and that gE as measured agrees with gE as computed.

Examples

1. Let it be required to divide the field of Example 4, Art. 117, in two equal parts by a line running from Sta. 6. Find the bearing and length of the line and the point of intersection with the opposite side.

2. Using the same example, let it be required to run the dividing line due east and west. Find the length of the line and its points of intersection with the boundaries of the field.

CHAPTER VI

SOME SURVEYING PROBLEMS

135. Obstacles. In the field work of the surveyor, various obstacles arise which must be overcome. A few of the more common difficulties, with the methods of surmounting them, will be given.

I. In Fig. 92 it is required to produce the line XA in distance and direction.

1. At A erect the perpendicular AC, to which erect the perpendicular CD, which make long enough to pass the obstacle. At the point D erect the perpendicular DB equal to AC, and at B erect the perpendicular BY, which is the line produced. The distance AB equals CD.

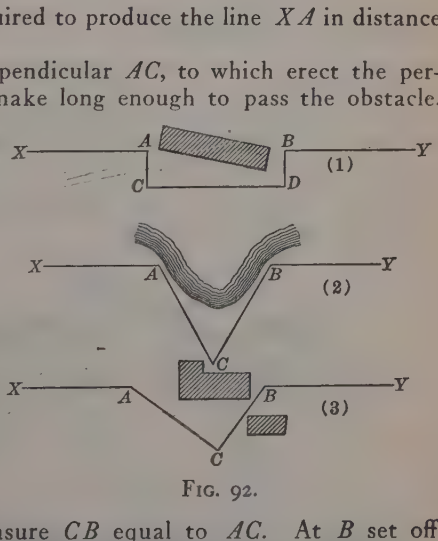


FIG. 92.

2. At A set off an angle of 60° and measure AC of such length that a line making an angle of 60° with AC will pass the obstacle. At C set off from AC an angle of 60° and measure CB equal to AC. At B set off the angle ABC 60°. The instrument will then be in the line XA produced, pointing toward A, and the distance AB equals AC or CB.

3. At A set off any convenient angle BAC and measure any distance AC. At C set off any angle ACB, making it 90° if convenient, and measure CB, a distance determined by solving

the triangle ABC, and at B set off an angle ABC determined in the same way.

II. If the object to be passed is small, as a tree that is not to be cut, a small angle α may be turned some distance from the obstacle, as at A in Fig. 93, and a point C set opposite the obstacle where an angle of 2α is turned and a point set at B, CB being made equal to AC. If the short distance OC be measured, then from Art. 13

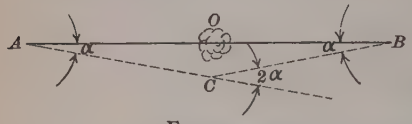


Fig. 93.

$$AB = 2\,AC - 2\,\frac{\overline{OC}^2}{2\,AC} = 2\,AC - \frac{\overline{OC}^2}{AC},$$

or if a table book is handy

$$AB = 2\,\overline{AC}\cos\alpha.$$

At B the angle α is turned from BC for the continuation of the line AB or, if azimuths are used, the transit at B is backsighted on C with the azimuth BC and turned to the azimuth of AB.

III. In Fig. 94 it is required to determine the length and bearing of the line AB.

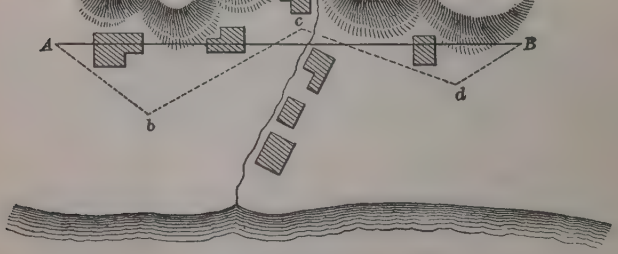


Fig. 94.

Run the random line $Abcd\,B$, noting bearings and distances. If the survey is treated as a closed field, AB will be the error of closure, or the closing line, or a wanting side which may be fully

determined in the manner previously given for finding the bearing and length of a wanting side.

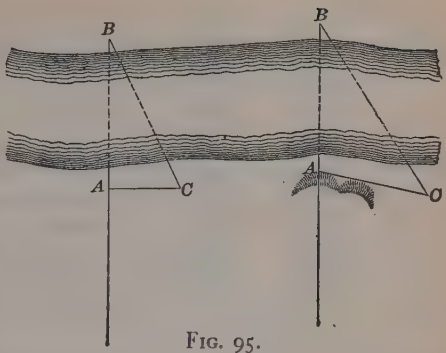


FIG. 95.

136. Problems. I. In Fig. 95 it is required to determine the distance *AB*, *B* being an inaccessible point, or the stream too wide to be spanned by a single chain or tape length.

1. At *A* set off the angle *BAC* equal to 90° and measure *AC* a convenient distance, not so short as to make the angle *ABC* less than 20°, unless this is unavoidable, and at *C* measure the angle *BCA*. Solve the right-angled triangle.

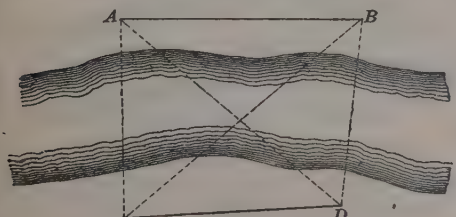


FIG. 96.

2. At *A* set off any convenient angle *BAC* and measure any convenient distance *AC*, observing the previous caution with regard to the angle at *B*. Measure the angle at *C* and solve the triangle *ACB*.

II. In Fig. 96 it is required to determine the length and bearing of the inaccessible line *AB*.

1. Measure the line *CD*, from each end of which both *A* and *B* may be seen, and determine its bearing. With the instrument first at *C* and then at *D* measure three angles at each point. Solve the triangle *ACD* for *AC*, *BCD*

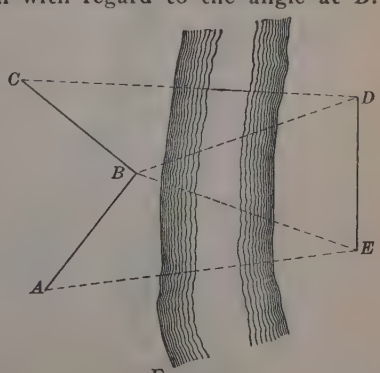


FIG. 97.

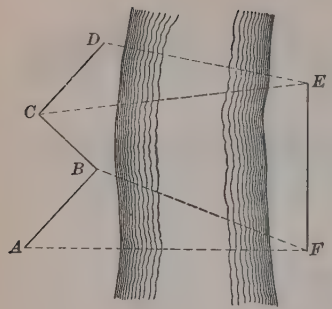


FIG. 98.

Measure AB and BC. Measure the angles at C, B, and A. Solve CBD for BD and BDC. Solve BEA for BE. Solve BDE for DE and BDE which with BDC applied to the bearing of CD will give the bearing of DE.

for CB, and ABC for AB, and an angle at A or B. Another combination of triangles could be used.

III. In Fig. 97 it is required to find the length and bearing of DE inaccessible, D and E being both visible from only one point, B. Select points A and C from which respectively B and E and B and D can be seen.

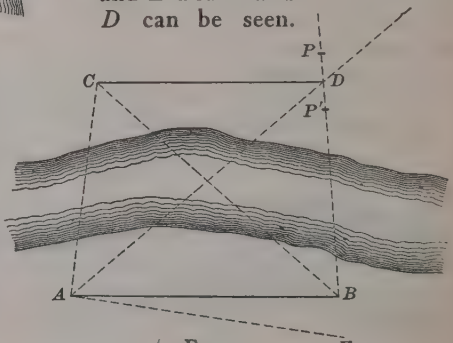


FIG. 99.

IV. In Fig. 98 it is required to find the length and bearing of EF inaccessible from A, B, C, and D, E being visible only from D and C, and F from A and B. Let the student solve and put the solution on the margin of the book.

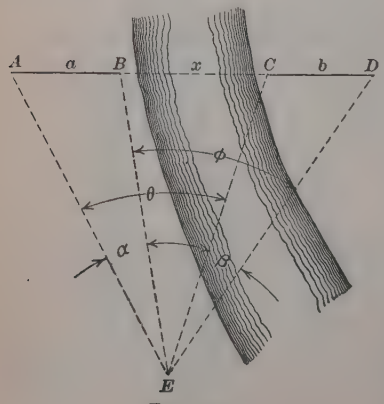


FIG. 100.

V. In Fig. 99 it is required to find a point in a line through C and parallel to AB, C being inaccessible from AB, but it being possible to have a man on the C side of the stream. Set

up at A and measure the angle CAB, and at B measure the angle CBA; lay off the angle $ABP = CAB$ and set two points P and P' on BP, one on either side of the required line through C as nearly as can be judged. Stretch a string from P to P'; set up at A, lay off $BAD = CBA$ and set D under the string joining PP'. CD will be the required parallel line. If CD is marked and B unknown, and it is required to run a line through A parallel to CD, select some other point E, measure AE and proceed to find the angle that AC makes with CD by the method of Problem II. CAB is its supplement.

VI. In Fig. 100 it is required to find $BC = x$, a and b being measured and all the angles at a selected point E to one side of the line AD.

In the triangles AEC and AEB,

$$BE = \frac{a \sin A}{\sin \alpha}$$

$$EC = \frac{(a + x) \sin A}{\sin \theta} \text{ and } \frac{EC}{BE} = \frac{(a + x) \sin \alpha}{a \sin \theta};$$

in the triangles CED and BED

$$EC = \frac{b \sin D}{\sin \beta}$$

$$BE = \frac{(b + x) \sin D}{\sin \phi} \text{ and } \frac{EC}{BE} = \frac{b \sin \phi}{(b + x) \sin \beta};$$

whence

$$\frac{(a + x) \sin \alpha}{a \sin \theta} = \frac{b \sin \phi}{(b + x) \sin \beta},$$

and $$(a + x)(b + x) = \frac{ab \sin \phi \sin \theta}{\sin \alpha \sin \beta}.$$

Let the student solve and get

$$x = -\frac{a + b}{2} \pm \sqrt{\frac{ab \sin \phi \sin \theta}{\sin \alpha \sin \beta} + \left(\frac{a - b}{2}\right)^2}$$

and tell which of the two signs to use.

VII. In Fig. 101 it is required to find the point D on AB that

is in a perpendicular to AB from C which is visible but inaccessible.

1. Set two transit points, A and B, a few feet apart one on either side of C, determined by judgment; stretch a string between them and set the transit over the string until by trial an angle of 90° turned from A or B will cause the line of sight to pass through C.

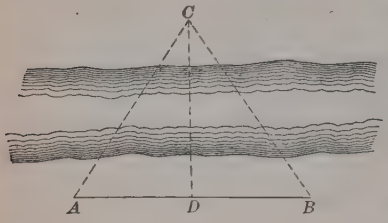


FIG. 101.

2. Set up at A and B and measure the angles CAB and CBA; measure AB; solve the triangle ACB for AC or BC and either ACD or BCD for AD or BD. Note that neither AC nor BC need be found; thus, using the sine formula for AC, $\log AD = \log AB + \log \sin B + \log \cos A - \log \sin C$. In making computations, unnecessary intermediate steps, such, in this case, as finding $\log AC$, should be avoided by always writing out the complete formula for the quantity required, when possible combinations to save time will appear.

VIII. In Fig. 102 let it be required to find the height of the ball at B, it being impracticable to measure to the point directly under B.

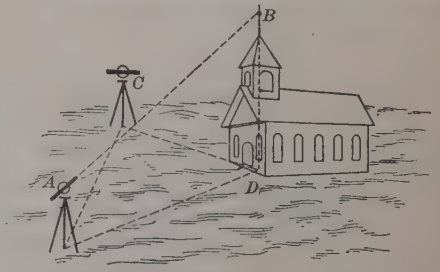


FIG. 102.

Set up at C and measure the horizontal angle DCA, using B for the sight over D. Set up at A and measure DAC. Measure AC, then AD may be computed. Now if the angle of elevation at A is measured, the height of B above A may be computed, and if the telescope is leveled and a mark made on the building, the distance down to the base may be measured from it and added to the height of B above A to get the height of the ball above the ground. The same method will determine the height of a mountain without ascending it, though if the moun-

tain is very far away, the effect of refraction and curvature must be considered and it is the height above the *point of observation* that is determined.

IX. In many surveys it is necessary to locate details of buildings for plotting a map. There are several ways of doing this according to the location, shape, and magnitude of the objects and the precision required. Thus, in running the line *AB*, Fig. 103, the building at (1) may be located by rectangular offsets at *e* and *h*, the right angle being estimated by eye, measured with an optical square, a chain or tape, or the transit, the positions of *e* and *h* being found by trial. It must be remembered that two points, or one point and a direction, must be known to determine any straight line. Therefore, in (1), if *eE* and *hH* are found, either *I* or *K* and *G* must be found by

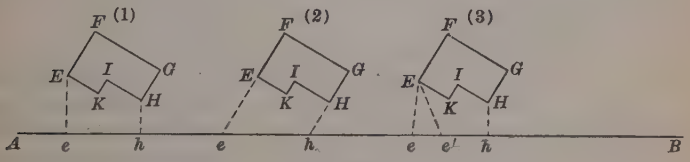


FIG. 103.

additional offsets to completely locate the building or, having *K* or *I* located, *FG* may be found by measuring *HG* or *EF*.

Query. If *eE*, *hH*, and the distances *KI* and *HG* are measured, *eh* being known from the positions of *e* and *h* on the line *AB*, can the building be mapped?

In (2) the building is determined by first finding the point *e* where the line *AB* is intersected by the plane of one end of the building produced. This may be done by eye for all but work requiring instrumental precision, as in some city work, the instrument man lining in a flagman between *A* and *B* while the flagman places himself by eye in the line *EF*. After finding the point *e*, the angle *EeB* is measured with the tape or instrument and the distance *eE* and the dimensions of the building are measured. Instead of measuring the angle, a second line *hH* may be determined. This will give the length of the building and the offset *KI*, so that it will be necessary to measure only *HG* and either *EK* or *IH* of the building dimensions.

In (3) Ee and Ee' are measured from convenient points to locate E; Hh is measured from a convenient point h; EK, KI, and IH are measured; EH is computed as the hypotenuse of a right triangle of sides $EK + IH$ and KI, and H located by arcs swung from E and h; HG or EF is measured to completely locate the building.

The second method is the quickest and simplest when it can be used.

In some cases several near-by buildings or objects are to be located. In the case of buildings this may be done by referring one building to another as suggested in Fig. 104, which the student may use to explain how to locate M and L after the building EFG is located, adding or omitting dotted lines as is found necessary.

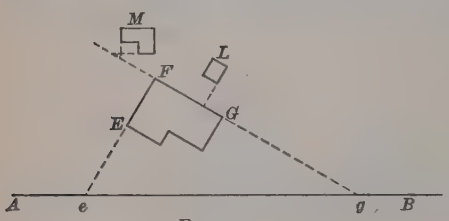


FIG. 104.

In the location of objects with reference to a transit line, no obscurity as to methods or necessary measurements need arise if it is always remembered that to determine a line two points, or one point and a direction, are necessary; to determine a point with reference to a line, its direction and distance from a known point on the line is necessary; and that if the hypotenuse and one side of a right triangle are given, the triangle may be constructed on the drawing board with a draftsman's compass and straight edge, or even by trial with a triangle alone.

137. Two common problems. I. In Fig. 105 it is required to determine the lengths and bearings of the sides of the field $ABCDEF$, the lines being through woods and no two corners visible, the one from the other. The positions of the corners are, however, known.

Run the random line $AabBcdCefDgEhikFlA$, and compute the latitudes and longitudes, or the coördinates, of the points $ABCDEF$, referred to the true or any convenient meridian, preferably the true meridian through the most westerly point of the survey. From these coördinates determine the latitude and longitude differences and the bearing and length of each course.

II. If a corporation, as a mining company, proposes to purchase a considerable tract of ground containing many small parcels owned by different individuals, and a description of each parcel with its area is wanted, there are two ways of obtaining the information.

1. Each parcel may be surveyed separately, determining the bearings and lengths of the courses and the areas.

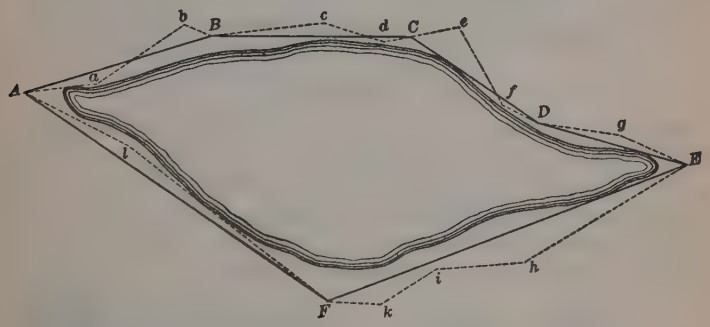


FIG. 105.

2. By far the quicker and better way is to run a random field, touching on the various corners of the various parcels, to compute the coördinates of these corners, and from these the bearings, or azimuths, and the lengths of the sides and the areas. If a separate description of each piece is not required, but only the area, the computation of bearing and length of course may be omitted and the areas may be obtained at once from the coordinates of the corners.

CHAPTER VII

SOME SPECIAL OFFICE INSTRUMENTS

THE PLANIMETER

138. Description. The most elegant and rapid method of obtaining the area of an irregular figure is to draw the figure to scale and measure the area with a planimeter. There are three kinds of planimeters, shown in Figs. 106, 107, and 108.

Figure 106 shows the polar planimeter, that most commonly used. Figure 107 is a suspended planimeter, which is a polar

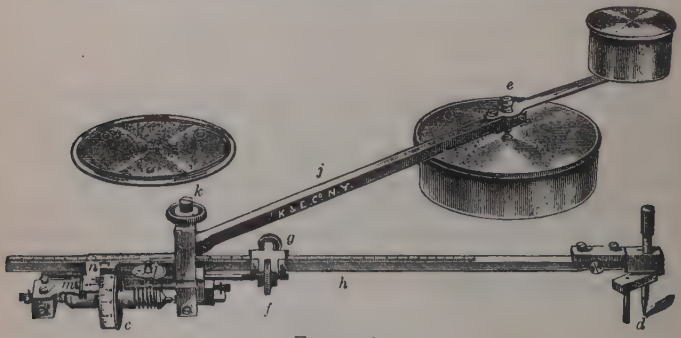


FIG. 106.

planimeter so arranged that the wheel *c*, Fig. 106, will roll on a polished surface instead of on the drawing. Because of the smooth surface on which the registering wheel rolls, this instrument is more accurate in its results than the polar planimeter. Figure 108 is a rolling planimeter. This is the most costly and the most accurate of the three forms. Its principle of action is somewhat different from that of the polar planimeter.

The polar planimeter consists of two arms, *h* and *j*, one of

which, j, is of fixed length, and the other is adjustable through the frame shown on the left. There is a clamp at g and a slow-

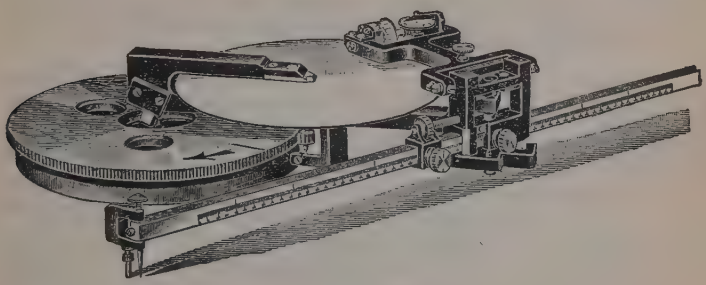


FIG. 107.

motion screw f for setting the adjustable arm to the required length. The arm j is pivoted at k, and at n on the frame is a

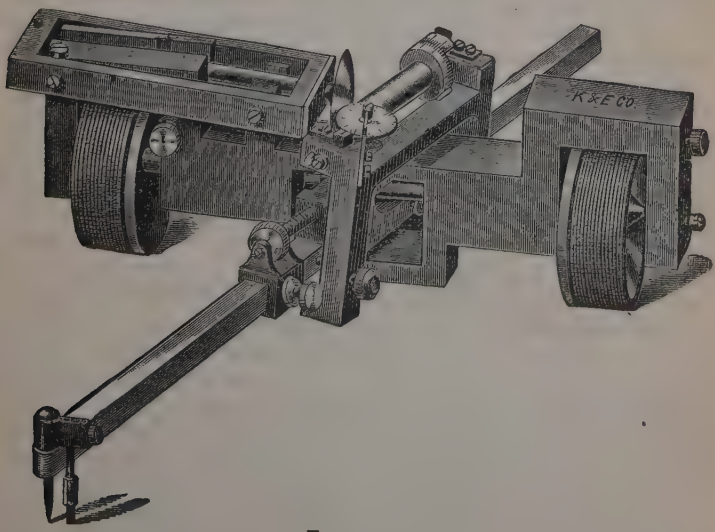


FIG. 108.

vernier graduation which indicates the length of the adjustable arm. There is also a wheel c, whose axis of revolution is parallel

to the arm *h*, and, in the discussion that follows, will be considered a part of that arm. The wheel is so mounted as to be almost frictionless. The disk *l* records entire revolutions of the wheel, while by the vernier *m* the fractional revolutions are read.

139. Use. The point *e* is fixed in the drawing board, preferably outside of the figure to be measured. The tracer *d* is then placed on a point in the circumference of the figure, and the wheel *c* is read. The wheel could be set to zero, but this is not easy to do, and, hence, it is read at whatever it happens to be. The tracer *d* is now moved carefully around the circumference and stopped at the beginning point. The wheel is read again, and the difference of the two readings indicates the number of revolutions made by the wheel. The instrument is made so that if the tracer is moved clockwise, the wheel will roll in the direction of its graduation, and hence the final reading will be greater than the initial reading, and *vice versa*. The wheel reads by its vernier to thousandths of a revolution. The principle of the instrument is such that the distance rolled by the wheel, or the number of revolutions times the circumference in inches, multiplied by the length of the arm in inches, is the area in square inches bounded by the path of the tracer *d*. The arm *h* is generally so set that ten times the number of revolutions is the area. If other units, as tenths of a foot, are used for the above-named linear units, the area will be given in square units of the same kind. In any event, it is the area of the drawing that is measured. If this drawing has been made to a scale of, say, forty feet to an inch, a square inch of paper is equivalent to sixteen hundred square feet, and the area given by the instrument must be multiplied by this quantity to get the area in square feet. If the fixed point is placed inside of the area to be measured so that the tracer *d* in circumscribing the required area makes a complete revolution about *e*, there must be added to the result in square inches a certain constant area — a constant for each setting of the arm *h* — called the area of the zero circumference. The drawing should be perfectly smooth. Change of paper with change of atmospheric conditions makes the scale vary, so that not too high a degree of precision should be expected, nor extreme uniformity of measurements made at different times. The polar planimeter will usually give results within one per cent of the truth if care is

used. The scale of the drawing should be tested and, if necessary, a correction can be made.

140. Theory. The following is a discussion of the theory of the instrument, written for those who are not familiar with the principles of the Calculus.

In the figures that follow, the essential parts of the instrument are lettered as in Fig. 106. The instrument is so constructed that neither d nor c can cross j. The length of the arm h is from tracer point d to pivot k, though the index for reading the length is to the left of k in Fig. 106. In circumscribing an area, the curved path of d may be conceived as divided into an infinite number of infinitesimal portions, each a straight line.

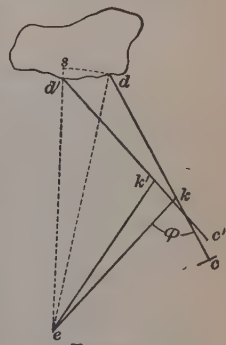


FIG. 109.

Each of the small portions may be conceived as made up of two parts or component motions, one radial with reference to e, and the other circumferential.

With the radial motion the value of the angle ϕ is constantly changing, while with the circumferential motion the value of ϕ and the length of the line ed remains fixed. In Fig. 109 the component motions sd and sd', and their resultant motion dd', are shown greatly enlarged. It will be evident, that, in circumscribing a closed figure, each minute movement of *d toward e* will have its corresponding movement *from e* with the *same value of* ϕ. Each element of right-hand circumferential motion will have its corresponding element of left-hand circumferential motion; but, since d will be farther from e for one than for the other, these corresponding elements will not be made with equal values of ϕ.

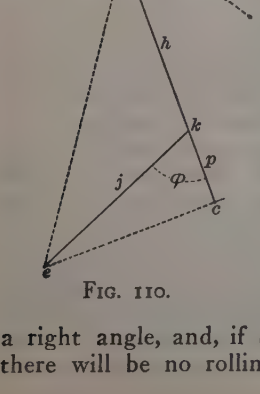


FIG. 110.

When the plane of the wheel c, Fig. 110, passes through e, the angle dce is a right angle, and, if d is revolved about e with ϕ constant, there will be no rolling of the wheel c, because the direction

of motion of all points of the instrument about e is circumferential, and the radius of motion of the point c is ec, and this is normal to the axis of the wheel, and hence the motion of the wheel is parallel to its axis, and the wheel simply slips and makes no record. The path described by d for the particular value of ϕ that brings about the above result, is known as the zero circumference. Its radius, ed, may be easily shown to be

$$R = \sqrt{j^2 + h^2 + 2ph}, \tag{1}$$

$$\phi = \cos^{-1}\frac{p}{j}. \tag{2}$$

It will be evident that if d could be moved *outward* till c should fall in the line ek, and then rotated clockwise, the motion of c would be all rolling motion, and would be, looking from c to d, clockwise. The wheel is graduated so as to record positively for this kind of motion. Between these two positions the motion of the wheel will be partly slip and partly roll, the amount of each depending on the value of ϕ; and the roll will all be clockwise. It will be further evident that, if d were moved *in* till c should fall in the line ke produced, and then rotated clockwise, the motion of c would be all roll and would be counter-clockwise or left-handed, when looking as before from c to d. For d between the zero circumference and the last-named position, clockwise motion will produce a motion of c partly roll and partly slip. The amount of each will depend on the value of ϕ, and the roll will be counter-clockwise. Hence, clockwise motion of c will be caused by positive motion of d *outside* the zero circumference, and by negative motion of d *inside* the zero circumference, and *vice versa;* and, since the amount of roll for a given motion of the tracer depends on the value of ϕ, any two equal infinitesimal motions in opposite directions with the same value of ϕ will produce no resultant roll of the wheel, while, if made with unequal values of ϕ, there will be a resultant roll of the wheel that can be read. For these reasons, the radial components in circumscribing a closed figure cause no resultant rolling of the wheel and may be neglected, while the circumferential components do cause a resultant roll and must be considered. It will be shown that the roll of the wheel for a given circumferential motion of d is proportional to the area included

between the path of d, the radial lines from e to the initial and final points of d's path, and the arc of the zero circumference included between those lines.

Let dd', Fig. 111, be a minute circumferential component of d's motion due to the movement of the instrument about e as a center, through the angle Δ, ϕ remaining constant. The wheel will move through the arc cc', and will roll partly and slip partly. The rolling component of its motion will, of course, be normal to its axis, and may be represented by the line cs, csc' being considered an infinitesimal right-angled triangle. Let A be the length in linear units of an arc of radius unity and central angle Δ. (For any other radius X the length of the arc for a central angle of Δ would be X times A.) The angle cec' is Δ, and the arc cc' is given by

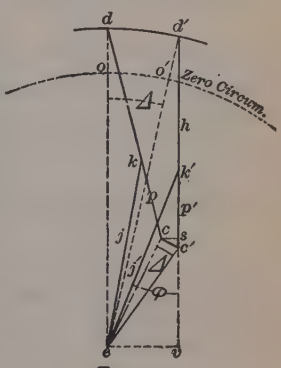


FIG. 111.

$$cc' = \overline{c'e} \cdot A.$$

The roll of the wheel is

$$cs = \overline{c'e} \cdot A \cdot \cos c'cs.$$

The angle Δ being very small, cc' may be considered perpendicular to $c'e$, whence

$$c'cs = ec'v,$$

ev being drawn perpendicular to $d'c'$ produced. Then

$$c'v = \overline{c'e} \cdot \cos c'cs;$$

whence $$cs = A \cdot \overline{c'v};$$

now $$c'v = j \cos \phi - p;$$

therefore, $$c'v = A\,(j \cos \phi - p), \qquad (3)$$

which is the roll of the wheel for the motion of d through the arc dd'. To show that this is proportional to the area $dd'o'o$, there must be deduced an expression for that area.

From Trigonometry

$$ed = \sqrt{j^2 + h^2 + 2jh \cos \phi},$$

$$dd' = ed \cdot A.$$

The area of a sector of a circle is the product of one half its arc by its radius, whence

$$\text{Area } edd' = \tfrac{1}{2} A (j^2 + h^2 + 2jh \cos \phi). \qquad (4)$$

Using the value of the radius of the zero circumference given in equation (1), there results for the value of the area eoo',

$$\text{Area } eoo' = \tfrac{1}{2} A (j^2 + h^2 + 2ph). \qquad (5)$$

Subtracting (5) from (4), there results

$$\text{Area } dd'o'o = Ah (j \cos \phi - p). \qquad (6)$$

This area is equation (3), the roll of the wheel, multiplied by the length of the adjustable arm, and hence is proportional to the roll of the wheel. Q.E.D.

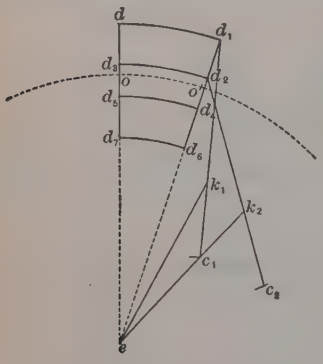


Fig. 112.

It is to be shown now that, in tracing a closed area, the record of the wheel is correctly summed. In Fig. 112 let the tracing point move about the area $dd_1d_2d_3d$ clockwise. Motion from d to d_1 will cause a clockwise roll of the wheel proportional to the area $dd_1o'o$. The motion from d_1 to d_2 will be neutralized by motion from d_3 to d. Motion from d_2 to d_3 will cause counter-clockwise roll of the wheel proportional to the area d_2d_3oo', and the resulting roll will therefore be proportional to the

$$\text{Area } dd_1d_2d_3.$$

The student may reason similarly for the other areas. Since the roll of the wheel is proportional to the area lying between d's path and the zero circumference and is positive, or clockwise, when d is outside the zero circumference and moves to the right, and negative when d is inside and moves to the right, it follows

that if an area is traced with e inside that area, so that d must complete a revolution about e, there must be added to the area obtained by multiplying the roll of the wheel by h, the area of the zero circumference. This area is

$$Z = \pi (j^2 + h^2 + 2\,ph).$$

141. To find the zero circumference. This area is usually furnished with the instrument when it comes from the maker, but may be found thus:

Measure a known area with the point e within it and compare the result by the instrument with what is known to be the correct area. The difference is Z. This should be done a number of times, and a mean value of the several determinations used.

142. To find the circumference of the wheel. If n is the number of revolutions, and c is the circumference,

$$\text{Roll of wheel} = nc.$$

In a given circumscribed area with e outside,

$$A = hnc.$$

If c is not known, measure a known area with any convenient length of arm and note the reading of the wheel, which is n. From the known quantities compute c. This should likewise be done a number of times.

143. Length of arm. It is very convenient to make h such a length as will reduce the work of multiplying hnc to a minimum. Most instruments are made so that the length of the arm may be such that

$$A = 10\,n.$$

To find what this length is for a given instrument in which c is known, let it be assumed that *one* revolution of the wheel shall correspond to ten square inches;

then $$10 = hc,$$

and $$h = \frac{10}{c}.$$

If the arm h is not graduated, it may be set by trial so that $A = 10\,n$ and the value c will not be required. Some cheap forms of the instrument are made with the arm h fixed in length. When so made they are usually proportioned so that $A = 10\,n$.

Examples

1. The wheel of a planimeter has a circumference of 2.26 inches. What should be the length of tracer that the number of revolutions multiplied by 10 shall give in square inches the area circumscribed?

2. An area is circumscribed by a planimeter with the fixed point inside, and the resulting reading is found to be 11.26 square inches. The area is known to be 120 square inches. What is the area of the zero circumference?

THE SLIDE RULE

144. Description. The slide rule is an instrument for determining products, quotients, powers, and roots of numbers mechanically. This is done by adding and subtracting logarithms graphically. Mantissas of logarithms of numbers from 1 to 10 which are the same for numbers from 100 to 1000 or 1000 to 10,000, are laid off to scale on two pieces of rule which are made to slide by each other so that the sums of the logarithms or their differences can be obtained mechanically. The scale is numbered from 1 to 10 at the points which mark the logarithms of the several numbers used. The common scale is 5 inches and the common rule is 10 inches long, so that the series of logarithms is put on twice and the numbering either repeated for the second set or continued from 10 to 100. The most common form of the rule is the Mannheim rule. On this rule, which is made essentially as shown in Figs. 113 and 114, there are two scales A and B just alike on rule and slide and C and D just alike on rule and slide. C and D differ from A and B in being graduated to twice the scale unit that A and B are graduated to, so that the logarithm of a given number on C and D is twice as long as the logarithm of the same number on A and B, and,

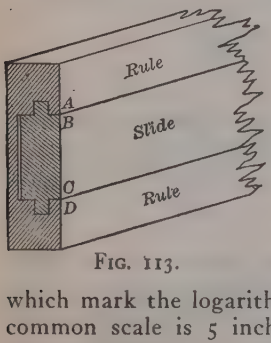


Fig. 113.

therefore, any number on the lower rule or slide is opposite its square on the upper rule or slide; and if the upper rule and slide be regarded as standard logarithmic scales, the lower rule and slide will be standard logarithmic scales of the squares of the numbers shown. The carpenter's rule has only one scale of squares, that on the lower rule, the other three scales being alike. On the reverse of the slide one usually finds logarithms of sines and tangents laid out for trigonometric calculations. The slide is taken out of the rule, turned over, and replaced for such work. There are various other scales on various makes of rules, all of which are explained in the pamphlet of instructions that should accompany each rule.

FIG. 114.

145. Use of the rule. The following illustrations will show the use of the rule:

1. *Multiplication.* Fig. 115. To find the product of two numbers, say 2×3, with the rule, set 1 of scale *B* opposite 2 of scale *A*, and opposite 3 of scale *B* find the product 6 on scale *A*. For in this way to log 2 on scale *A* we add log 3, found on scale *B*, giving log 6 on scale *A*.

If 2 is to be multiplied by any other number, as 4, the same setting is made, and opposite 4 of *B* we find the product on *A*.

If any constant number *b* is to be multiplied by a series of numbers, so that we wish to find the value of bx for different values of x, opposite *b* of *A* set 1 of *B*, and opposite x of *B* read the product bx on *A*.

2. *Division.* Fig. 115. To divide a number by another as $8 \div 4$, set 4 of *B* opposite 8 of *A*, and, opposite 1 of *B*, read the quotient 2 on *A*. For, in this way we take away log 4 from log 8 and find the number on *A* that corresponds to the remainder.

Fig. 116. To divide a series of numbers x by a constant number *a*, opposite 1 of *A* set *a* of *B*, and opposite x of *B* read the quotient $x \div a$ on *A*.

3. *Proportion.* An expression in the form $\frac{bx}{a}$, as $\frac{4 \times 3}{2}$, may be written $\frac{4}{2} \times 3$. In Fig. 115, opposite 4 of *A* set 2 of *B*, and the index (1) of *B* will be opposite the quotient $4 \div 2$ on *A;* then opposite 3 of *B* read the result 6 on *A*, for in this way we add log 3 to the logarithm of the quotient.

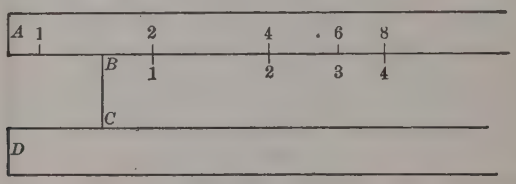


FIG. 115.

In any case, since mantissas only are used, one will get simply the proper figures in proper sequence, and must determine the decimal point from the nature of the problem or a rough calculation.

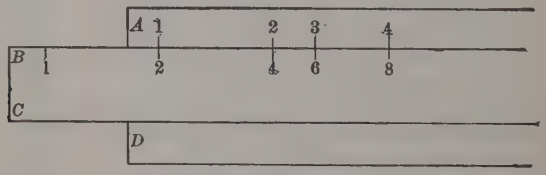


FIG. 116.

146. Shifting the slide. As the mantissas only are used, and as the addition or subtraction of the distance between the indices of the scales would affect the characteristic alone, the position of the slide can be changed, by bringing one index into the position previously occupied by another index, without affecting the sequence of figures in the result. This is necessary sometimes when the result from one setting falls off the rule. If the index is shifted through one length, the result will be found on the rule.

147. Use of the runner. The runner found on all rules is used to mark the result of a part of a computation that must

be divided into two or more computations. Thus, if $y = \frac{abc}{d}$ is to be calculated, $\frac{ab}{d}$ is all that can be done with a single setting. The runner is brought to the result $\frac{ab}{d}$, the index of the slide is brought to the runner so that log c may be added to log $\frac{ab}{d}$. The runner may also be used in finding squares and square roots. Thus, if the square of 6.2 is desired, the runner is brought to 6.2 of scale D and the square is found at the runner on scale A; and conversely, if the square root of a number is wanted, the runner is brought to the number on scale A and its square root will appear at the runner on scale D. This condition must be noted: If the number has an odd number of digits to the left of the decimal point, the square root is found on the left half of the rule, if an even number, on the right half, and if the number is wholly fractional, then if there be an odd number of ciphers following the decimal point, the root is on the left half, and if an even number (including none), the root is on the right half. If this rule cannot be remembered, the first figure of the root may be determined mentally, and the proper half of the rule found by inspection.

148. Problems with the slide direct. The following problems may be solved with the slide direct; that is, in its usual position in the rule, its graduations corresponding to those on the rule when the indices are brought together. Set for the constants first, then any number of results may be had without changing the setting for different values of x. In all cases set the denominator on the slide at the constant numerator on the rule, using the upper or lower rule and slide according as the number is first or second power.

1. $y = \frac{bx}{a}$ answer found on rule A.

2. $y = \sqrt{\frac{bx}{a}}$ answer found on rule D.

3. $y = \frac{b^2x}{a}$ answer found on rule A.

4. $y = \sqrt{\frac{b^2x}{a}}$ answer found on rule *D*.

5. $y = \frac{bx}{a^2}$ answer found on rule *A*.

6. $y = \sqrt{\frac{bx}{a^2}}$ answer found on rule *D*.

7. $y = \frac{bx^2}{a}$ answer found on rule *A*.

8. $y = \sqrt{\frac{bx^2}{a}}$ answer found on rule *D*.

Find x on the lower slide.

9. $y = \frac{b^2x^2}{a^2}$ answer found on rule *A*.

Perform the whole operation on the lower slide and rule, but read the answer on the upper rule opposite x on the lower slide.

10. $y = \sqrt{\frac{b^2x^2}{a^2}}$ is the same as No. 1.

Thus far it has been assumed that all ordinary calculations are made with scales *A* and *B*, and the rules are generally made with the expectation that these scales will be used. But the greater degree of precision to be obtained by using scales *C* and *D* for ordinary multiplication and division, and the greater ease with which they are set owing to the greater scale unit, has led most practising engineers to use these scales instead of the upper scales *A* and *B*. The results more frequently fall off the rule; but this can be largely obviated by giving a little thought to what the result is to be. A little experience enables the operator to set the rule right for most cases.

149. Problems with the slide inverted. When a series of values is to be obtained for a proportion involving a constant product in its numerator and a variable quantity in the denominator, it is most convenient to use the slide inverted, since

a single setting will give the product of the two quantities of the numerator opposite the index of the slide so that any number of logarithms may then be subtracted without changing the setting. The runner must be used, since slide B and C will no longer be adjacent to rule A and D respectively. Thus, with the slide inverted, to solve the expression $\frac{ab}{x}$ set b on slide B opposite a on rule A, when the two indices of the rule and slide will be apart by the sum of the logarithms of a and b, and logarithm x may be subtracted from either the rule or the slide so that the answer may be obtained on either; looking for x on the slide B the answer is opposite on rule A, or looking for x on rule A the answer is opposite on slide B. Care must be taken not to confuse 6's and 9's.

If the x and a in the problems given for the slide direct change places, the computations may be made with the slide inverted, always setting for the constant product of the numerator first.

In the form of rule shown in Fig. 114, a reversed scale corresponding to D is shown in the middle, and this may be used with the runner without inverting the slide. This is a modern and most convenient form of rule.

150. Use of the runner in complicated expressions. If we have found the value of an expression, say $\frac{ab}{c}$, and marked its place on the rule by placing the runner there, then, to multiply it by a fraction $\frac{d}{e}$, we bring the denominator e on the slide to the runner and read the result on the rule opposite d on the slide, for this subtracts log e from log $\frac{ab}{c}$, and then adds log d to the remainder.

$$y = \frac{abcde}{ghk} = \frac{ab}{g} \cdot \frac{c}{h} \cdot \frac{d}{k} \cdot \frac{e}{1}.$$

Denoting the runner by R, we have

A Opp. a.	Read y
B Set g. R to b. h to R. R to c. k to R. R to d. 1 to R Opp. e	

EXAMPLES

$$\frac{4\cdot5\cdot6\cdot7}{2\cdot3\cdot4}=\frac{4\cdot5}{2}\cdot\frac{6}{3}\cdot\frac{7}{4}=35.$$

$$\frac{5\cdot6\cdot7\cdot8}{2\cdot4\cdot5\cdot6}=\frac{5\cdot6}{2}\cdot\frac{7}{4}\cdot\frac{8}{5}\cdot\frac{1}{6}=7.$$

$$\frac{8\cdot9}{2\cdot3\cdot4\cdot6}=\frac{8\cdot9}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{6}=0.5.$$

An expression of the form

$$y=\frac{a(m^2+n^2+r+sp+q^3)}{b}$$

may be written

$$y=\frac{a}{b}m^2+\frac{a}{b}n^2+\frac{a}{b}r+\frac{a}{b}sp+\frac{a}{b}q^3,$$

and the values of the several terms found separately. Notice that one setting will answer for the first three terms.

151. Gage points. When a formula is used often, as that for computing the horse power of an engine, it will be found convenient to combine the constant factors, sometimes using the resulting constant, and sometimes its reciprocal. Such a constant is called a *gage point*. There are various commonly used points, such as π, marked on the different rules, the particular points being named in the descriptions accompanying the rules.

152. Extraction of cube roots. $y=\sqrt[3]{b}$. Since $\sqrt[3]{8.000}=2$, $\sqrt[3]{80.000}=4+$, $\sqrt[3]{800.000}=9+$, the cube root of any sequence of figures will depend upon the position of the decimal point, so that the first figure of the root should be known approximately in order that the wrong number may not be taken. This first figure may be found by dividing the given number into sections containing three digits each, commencing at the decimal point, and extracting the cube root of the left section. Then, with the slide *inverted*, opposite b of A, set one of the extreme indices (1) of the slide, and find the number on D that is opposite the same number on B. This will be the cube root required. Sometimes the right index of the slide must be used, at other times the left index.

Otherwise: With the slide direct, bring the first figure of the root on B under the cube on A; note that the index of the slide C is near the first figure of the root on D, and move the slide until the same numbers on B and D are opposite the cube on A and the index on C, respectively. On some rules there is a cube root scale on the edge of the rule.

153. Slide reversed. The reverse of the slide sometimes has a scale of logarithmic sines along one edge and a scale of logarithmic tangents along the other, the scales being so arranged that the numbers (degrees) increase from left to right when, the slide being reversed, the corresponding edge of the slide is adjacent to A. Sometimes the scale of sines fits rule A and that of tangents rule D.

To multiply by $\sin x$ or $\tan x$, the index of the slide is brought to the number to be multiplied on the rule and the result is read on the rule opposite x on the slide.

To divide by $\sin x$ or $\tan x$, bring x of the slide to the dividend on the rule and read the result on the rule opposite the index of the slide.

To multiply and divide by the $\cos x$, use the sin of $90 - x$.

To multiply or divide by the cotangent of x, divide or multiply by $\tan x$.

For tangents and cotangents when x is greater than 45°, use the complement of x with the complementary function; thus, for $a \times \tan 60°$, write $a \times \cot 30°$. When a result comes off the rule shift the slide through its length, which merely changes the characteristic and not the sequence of the figures.

To solve $y = \dfrac{a \sin x}{\sin l}$ bring l of the slide to a of the rule and read the result on the rule opposite x on the slide. This is useful in solving triangles by the sine formula.

On the reverse side of the slide will be found generally a scale of equal parts like that used in scale D on the rule. This may be used for getting the logarithm of a number, or for getting the value of such an expression as $2.3^{1.53}$. To find the logarithm of a number, as 2.3, bring the index of the slide to 2.3 on scale D, turn over, and, opposite an index mark near the right end of the rule, read the logarithm on the scale of equal parts. To find the value of $2.3^{1.53}$, bring the index of the slide to the number on scale D just found for log 2.3 and opposite 1.53 on the

slide read log $2.3^{1.53}$ on scale *D*. Set this quantity on the scale of equal parts at the index on the back of the rule, and opposite the index of the slide read $3.576 = 2.3^{1.53}$.

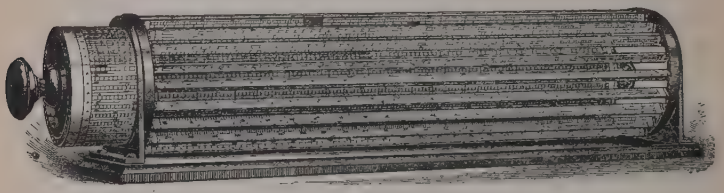


FIG. 117.

154. The Thacher rule. This rule (Fig. 117) consists of a wooden base bearing two upright metallic standards with a large circular opening in each, the line joining the centers of the openings being perpendicular to the planes of the standards. Attached to the standards are two circular plates of metal, each with a large circular opening concentric with those in the standards, so arranged that they can revolve around the line joining the centers of the openings as an axis. These plates are united by twenty bars a little more than 18 inches long, triangular in section, and perpendicular to the plates. These bars are arranged at equal distances around the circular openings, with their vertices outward so that their bases form a cylindrical envelope, the distances between the bars being equal approximately to the width of their bases. A cylindrical slide fits in this cylindrical envelope, moving with either a rotary or a longitudinal motion. The system of bars and plates may be rotated about the axis of the cylindrical envelope without disturbing the relative position of the bars and the slide.

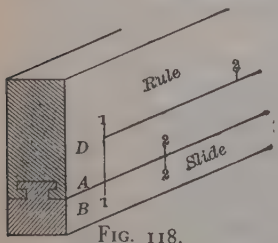


FIG. 118.

The system of bars is the rule, and the cylinder is the slide. The latter bears only one scale *B*, while the former contains the scales *A* and *D*. The Thacher rule is equivalent to a carpenter's rule in which the slide bears only one scale. The arrangement of the scales is practically the same as that in a straight rule of the form shown in Fig. 118.

The Thacher rule gives results that should never be in error by more than one unit in the fourth significant figure, while the fifth figure can often be found with only a small error. The error should not exceed one part in ten thousand, so that this rule is intermediate in accuracy between a four- and a five-place logarithmic table. It is essentially a 30-foot slide rule. The settings to be made for various computations are printed on the base of the rule.

155. Fuller's slide rule. This rule is shown in Fig. 119. The hollow sleeve *C*, which bears the graduations, is capable of sliding along and revolving about the continuous cylinder *H H*, the latter being held by the handle attached to it. *F* is a fixed index fastened to *H*, and by moving the sleeve, any division of the scale may be made to coincide with *F*. The

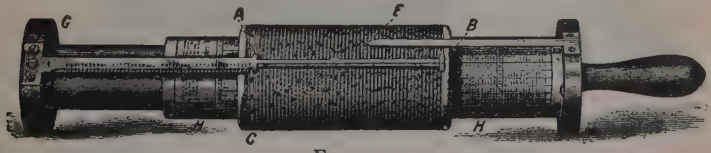


FIG. 119.

cylinder *H* is hollow, forming a guide for the motion of a third cylinder that is attached to the flange *G*, its axis being coincident with the common axis of *C* and *H*. *A* and *B* are two indices fixed to and moving with *G*, the distance *AB* being equal to the axial length of the scale. This scale, 500 inches in length, is wrapped on the sleeve *C* in the form of a helix, its beginning being at the end towards *G*. Stops are provided, so that the indices may be readily made to coincide with the beginning of the scale.

156. Remark. Many forms of slide rules are made for special purposes, as sewer calculations, bridge calculations, etc.

The 10-inch Mannheim rule which is readily carried in the pocket is about equivalent to a three-place logarithmic table, is much more rapidly used, and is as valuable in the field as in the office. In selecting a rule, careful examination should be made to see that the graduations of the corresponding scales on slide and rule fit exactly, trying both sides of the slide, that the runner runs true, and that the rule is not warped.

157. Reckoning machines. There are several reckoning machines on the market, one of which, the Bunzel, is shown in Fig. 120. These machines are made in different sizes, usually to give 12, 16, or 20 figures in a product or dividend. The

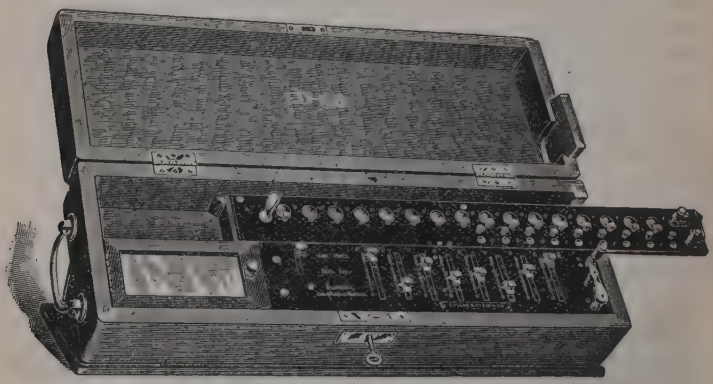


FIG. 120.

machines add, subtract, multiply, divide, raise to powers, and extract roots mechanically, and are simple of operation. Where there is much computing to do, requiring five or more places, as in tabling work, they are a valuable part of an office equipment. They are operated by setting the shifters shown in the grooves and turning the crank handle. They cost from $250 to $375.

158. The pantograph. The pantograph is an instrument for enlarging, reducing, or simply copying drawings. It is made in various forms, the simplest of which is shown in Fig. 121. It is essentially a parallel link motion so constructed that for any proportional scale for which the instrument may be set, the fixed point, F, tracing point, T, and drawing point, D, are in a straight line. T and D will then revolve around F in concentric circumferences, whose radii are the variable distances FT and FD which change as the parallelogram is extended or closed, but which always bear the same ratio one to the other. If T is the tracing point and D the drawing point, the instrument is set for enlarging; if the pencil is moved to T and the tracing point to D, the instrument is set for reducing; if T is made the

fixed point and *F* and *D* the tracing and drawing points, as is possible in some instruments, the instrument can be set to copy without change of scale.

FIG. 121.

The essential requirement of the instrument is that the fixed point, tracing point, and drawing point, shall always be in a

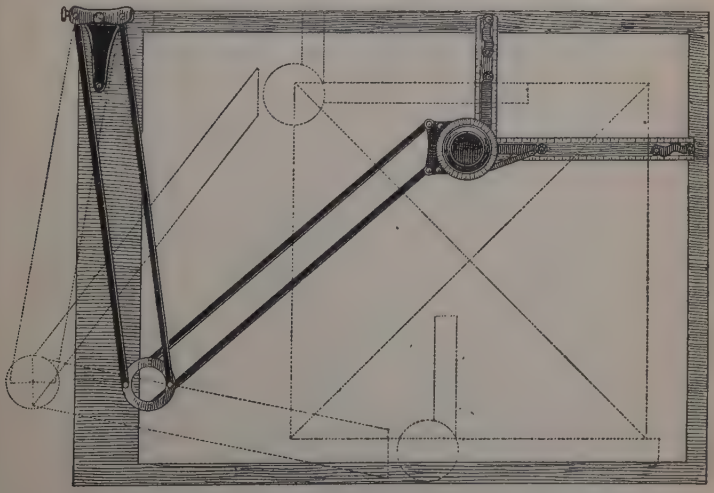


FIG. 122.

straight line on three or more sides of the jointed parallelogram. In Fig. 121 they are on four sides.

159. Universal drafting machine. This instrument shown in Fig. 122 is a most useful office tool. With it, lines may be drawn to exact length at once at any angle that can be set on the protractor and at right angles to that setting. The scales are interchangeable, so that any desired scale can be used. The protractors are also interchangeable, and one graduated especially for architects, or one graduated for engineers, a full circle and vernier reading to minutes, may be used. The instrument is anchored either at the corner of the drawing board, as shown in the figure, or near the middle of one side. The mid-anchor allows more of the board to be used. Several patterns are made. The instrument is a combination of protractor, parallel ruler, triangle, and scale.

CHAPTER VIII

STADIA MEASUREMENTS

160. Defined. The stadia is a device for reading distances by means of a graduated rod and auxiliary wires in a telescope. In many farm surveys, where the ground is very rough and not very valuable, the measurements may be made with the stadia. When so made, they will probably be somewhat more accurate than if made with a chain or tape, with the care that would ordinarily be taken in the kind of work mentioned. In fact, with the use of care and judgment, the stadia will serve for almost all farm surveys.

The stadia is the best distance measurer for extensive topographical surveys. The discussion of the stadia that follows will perhaps indicate its limitations. The term "stadia" has been very loosely used in this country. The word is the plural of the Latin word "stadium," which means a standard of measure. "Stadia" was the word adopted by the Italian engineer Porro (who invented the method to be described) to indicate the rod used by him in the application of his method. The English have retained this use of the word and apply the word "tacheometer" to the transit equipped with the additional wires used in the method. "Tacheometer" means quick measurer, and is applied by at least one American manufacturing firm to their high-class instruments that are equipped with vertical circle, level to telescope, and stadia wires. The word as used by them is "tachymeter." As generally used in this country the word "stadia" means the combination of instrument and rod, but it is thought that it should be applied to the rod only, and the word "tacheometer" or "tachymeter" is a very suitable word to apply to the complete transit instrument equipped for stadia work. Such a transit should have an inverting telescope of comparatively high power for the best work. The term "stadia wires" is properly used to designate the wires used with the stadia.

161. Method explained. Stadia measurements depend simply on the proportionality of the corresponding sides of similar triangles.

The telescope is fitted with two wires in addition to those already described, one above, and one below the horizontal wire, and both parallel to it. When a rod is held at some distance from the instrument, and the telescope is properly focused on the rod, an image of the rod will be formed in the plane of the wires, and a certain definite portion of this image will be seen between the two stadia wires. The rod is usually held

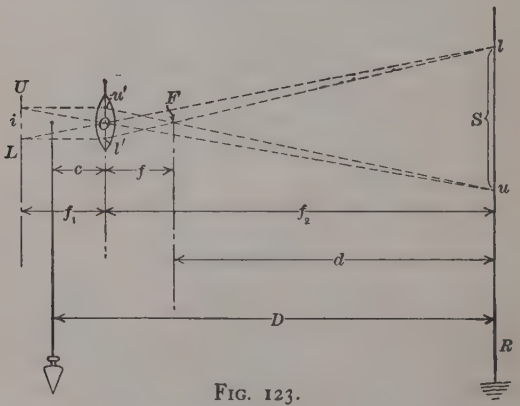


FIG. 123.

vertical. If the telescope is horizontal, the conditions will be as shown in Fig. 123. The two wires are shown at U and L. By drawing lines from U and L through the optical center of the objective O, to the rod R, the space on the rod whose image is included between the wires is seen to be lu.

The focal length of a lens is the distance from its optical center to the point in which parallel rays of light are focused. If f be the focal length of the objective, O, rays of light proceeding from U and L parallel to the axis of the lens will be focused at F, and if continued, will cut the rod also at u and l respectively. Since the distance between the wires does not change, the "space intercepted" on the rod by the wires will always be that lying between the two lines Fl and Fu. As the rod comes nearer to the lens the wires must be moved back or

the lens forward to keep the wires in the changing conjugate focal plane of the lens. Conjugate foci and conjugate focal lengths are those points and distances respectively on either side of a lens at which images and objects are when the rays of light from the object are not parallel; *i.e.*, the object is not at an infinite distance. f_2 and f_1 are conjugate focal distances.

Considering the similar triangles *Ful* and *Fu'l'* it is seen that

$$\frac{d}{f} = \frac{s}{i}; \tag{1}$$

whence

$$d = \frac{f}{i}s. \tag{2}$$

The distance from the rod to the objective is

$$f_2 = d + f = \frac{f}{i}s + f, \tag{3}$$

and to the center of the instruments it is

$$D = \frac{f}{i}s + (f + c). \tag{4}$$

If, then, a graduated rod is held at an unknown distance from the instrument, the distance will become known by multiplying the space intercepted on the rod by the ratio of f to i and adding to the product the approximately constant quantity $(f + c)$.

Since it is usual to move the objectives in focusing, C is not quite constant, but the error is neglected.

162. Spacing of the wires. The value of $\frac{f}{i}$ must be known. Sometimes the wires are made so that the space between them is adjustable. This is not considered by the author as good practice. The wires should be fixed. They are attached to the same ring that carries the cross wires, and should be spaced at equal distances from the horizontal wire. This is not necessary, but is very convenient in reading, because it will happen occasionally that but one stadia wire and the horizontal wire can be read. This is likely to occur in brushy country, but

should not be permitted when it is possible, at an expense commensurate with the importance of the work, to avoid it. In case it should occur, it is convenient to have the wires equally spaced, because the reading of one wire multiplied by two will give a sufficiently accurate determination of the distance. In case the wires are not equally spaced, separate values of the intervals may be determined for the wires.

It is thought best to have the ratio $\frac{f}{i} = 100$ for convenience. If one is introducing stadia wires in his transit, he will find that it is practically impossible to accomplish this result accurately himself. It can rarely be done outside of the maker's shop. However, it is not absolutely necessary to make this ratio 100, as the rod may be graduated to suit the instrument.

163. To approximate the value of f. Focus the telescope on a distant point (theoretically the point should be infinitely distant, as a star) and measure the distance from the center of the objective to the plane of the wires. This is the value of f.

164. The value of c. This is not quite constant in most instruments, since the objective is moved in and out in focusing for different distances. For no work done by the stadia will the error from this cause exceed one quarter of an inch, which is inappreciable compared with the smallest unit readings taken by this method. Single distances may not be read in careful work much closer than the nearest half foot; and, in a very large percentage of work done with the stadia, the distances are read only to the nearest yard or meter.

The value of c may be determined by focusing on an object at, say, two hundred feet distance, and measuring the distance from the objective to the center of the horizontal axis. In some inverting instruments the focusing of the image on the wires is accomplished by moving the eyepiece and wires together, instead of the objective. In another form, focusing is accomplished by an internal movable focusing lens. In such instruments c is a constant.

165. The value of $\frac{f}{i}$. If the value of $\frac{f}{i}$ is not known, it may be determined as follows: Select a strip of level ground, drive a

stake and center it with a tack. Set the transit over this point. From this point measure $f+c$, determined as in Arts. 163 and 164, and from the point thus found measure several distances from 100 feet to 500 feet; read the spaces intercepted on the rod held at the several distances; then, if D represents a measured distance and S the corresponding space intercepted,

$$D = \frac{f}{i} S, \qquad (5)$$

in which D and S are known, and hence $\frac{f}{i}$ may be found. Determinations should be made for the several distances and averaged. In order to avoid bias it is well that some or all of the distances should be unknown to the instrument man when he reads them with the stadia. It is necessary sometimes to read the space intercepted between only the middle wire and one extreme wire, and it is desirable, therefore, to know the coefficient of S for the upper and lower wire spaces. This will be approximately $\frac{1}{2}\frac{f}{i}$.

166. The rod. While a rod graduated by lines to feet, tenths, and hundredths, as the ordinary leveling rod (see page 56), may be used as a stadia, it is more convenient to use a rod that is graduated so as to give the distance intercepted by the wires directly by inspection, without reading the lower and the upper wire and making a mental subtraction, as would be necessary in the use of the level rod. Various patterns of rods have been devised by different surveyors. A rod suitable for a topographic survey covering a large area and mapped to a very small scale would not be suitable for a similar survey of a small tract to be mapped on a large scale. In the former case the units on the rod should be yards or meters, while in the latter case the units should be feet. The reason for this difference will be apparent. For any work connected with land surveys the rod should be graduated to feet, tenths, and hundredths.

FIG. 124.

An excellent form of rod is that designed by Mr. T. D. Allin of Pasadena, Cal.[1] Figure 124 is a drawing of a rod designed on the same principle as that of Mr. Allin. The form of the figures is somewhat modified, to the improvement of the rod. The essential features of a stadia rod are that it shall be well graduated in such designs as will render it easily and quickly read. There should always be some of the white rod showing at every graduation, to give definiteness to the position of the wire. It will be noticed that this has been accomplished in the rod shown in Fig. 124. The peculiarity of this rod is that the different divisions are very distinctly marked so as to make the reading of the distance very rapid and easy. It is assumed that the value of $\frac{f}{i}$ is 100, though this is not necessary, as a rod could be graduated with these same diagrams to fit any instrument. In the rod shown, the distance from point to point of the finest divisions is two one-hundredths of a foot, corresponding to two feet of distance. The distance from point to point of the larger divisions shown on the right of the rod is one tenth of a foot, corresponding to ten feet distance. On the left side of the rod the half-foot points stand out plainly, corresponding to fifty feet distance, and each whole foot is plainly seen also, giving one hundred feet distance. These whole feet are again marked in sets of three, giving three hundred feet distance. To read the rod, set the upper wire on the fifty-foot point nearest it and then run down the rod to the lower wire, taking in at once all the three-hundred-foot spaces, adding the hundreds and the final fifty, then the tens, and lastly the single units.

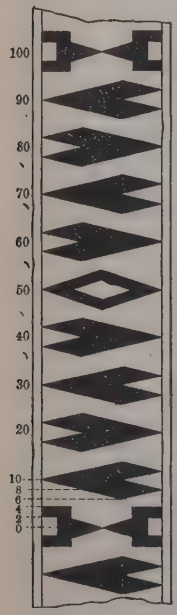


FIG. 125.

Another good rod for long-distance work is shown in Fig. 125. The manner of reading this rod is seen from the figures at the side of the rod. This rod is better suited for yard or meter units than the Allin rod, and the author has used it with feet units with success.

[1] See "Engineering News," August 2, 1894.

167. Inclined readings. It has been assumed thus far that all readings taken with the stadia are horizontal. It will be evident that probably there will be more readings that are inclined than horizontal. When the readings are inclined, the formulas already derived must be modified.

In Fig. 126 let the distance from I to A be required. If the rod were held perpendicular to the line of sight, the reading EF would at once give the inclined distance IC, which then multiplied by the cosine of the angle α would give the required horizontal distance. The angle α is determined by setting the middle horizontal wire on a point on the rod as much above A as the center of the telescope is above the ground at I, and then reading the vertical circle of the transit. The rod is usually

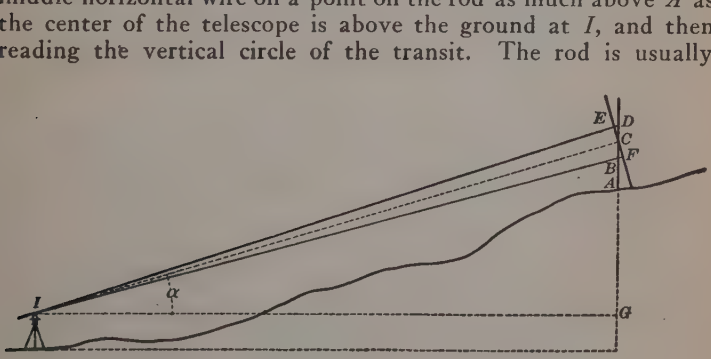


FIG. 126.

held vertical. Therefore, the reading obtained will be DB. If the angles at E and F are considered right angles, an approximation sufficiently exact for the purpose EF is given by

$$EF = DB \cos \alpha. \tag{7}$$

If, then, R is the reading DB, the distance IC is given by

$$IC = \frac{f}{i} R \cos \alpha + (f + c). \tag{8}$$

But it is the distance IG that is required.

$$IG = IC \cos \alpha,$$

$$= \frac{f}{i} R \cos^2 \alpha + (f + c) \cos \alpha. \tag{9}$$

From this equation the distance is obtained. If it were necessary to perform this multiplication for every sight taken, very much of the usefulness of the stadia would be offset by this great labor. The value of $(f + c)$ varies from, say, nine inches to fifteen inches or more, according to the construction of the instrument. Where such an error is unimportant, the final term may be neglected. For vertical angles under 6°, sufficient precision will usually be obtained if $(f + c)$ is omitted and the $\cos^2$ reduction neglected. The distance would then be recorded as read. If $(f + c)$ is one unit, this method gives precise results when $R = 100$ and $\alpha = 5° 44'$; when $R = 500$ and $\alpha = 2° 32'$; when $R = 1000$ and $\alpha = 1° 46'$; etc.

For the more ready computation of the results, tables have been computed giving the value of $\frac{f}{i} R \cos^2\alpha$, for $R = 1$, and angles from 0° to 30°. Table XIV, Appendix, pages 499–501, is such a table. To explain the use of the table, suppose a reading of 1.52 feet, usually read at once 152 feet, is had with a vertical angle of 5° 30′. Under 5° 30′ Hor. Dist. find 99.08, which multiply by 1.52, and add the correction found at the bottom of the page for c, which is $(f + c)$. This multiplication may be hastened by the use of a slide rule,[1] which is an instrument that every surveyor should own and use. The column in the table headed Diff. Elev. is used when it is desired to know the elevation of the position of the rod above the instrument.

It is often quicker, to find the correction, to subtract from the distance read than to find the reduced distance; thus, in the preceding example, $100 - 99.08 = 0.92$, which, multiplied by 1.52 gives 1.4, which, less, say, 1.00 for $(f + c)$ gives 0.4 only to be subtracted from 152.

168. Difference of elevation. In Fig. 126, since the instrument is directed to a point on the rod as much above A as the center of the instrument is above the ground at I (found by standing a rod alongside the instrument), the difference in elevation between the ground at I and at A will be CG, and

$$CG = IC \sin \alpha = \frac{f}{i} R \cos \alpha \sin \alpha + (f + c) \sin \alpha$$

[1] See Chapter VI.

$$\text{or } CG = \frac{f}{i} R \frac{\sin 2\alpha}{2} + (f + c) \sin \alpha.$$

For any reading and vertical angle the value of CG is found from the table in the same way as was the horizontal distance.

169. Diagram. A still more rapid method of reducing the observations is by means of diagrams. Omitting the second term in equation (9), the distance for any given vertical angle is proportional to the reading. To make a diagram to give the correction to apply to the readings to get the required distance, considering only the first term, a distance equal to the longest probable reading is laid off along one side of a sheet of cross-section paper ruled to tenths of inches or other small units. Suppose the distance to be 1000 feet. A space of ten inches may then be laid off, making the scale of distance 100 feet to an inch. Select an angle, as, say, 10°, and find from Table XIV the true distance for a reading of 1000 feet and a vertical angle of 10°. The difference between this value and 1000 feet will be the correction to apply to the reading to get the true distance.

At the extremity of the distance line AB, Fig. 127, lay off at right angles to AB, and to a scale of, say, 10 feet to an inch, the correction just found for the distance and angle. Suppose the point found is C. Draw a straight line from C to A. Since the correction for this angle of 10° will be proportional to the reading, the correction for any other reading than 1000 feet will be given by the ordinate to the line just drawn at the proper distance from A. This will be evident from the similarity of the triangles involved. Thus, the correction for any reading, as, say, 450 feet, is found by looking along AB to the 450-foot point, and taking to the scale of 10 feet to an inch the ordinate at the 450-foot point to the 10° line. A similar line should be drawn in a similar manner for all other angles. It will be sufficiently exact to draw the lines for degrees only, estimating the minutes by eye.

It should be noticed that the lines marked 5°, 10°, 15°, etc., do not make angles of 5°, 10°, 15°, etc., with the horizontal line, but are drawn to the points found by laying off on the vertical the correction to the reading for the given angle, as found in the tables or as worked out from the equations.

A diagram for differences in elevation may be made in the same manner (see Fig. 128), except that it will be necessary to draw the ten-minute lines, as the elevations may not be estimated with sufficient exactness without these lines.

The differences, as found in the table for the given distance and angles, are laid off on the extreme vertical line. The ten-minute lines may be drawn ordinarily with sufficient exactness

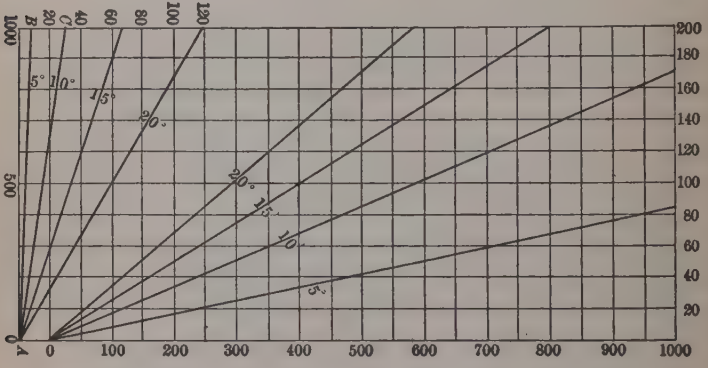


FIG. 127.

FIG. 128.

by dividing the space between two adjacent degree lines into six equal parts. These two diagrams may be drawn on one sheet of paper.[1] There are other forms of diagrams made for the same work.[2]

The second term of the equations must be applied separately, when it may be necessary to apply it at all.

170. Slide rule. The most rapid method of reducing the differences of elevation is by means of a special slide rule designed for this work by Mr. Colby of St. Louis. A cut of this rule is shown in Fig. 129. It is about four and one half feet long by one and one half inches high by three inches wide. The slide is a thin strip of wood with a varnished paper scale. Full instructions for use accompany each rule. With a little prac-

[1] Such diagrams are printed and published by John Wiley and Sons, New York.

[2] See "Engineer's Surveying Instruments" by Professor Ira O. Baker, and A. H. Abbott and Co.'s (Chicago) instrument catalogue.

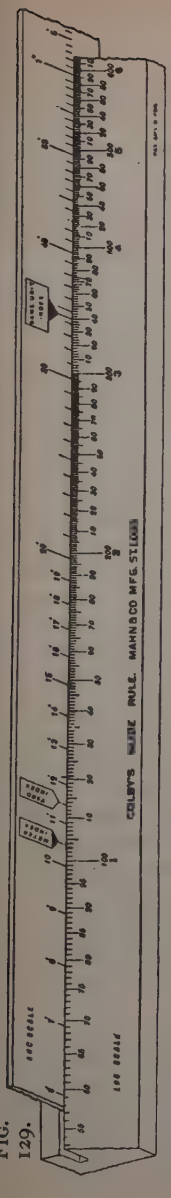


Fig. 129.

tice with this rule, rather more than 300 observations can be reduced in one hour. Webb's rule is cylindrical and smaller so as to be used conveniently in the field for immediate reductions. Keuffel and Esser also make a small rule for field and office use.

171. Graduating a stadia. The stadia is usually not kept in stock by instrument makers, probably because there is no one design that is generally adopted by surveyors, each surveyor having his own idea as to the best form. Any instrument maker will make these rods after a model pattern furnished by the surveyor, but the surveyor may make his own rods very much cheaper than he can get them made.

Select a straight-grained piece of any light wood that is not too soft, have it dressed to the required length, width, and thickness. These dimensions are usually twelve or fifteen feet by three to four inches by seven eighths of an inch. Generally the wood is made uniform in dimensions throughout, but may be made thinner toward the top and may be shod with a strip of iron or brass at the bottom. Some prefer to have no top or bottom, but to have the rod graduated symmetrically from the center out to the ends, so that whichever end is turned up the reading is made equally well. There seems to be no great advantage in this. The rod shown in Fig. 124 is three inches wide inside of the side strips.

Give the rod three light coats of white paint, letting each coat become thoroughly dry before the next is put on. After the final coat is dry, graduate the rod. The graduations should be to feet, yards, or meters, and their decimal parts. Having marked the graduations with fine knife cuts, the diagrams to be used are drawn on with a hard pencil and painted with a good quality of black paint. Both the white and

black paint used should be " killed " with turpentine so that it may dry without gloss.

Occasionally one may have to put wires in an instrument and be unable to space them just $\frac{1}{100} f$ apart. In this case the rod should be graduated to fit the instrument. To do this, set up the transit that is to be used with the rod, on a level strip of ground, and, having determined the value $(f + c)$, lay this distance off in front of the point of the plumb bob. From the point thus obtained, measure carefully with a steel tape distances of one hundred feet, two hundred feet, three hundred feet, and more if desired. (If the rod is to be graduated to yards or meters, the distances should be one hundred, two hundred, and three hundred, of the units to be used.) Hold the blank rod at these distances and carefully mark with a hard pencil the points cut by the two wires at the different distances.

The spaces cut should, of course, be strictly proportional to the distances; but owing to inaccuracies of observation they may not be exactly so. Measure the several spaces intercepted and divide the three-hundred space by three, the two-hundred space by two, and average the quotients with the one-hundred space to determine the average space intercepted at one hundred feet. These spaces should be measured with great care, using either a good steel tape or a standard draughtsman's scale. This value of the space intercepted at one hundred feet having been determined, it is divided into as many equal parts as may be required, and the rod is spaced off for the diagrams that have been selected for use. When the rod is thoroughly dry, strips of hard wood, about one eighth to one fourth of an inch wider than the rod is thick, are screwed to the edges of the rod. These strips, which are about a quarter of an inch thick, are to protect the graduated surface of the rod. They are very necessary to a rod that is to be much used. The rod should be shod with brass or iron about one eighth of an inch thick, at both ends if it is to be used either end up, and at the bottom if it is to have a bottom.

172. Smith's observations. Professor L. S. Smith, C. E., has shown [1] that systematic errors are introduced into work when it is assumed that a wire interval determined at any one time is

[1] Bulletin of the University of Wisconsin, Engineering Series, Vol. I, No. 5.

correct for all future work. This is due to the fact that the effect of refraction is a variable quantity depending on the relative temperatures of air and ground. The effect of refraction is very much greater near the ground than a few feet above. It is much greater at noon than before or after. The effect varies for different distances. It is also shown that observers have different "personal equations."

It follows from the above that it is better to graduate the rod to standard units and depend on the maker to place the wires at the interval $\frac{f}{100}$. It will be sufficient for many surveys to consider this as correct and to use the reduction tables or diagrams at once.

But if close work is to be done, it will be better to determine the wire interval by observations at all hours of the working day, and on several days, so that the average result shall conform to the average to be expected from the average conditions to prevail during the survey.

Thus it will be found that wires set $\frac{f}{100}$ apart will intercept on the rod held at distances of $100 + (f + c)$, $200 + (f + c)$, etc., from the instrument, varying lengths, from possibly a little more than one unit to a little less than one unit for each 100 feet. The average result of all the observations would give a factor by which all observed distances on the survey should be affected before entering the reduction tables or diagrams. Thus, suppose the average intercept for $100 + (f + c)$ distance as determined from all observations should be 1.0039, then all distances read would be read too long, and should be multiplied by $\frac{1}{1.0039}$ before entering the reduction tables. The multiplication is performed very quickly by the aid of an auxiliary diagram, or table, or with the slide rule. It is not strictly correct to say that all distances will be read too long. Some will be too long and some too short, but the average result will be too long. By supplying this factor perhaps no one of the distances is made correct, but the errors introduced are no longer cumulative, as they would be were the factor not used. They are now compensating. The factor should be determined by the person who is to make the observations on the survey.

By adopting these precautions and determining a factor for each of several average conditions, — say for each of the four seasons, — a degree of precision of one in two thousand or better may be obtained with the stadia in a long line or large closed survey, even though no single distance may be read closer than to the nearest foot, which in a short line would mean a large proportional error.

173. Notes. Owing to the fact that vertical angles must be observed in traversing with the stadia, the form of notes to be kept in such work differs from that used with the transit and tape or chain. The form shown below is a good one.

Traverse of Mud Turnpike, June 21, 1895					Right Page for Remarks and Descriptive Notes and Sketches
Station	Azimuth	Distance feet	Vert. Angle	Needle	
etc.	etc.	etc.	etc.	etc.	
B	63° 26′	324	— 0° 26′	63° 25′	Describe B
(Back)	(217 42)	(478)	(— 3° 59′)	(217 40)	
A	37° 42′	476	+ 4° 00	37° 45′	Describe A

When elevations are to be kept, the form of notes shown on page 319 is better. If it is necessary to correct the distances, the corrected distances may be written in red in the distance column, leaving the original distances for possible future checks.

CHAPTER IX

MERIDIAN, LATITUDE, AND TIME

174. Methods of determining the meridian. The direction of the meridian, or North and South line, is determined in several ways, one of which, very old and only roughly approximate, is to mark on the ground or on a flat surface, the extremities of the shadows cast by a vertical rod or other object when the sun is at equal altitudes above the eastern and western horizons. This is done by noting the shadow at about 9 or 10 o'clock in the morning and drawing a circle through the extremity of the shadow, taking the foot of the rod as a center, and then marking the point on this circle that is just touched by the shadow in the afternoon. The line joining the center of the circle and the point midway between the two positions of the extremity of the shadow lies in the meridian.

The other methods used by surveyors are:

1. By observations with the transit on Polaris, the north star.
2. By observation on any star.
3. By observations with the transit on the sun.
4. By the use of the solar transit.

An "observation" on a heavenly body for the determination of a North line, or "the meridian," as is more commonly said, consists in pointing the telescope of the transit at the body, at a given known instant and transferring the line thus determined to the ground. This may or may not be the meridian according to the time of the observation. If it is not, its angle with the meridian will be found or known as hereafter described.

The best and most precise of these methods is a transit observation on Polaris, but it is not possible in all latitudes. Why?

This star is not directly at the pole, but at present is about one and one sixth degrees from the pole about which it seems to revolve in 23 hours and 56+ minutes, thus bringing it to the meridian twice every day.

When it is on the meridian above the pole it is said to be at upper culmination; when on the meridian below the pole, at lower culmination; when it is farthest east or west it is said to be at eastern or western elongation. It is approximately on the meridian when a plumb line traverses it and the star next the end of the handle of the big dipper or Zeta Ursæ Majoris, that is, the star Zeta in the constellation of the Great Bear, which is the name of the constellation including the dipper. The stars in every constellation are lettered or numbered. Polaris is known as Alpha Ursæ Minoris, or Alpha of the constellation of the Lesser Bear.

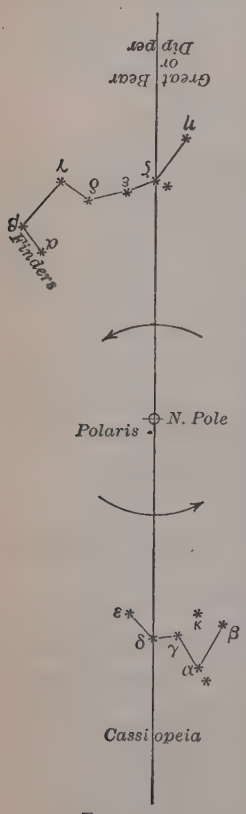


FIG. 130.

The position of Polaris, the north pole, and the dipper, together with the constellation of Cassiopeia, are shown in the figure (130) which is taken from the Manual of Instructions issued by the General Land Office. The figure shows Polaris at lower culmination. Polaris is generally found in the heavens by finding the dipper and then looking along the line of the two stars forming the outer edge of the bowl, called the finders. If the figure is turned upside down, it will show the relative positions of the several stars for upper culmination; if looked at from the left side, it will show the relative positions for Eastern Elongation, and from the right side, the positions for Western Elongation. The arrows show the apparent direction of rotation about the pole.

About six and one third minutes (1914) after Polaris and Zeta Ursæ Majoris are in a vertical line, Polaris is on the meridian. For years after 1914 add 0.35 of a minute for each year.

It is not best to observe the star at culmination because at that time its motion appears to be all horizontal, that is, in

azimuth, is quite rapid, and a small error in time makes a considerable error in the work. It is better observed at elongation because then its motion appears to be all vertical and for several minutes it is within a few seconds of arc of the vertical through its elongation point.

An observation on Polaris consists in pointing the telescope of the transit to the star, and ranging out on the ground the direction of the vertical plane thus found. If the star is at culmination, this direction is that of the meridian, if at any other part of its apparent path about the pole, the angle of the vertical plane of the star with the meridian plane, called *the azimuth of the star for the instant of observation*, must be found by solving a spherical triangle.

An observation on any star may consist in one of two procedures: the vertical planes containing the star at equal distances east and west from the meridian may be found and their angle bisected for the meridian, called the method of equal altitudes, or a single altitude of the star may be measured and the angle between the vertical plane through it and the meridian plane, *i.e.*, the azimuth of the star at the time of observation, may be calculated by solving a spherical triangle.

An observation on the sun consists in measuring its altitude at a given time and calculating its azimuth for the time by solving a spherical triangle.

These methods will be mentioned more fully, and, together with the use of the solar transit, will be explained in the following articles.

175. The spherical triangles. Let a celestial sphere be imagined concentric with the earth and of infinite radius, a conception sufficiently near the truth for the purpose in hand. If the earth's equatorial plane and its axis be extended to the celestial sphere, they will cut from that sphere a celestial equator and two celestial poles 90° from the equator. If one were at a point on the earth's equator, his horizon, being at an infinite distance so that the earth's radius may be neglected, would include the celestial poles, and if he were to go north or south through some number of degrees, by just so many degrees would his horizon be depressed below the pole toward which he might go. Thus, in Fig. 131, the small circle at E is the earth, the great circle NZQ a meridian circle through e, extended to the celes-

tial sphere; Z is the zenith of the place e on the earth having north latitude ZEQ. N is the north horizon of a person at e, and P is the north celestial pole. $PEN = ZEQ$, or P is above N by the latitude of e, the place of observation. Therefore, the angular elevation of the celestial pole above the horizon equals the latitude of the place of observation; the distance from the pole to the zenith is the colatitude; from the zenith to the equator the latitude; and equator to nearest horizon (north or south) the colatitude.

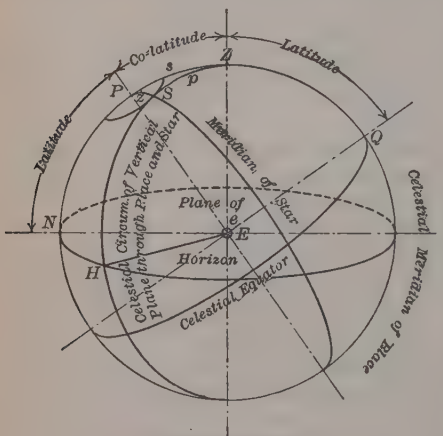


FIG. 131.

On the celestial sphere the measurement corresponding to terrestrial latitude is called *declination*, so that one speaks of the declination of a star, or a planet, or the sun, rather than its latitude when speaking of its angular distance north or south of the equator. The other measurement on the celestial sphere corresponding to terrestrial longitude is called *right ascension*, and is measured from an arbitrary celestial meridian passing through the "first point of Aries." With this measurement the surveyor has no immediate interest.

When observing the pole star it is customary to use its "pole distance," or codeclination, for the computations. When Polaris is observed at elongation by an observer at E, Fig. 131, the plane $ZESH$ is determined and the angle $NEH = PZS$ is required. The circle about P represents the apparent path of the star about the pole, and S is the position of the star at western elongation. This position is the point on the circle at which the plane ZEH is tangent to it, and, hence, is such that PSZ is a right angle. PS is the pole distance of the star. Then in the right triangle ZPS, PZ, the hypotenuse, is the colatitude of the place of observation, E; PS is the pole distance of the

star, and the angle Z is obtained from the formulas of spherical trigonometry

$$\text{Sin } Z = \frac{\text{Sin } PS}{\text{Sin } PZ} = \frac{\text{Sin Pole distance}}{\text{Cos latitude}}. \qquad (1)$$

The pole distance is found in the American Ephemeris or Nautical Almanac and is also given in Table IV, page 476 of this book. It is slowly diminishing, and will continue to diminish until A.D. 2095 after which it will slowly increase.

The latitude may be obtained from a good map since, south of 50° north latitude, an error of one degree will give an error in Z of from $\frac{1}{2}$ minute in latitude 20° to $2\frac{1}{2}$ minutes in latitude 50°, and it should be possible usually to find the latitude on a map to within a few minutes. But if not so determinable it may be found by measuring the altitude of Polaris as hereafter explained.

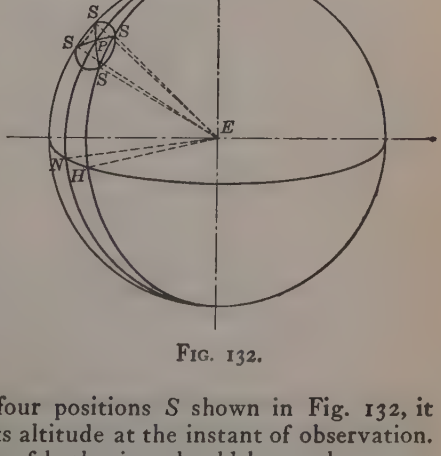


FIG. 132.

If Polaris is observed at any time between elongation and culmination, as at any of the four positions S shown in Fig. 132, it is necessary to measure its altitude at the instant of observation. Therefore the intersection of both wires should be on the star at that instant, the vertical angle should be read, and the direction fixed by ranging out a line at once for reference after the calculations have been made. In the oblique spherical triangle ZPS the pole distance PS, colatitude PZ, and coaltitude SZ will be known. From these the angle at $Z = NEH$ may be found by the equation

$$\text{Sin } \tfrac{1}{2} Z = \pm \sqrt{\frac{\text{Sin } (s - a) \text{ Sin } (s - l)}{\text{Sin } a \text{ Sin } l}} \qquad (2)$$

in which $s = \frac{1}{2}(d + a + l)$, d being the pole distance, a the coaltitude, and l the colatitude. This is the usual equation for an angle when the three sides are known.

When the sun is observed, its altitude HS (Fig. 133) is measured, giving SZ the coaltitude. SD, the declination, is found from the Nautical Almanac for the time of observation, and the latitude NP is determined from a map or an observation on Polaris, or a noon altitude measurement of the sun. Then, in the triangle ZPS, PZ is the colatitude, l, ZS is the coaltitude, a, and PS is 90° ± the declination according as the sun is below or above the equator. In the northern hemisphere it is above from March 21 to September 21 and below the rest of the year, and *vice versa* in the southern hemisphere. The direction of the sun for the instant of observation being ranged out in EH, the angle $NEH = Z$ is the angle the vertical plane through the sun at the instant of observation makes with the meridian, and the arc NQH measures the azimuth of the sun. When this is found the meridian may be determined by laying off this angle from the line in EH. In the morning it is less than 180°, in the afternoon more, being 180° − Z or 360° − Z.

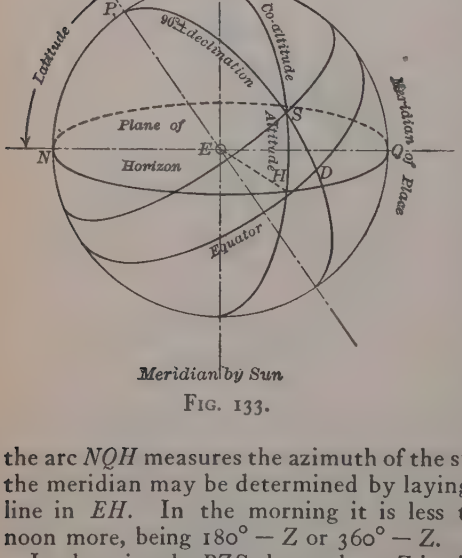


FIG. 133.

In the triangle PZS the angle at Z is given as before by

$$\text{Sin } \tfrac{1}{2} Z = \pm \sqrt{\frac{\text{Sin } (s - a) \text{ Sin } (s - l)}{\text{Sin } a \quad \text{Sin } l}} \qquad (2a)$$

in which $s = \frac{1}{2}(d + a + l)$, a is the coaltitude, l is the colatitude, and d is the pole distance of the sun. If instead of the complementary angles, we use h, the altitude, ϕ, the latitude, and

δ, the declination, and $s' = \frac{1}{2}(90 - \delta + \phi + h)$, then by substitution and reduction, δ being minus when south, and plus when north,

$$\text{Sin } \tfrac{1}{2} Z = \pm \sqrt{\frac{\text{Sin } (s' - \phi) \text{ Sin } (s' - h)}{\text{Cos } \phi \text{ Cos } h}}. \qquad (3)$$

Any star whose declination is known may be used instead of the sun. The Nautical Almanac gives the positions of many well known stars, but it is beyond the province of this book to carry the discussion into this field.

Examples

1. On a certain day when the sun's declination was $-17° 32.2'$ its altitude at a given instant in the forenoon was measured and corrected to be $25° 26'$ at a place whose latitude is $41° 40'$. What was the azimuth at the instant of observation?

2. At the same place an observation was made on Polaris when in its upper left hand quadrant and its correct altitude was found to be $42° 05'$. What was the azimuth at the time of observation, if the pole distance of the star was $1° 09.8'$?

3. On the same day at the same place what would have been the azimuth of the star at western elongation?

176. Latitude by Polaris. Since the altitude of the pole is the latitude of the place of observation, the latitude may be determined by measuring the altitude of Polaris at culmination and adding or subtracting the pole distance according as lower or upper culmination is observed. If both upper and lower culmination are observed, the latitude is the mean of the two altitudes. The procedure for a single observation is as follows:

With the transit carefully adjusted, set up a little before the time of culmination, found from Table VI, page 478 of this book, and bring the horizontal wire of the telescope on the star. If lower culmination is being observed, the star will seem to move rapidly eastward and very slowly downward. (In an inverting instrument the motion is opposite.) With the horizontal axis clamped follow the star by using the axis slow motion screw until it no longer seems to move downward. To be sure, hold the wire on the star until it seems to begin to move upward. Read the vertical circle and subtract from the reading the re-

fraction correction found from Table VII, page 480. Add the pole distance of Polaris found in Table IV, page 476 and the result is the latitude. If upper culmination is observed the procedure is the same, except that both the refraction correction and the pole distance are subtracted.

If the transit has a full vertical circle, it is well to read the altitude angle as soon as the star seems to cease going downward, reverse in azimuth, transit the telescope, set on the star and read again. There will be time for two complete double readings while the star seems stationary in altitude if the work is done rapidly. If there is a difference in the readings of a single set the mean should be taken, and if there is much difference, the second set will serve as a check on the accuracy of the work.

To see a star and the wires at night the wires must be illuminated. This is done by holding a light, lantern, or protected candle, near the object end of the telescope, shading it from the eyes of the observer. If held too near, the field will be so bright that the star will not be plainly visible. Some instruments are provided with a reflector that fits on the end of the telescope tube and throws the light of a near-by candle or lantern into the barrel of the telescope. Such a reflector is shown in Fig. 134.

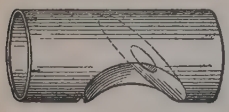


FIG. 134.

Some instruments, notably instruments for mine surveying, have a hollow horizontal axis with a window at the end and an internal reflector.

If the instrument has only a vertical arc so that reversed and inverted readings cannot be taken, great care must be exercised in the adjustment of the horizontal wire, telescope and plate bubbles, and vertical circle, or else the altitude will be incorrectly measured.

Since the construction of the transit prevents the reading of vertical angles of more than about 50° (the precise value depending on the particular instrument) without a prismatic eyepiece and above about 60° even with such an eyepiece, higher latitudes than 50° to 60° cannot be determined by this method.

177. Meridian by Polaris at elongation. The time of elongation is determined from Table VI, page 478. The transit being set up over a centered stake a few minutes before the time, and

provision being made for illuminating the wires as explained in Art. 176, the telescope is turned on the star and both plates clamped. Finding the star will be made easier if the latitude is set off on the vertical circle. The vertical wire is made to cover the star as nearly as possible, and by the tangent screws of alidade and vertical circle is made to follow the star as it appears to move to the right or left, that is, in azimuth, until it seems to be stationary in azimuth and to be moving vertically. The telescope is then plunged, and a stake is set some distance away and centered with a tack, and the work may then be left till morning.

Setting the distant stake requires some patience, and is best accomplished by holding a plumb line or surveyor's pin in front of a lantern flame or an illuminated white surface, as a piece of white paper on which a strong light is thrown from a lantern or bull's-eye. If nothing more than a candle is available, it may be used by being protected in a small box. In the morning the transit may be set again over the point used the night before and the azimuth of the star at elongation, calculated as directed in Art. 175, set off from the line ranged out, to the right or left according as western or eastern elongation was observed. A stake set in the line thus obtained, with that over which the transit is set, will define the meridian.

There is usually time to make two or three determinations of the position of the stake set in line with Polaris while the star remains practically at elongation, since for several minutes its azimuth motion is less than can be observed with the transit. Therefore, after first centering the stake, if it is accomplished quickly, reverse in azimuth, transit the telescope, set again on the star, and set a point beside the first one. If the two disagree, reverse again and repeat a third time for a check. If much time is consumed in setting the stake the first time, the check is only rough, and there need be no reversal in azimuth and the first setting should be adopted unless considerable error is shown, in which case the work should be repeated the next night. If possible within about ten minutes, two double observations should be made if the first double observation does not give identical points, to determine whether the discrepancy is due to error of observation or of instrument adjustment. If the latter, the middle point between the two determinations of the first double observation is the point to use; if the former, the check observations will tell which determination is correct.

Exercise

Determine the direction of the meridian by an observation on Polaris at elongation.

178. Meridian by Polaris at any time. At any time in the evening set up the instrument over a centered stake. Set a centered stake approximately north; set the alidade at zero; with the lower motion turn the line of sight on the tack of the distant stake, and loosen the alidade. Turn the telescope on Polaris, making the intersection of the wires exactly cover the star by using the clamps and slow motion screws of alidade and vertical circle; read the vertical circle and the azimuth angle (the small angle either east or west) between the vertical planes through the stake and the star; plunge the telescope and set and center a stake some distance away. Correct the observed altitude of the star for refraction as in Table VII, page 480, and, using the corrected altitude, the pole distance obtained from Table IV, page 476, and the latitude obtained to the nearest minute from a good map or from observed Polaris altitude, substitute in Equation (2) of Art. 175 for the sine of half Z, the azimuth angle, and do not forget that the result is only half Z. The instrument must be in the best possible adjustment throughout.

Since there are two equal azimuthal angles — one east, the other west — for any given altitude, and four positions of the star that will give the same Z angle — two east and two west of the meridian — the observer must know from the observed motion of the star in which quadrant of its apparent revolution it is, or he may know from the time of observation and the table of times of elongation and culmination at page 478.

The computed azimuth angle will differ from that measured from the reference stake by the azimuthal angle of the line of stakes; thus, if the angle from the reference stake to the star is 3° 36′ left, and the star's computed azimuth is 1° 22′ west, the line to the reference stake is to the east of the true meridian by 2° 14′. Instead of a special north reference stake and transit stake, two consecutive transit stations of a given survey may be used, in which case the azimuth of the line between them is what is found. The stake under the star is set for possible check measurements and for future reference.

Several determinations may be made at intervals of a few minutes and the calculations made at once or next day. The mean of the several determined azimuths of the reference stake should be used. Only one stake need be set under the star. This method may not commend itself for astronomical or geodetic work, but it is sufficiently good for any purpose for which the surveyor is likely to wish to use it.

If preferred, the reference line may be omitted, no azimuths read, but simply the stakes set, one for each observation, under the star. If a broad stake is used, it may be possible to get all lines centered on the one stake. The next day the computed azimuths of the several pointings may be laid off to a meridian line, the average of the meridian determinations being assumed to be correct, or the angles from the several points to a line of a survey may be read and the azimuth of the line found by applying the determined azimuth angles of Polaris to the measured angles.

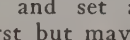
Another method often given is to observe the azimuth position of Polaris at any time, and, with the time correct to the nearest minute, apply a set of tables which give the azimuth of Polaris at any time for different years. As it is often difficult to be sure of the time to the nearest minute, and as there is little, if anything, to be gained in precision over the method already given, the method of using the hour angle will not be described. It can be found in the Manual of Instructions issued by the General Land Office at Washington.

Exercise

Follow the instructions of the article and find the direction of the meridian by an observation on Polaris between culmination and elongation, checking the determination by observation at elongation.

179. Meridian by equal altitudes of a star. In high or low latitudes in the northern hemisphere, or in the southern hemisphere, Polaris cannot be used, and resort is had to equal altitudes of any star that may be observed some distance on both sides of the meridian at reasonable altitudes of between 20 and 40 degrees. To do this, select the star; set up the instrument; clamp one horizontal motion, and with the other and the vertical motion of the telescope bring the intersection of the wires approximately

on the star; clamp both motions and bring the intersection of the wires exactly on the star by both slow motions manipulated together. Read the vertical circle with much care and record the reading. Loosen the vertical motion; plunge the telescope, and set a point some distance (150 to 200 feet) away. At intervals of ten or fifteen minutes repeat the whole procedure twice, marking the points set 1, 2, 3.

Wait until the star is about as far west of the meridian as it was east when last observed; set the vertical circle to read the altitude angle last observed; watch the star, following it with the alidade slow motion after it has nearly reached the horizontal wire, until at the instant it crosses the horizontal wire have it also on the vertical wire; loosen the vertical motion; plunge the telescope, and set a point about as far away as those set before culmination. Set the vertical angle in succession to read the second and first altitude angles observed before culmination and set stakes in the two lines determined as before, marking the three stakes 3, 2, 1. In the morning bisect the angle 3–instrument–3 and set a stake; bisect the angle 2–instrument–2, and set another point which should coincide with the first but may not exactly; bisect the angle 1–instrument–1 and set a third point. The average of these points should be in the meridian with the instrument. To find the average assume the most easterly (or westerly) point as a point of reference; measure the distances of the other two from it, add them and divide by three; the result is the distance of the average point from the point of reference. Thus, if the points lie  and E is chosen for a reference $2 + 3 \div 3 = 1\frac{2}{3}$ is the distance of the average point from E.

If the instrument has a full vertical circle the observation after culmination should be made with the telescope inverted, but the horizontal wire, telescope bubble, and vertical circle, must be in essentially perfect adjustment.

Probably it is wise to get the approximate altitudes of the star for the first three observations, then set the vertical circle to read an exact minute just a little higher than the star and keep the vertical wire on the star until the instant it crosses the horizontal wire. The altitude determinations are somewhat more exact than if an altitude that is probably not a whole number of minutes is read to the nearest minute.

This method requires some patience, since one should make it at about 2 hours or more on each side of the star's culmination.

If the star is near either pole and the latitude is low, the star will be observed on either side of upper culmination and the first observation will be east of the meridian. If the star is near the pole and the latitude high, the star will be observed either side of lower culmination and the first observation will be west of the meridian.

This method has the advantage of requiring no tables or calculations of any kind. It may be used in north middle latitude when elongation of Polaris occurs at an inconvenient time. A star that reaches culmination early in the night and that is not low on the horizon, should be chosen. Approximate north and south will be known by the needle or by the daytime position of the sun.

EXERCISE

Determine the meridian by the method of this article and compare the result with those of the two previous exercises.

180. The sun's declination. Apparent noon is the instant the sun appears to cross the meridian. An apparent solar day is the interval between two successive apparent culminations of the sun. It differs from the mean solar day, which is that measured by the clock, because the length of an apparent solar day is not a uniform quantity, while the clock must be made to carry a uniform day. The average of the solar day for the year is what is carried by the clock, and a clock is sometimes fast and sometimes slow of the sun. The difference is called the equation of time. The clock is slow of the sun in winter and summer and fast in spring and fall. The precise rates of change and the amount of the equation of time are found in the Ephemeris and the instrument maker's reprints.

To find the sun's declination for any longitude and time, look up the declination for the day in the Nautical Almanac or an instrument maker's reprint and note that it is the declination for noon at Greenwich, England, and that the hourly change is given, + meaning north change and − south change. The declination is given on one page for apparent noon and on the opposite page for mean noon. Generally mean noon will be used.

Convert the longitude of the place of observation, the nearest degree will be close enough, or the meridian of the watch time carried, if standard time is used, into time; add the result to local time or standard watch time carried to find the Greenwich time of the observation. In the eastern hemisphere the result is subtracted.

Multiply the hourly change by the elapsed time since Greenwich noon and add or subtract, as the case may be, to or from the tabular declination for the day, for the declination at the time of observation. Thus, if the place is 6 hrs., 20 m. west longitude, local time noon at the place of observation is $6\frac{1}{3}$ hours after noon at Greenwich, and if the observation is made at 10 o'clock a.m., it is made at $4\frac{1}{3}$ hours after noon at Greenwich, and $4\frac{1}{3}$ times the hourly change will be applied to the tabular declination. But if 90th meridian, 6 hrs., time is carried by the watch, as would be usual, instead of local time, the observation would be made just 4 hours after noon at Greenwich, and 4 times the hourly change should be applied to the tabular declination.

A page of one of the reprints of the Ephemeris is shown here. Let it be required to know the declination at 3 p.m., watch time of January 18, 1911, at a place in the Mississippi Valley where central time is used. Central time being 6 hours slow of Greenwich time, noon by watch time at the given place is 6 p.m. at Greenwich, and 3 p.m. at the given place is 9 p.m. at Greenwich. The hourly change in declination for January 18 is 29.78″, the sun is going north. $29.78 \times 9 = +\ 268.02'' = 4'\ 28.02''$, which applied to

$$-\ 20^\circ\ 42'\ 46.0''$$
$$+\quad\ \ 4'\ 28.0$$

gives $-\ 20^\circ\ 38'\ 18.0''$

The equation of time is not used in the foregoing example because the Ephemeris reprint gives *mean* noon values and the watch carries *mean* time, hence no correction is necessary. Some instrument makers use apparent noon values in their publications, and these must be changed to mean noon values if watch time is used.

181. Latitude by the sun's altitude. From the discussion of Art. 175 it will appear that the altitude of the intersection of

AT GREENWICH MEAN NOON

JANUARY, 1911

DAY OF WEEK	DAY OF MONTH	SUN'S SEMI-DIAMETER 16′ 17″				EQUATION OF TIME TO BE SUBTRACTED FROM MEAN TIME	
		Sun's Apparent Declination			Diff. for 1 Hr.		
		°	′	″	″	m	s
Sun.	1	S. 23	4	39.5	+ 11.46	3	19.35
Mon.	2	22	59	50.7	12.61	3	47.89
Tues.	3	22	54	34.4	13.75	4	16.10
Wed.	4	22	48	50.7	+ 14.89	4	43.94
Thur.	5	22	42	39.8	16.02	5	11.38
Frid.	6	22	36	1.9	17.14	5	38.38
Sat.	7	22	28	57.2	+ 18.25	6	4.91
Sun.	8	22	21	25.8	19.36	6	30.95
Mon.	9	22	13	28.1	20.45	6	56.48
Tues.	10	22	5	4.2	+ 21.53	7	21.45
Wed.	11	21	56	14.5	22.60	7	45.84
Thur.	12	21	46	59.2	23.66	8	9.64
Frid.	13	21	37	18.7	+ 24.71	8	32.83
Sat.	14	21	27	13.1	25.75	8	55.39
Sun.	15	21	16	42.8	26.78	9	17.31
Mon.	16	21	5	48.0	+ 27.79	9	38.58
Tues.	17	20	54	28.9	28.79	9	59.18
Wed.	18	20	42	46.0	29.78	10	19.09
Thur.	19	20	30	39.5	+ 30.76	10	38.31
Frid.	20	20	18	9.7	31.72	10	56.81
Sat.	21	20	5	17.0	32.67	11	14.59
Sun.	22	19	52	1.8	+ 33.60	11	31.63
Mon.	23	19	38	24.2	34.52	11	47.93
Tues.	24	19	24	24.8	35.43	12	3.48
Wed.	25	19	10	3.8	+ 36.32	12	18.25
Thur.	26	18	55	21.7	37.19	12	32.24
Frid.	27	18	40	18.8	38.05	12	45.44
Sat.	28	18	24	55.5	+ 38.89	12	57.84
Sun.	29	18	9	12.2	39.71	13	9.44
Mon.	30	17	53	9.2	40.52	13	20.23
Tues.	31	17	36	47.1	41.31	13	20.20
Wed.	32	S. 17	20	6.2	+ 42.09	13	39.34

Note. The sign + prefixed to the hourly change of declination indicates that south declinations are decreasing.

the celestial equator and meridian is the colatitude of the place of observation. If the sun has north declination, its altitude when on the meridian will be the colatitude plus the declination; when the declination is south, the altitude will be the colatitude minus the declination. Therefore, if the sun's altitude be measured when it crosses the meridian the latitude may be found.

It is usual to require the latitude for finding the meridian, and it may be determined without knowing the meridian because the sun is on the meridian when its altitude is a maximum. Moreover, it remains at its highest altitude for some minutes so far as this is determinable with a surveyor's transit.

To find the latitude, the transit is set just before noon at a convenient point not necessarily over any particular stake, the line of sight brought to the sun, and the horizontal wire made tangent to its lower or upper limb.[1] As the sun rises, the wire is kept tangent to the image by the vertical slow motion screw and the azimuth motion until the sun reaches its highest point. When the sun seems to stop rising, the vertical angle is read, corrected for refraction by Table IV, page 476, 16′ is added or subtracted, the declination is added if south and subtracted if north, and the result is the colatitude of the place. Since the sun is observed at apparent noon, apparent noon declination should be used rather than the mean noon declination.

If the instrument has a full vertical circle, it may be reversed in azimuth and the telescope transited after the first reading, and a second reading taken to the opposite limb to that first observed. The mean of the two readings corrected for refraction will be the true altitude. If only a single altitude is read, the horizontal wire, telescope and plate bubbles, and vertical circle, must be in essentially perfect adjustment.

Since altitudes greater than about 50° cannot be measured with the engineer's transit without a prismatic eyepiece, nor greater than about 60° with such an eyepiece, this method is limited in the northern hemisphere to latitudes whose complementary angles, plus the declination at the time of observation, equal 50° more or less if no prismatic eyepiece is used, and 60° if such an eyepiece is used. This makes the southern limits

[1] The image of the sun seen in the telescope is not a point but a circle, the vertical diameter of which subtends 32 minutes at the earth; hence the center of the sun is always 16 minutes higher than its lower edge or "limb," as it is called. The apparent diameter really varies from about 32′ 34″ at the winter solstice to about 31′ 32″ at the summer solstice because of the variable distance from the earth.

in the northern hemisphere from 63.5° to 53.5° at the summer solstice and 16.5° to 6.5° at the winter solstice. Any latitude north of these limits in which the sun gets well above the horizon can be determined at the times named, and intermediate limits correspond to intermediate seasons.

EXERCISES

1. What are the limits in the southern hemisphere?
2. With an Ephemeris giving the sun's declination find that part of the year during which latitude can be found by this method for the place of residence of the student.
3. If practicable, find the latitude by the method of this article.

182. Meridian by the sun's altitude. In precisely the same manner as described for Polaris at any time, the sun's altitude may be measured when not too near the horizon or meridian, preferably at least one hour either side of the meridian so that its altitude motion may be clearly perceptible and reasonably rapid, and two hours above the horizon to avoid irregularities of refraction occurring at low altitudes.

It is necessary to know the sun's declination at the time of observation, and the latitude of the place. The altitude is observed and corrected for refraction according to Table IV, page 476, using the lower part of the table and remembering that the sun appears too high.

The method of procedure is as follows: Drive and center two stakes some distance apart in an approximately north and south line (or use two consecutive transit points of a given survey). Set the transit over the southern stake, and with the verniers at zero bring the line of sight on the northern stake with the lower motion. Unclamp the alidade, and with horizontal and vertical motions bring the line of sight to the sun; clamp both motions, and with alidade and vertical slow-motion screws set as precisely as possible. Read the vertical angle and the azimuth angle. The first is the apparent altitude of the sun and the second the azimuth of the sun from the line of stakes.

A small piece of red glass placed inside the eyepiece cap will enable the observer to look at the sun; or, with an erecting eyepiece (if it is properly focused) drawn clear out, the image of sun and wires may be received on a card held back of the eyepiece. The sun's image will be a small circle which can be

quartered by the wires of the transit with sufficient precision, but to observe more precisely, bring the wires tangent to the sun's image. The image and wires are reversed and inverted. If the image is quartered, it will be necessary to manipulate the alidade and vertical slow motions together, so that if a card is used back of the instrument it must be held by an assistant. If the wires are made tangent to the edges only one screw need be moved, since one wire may be placed on the sun's image near the edge that is soon to move off, and the other wire kept tangent to the image until the instant that the edge covered by the first mentioned wire becomes tangent to that wire.

FIG. 135.

Since the sun appears to have a diameter of about 32 minutes of arc, if the wires are made tangent to its limbs (edges), a correction is required to the measured altitude and azimuth. For altitude, the correction is simply the sun's semi-diameter subtracted or added as the upper or lower limb is observed, or 16 minutes, but for azimuth it is $16' \times \sec h$, h being the corrected altitude.[1]

[1] This may be seen if it is remembered that the sun's diameter is $32'$ measured in the inclined plane containing the sun and place of observation, or, in Fig. 135 the semi-diameter is the arc AB measuring the angle $AO'B$ with radius R.

If the arc AB drawn with the radius r is considered equal to the arc AB drawn with the radius R, as is practically true, then, $A'B'$ and AB subtending the same angle Z,

$$\frac{A'B'}{AB} = \frac{R}{r};$$

but since $OBO' = B'O'B = h$

$$r = R \cos h$$

and $$A'B' = \frac{AB \times R}{R \cos h} = AB \sec h.$$

Substitute the corrected altitude, the latitude, found from a good map or one of the methods of Arts. 176 or 181, and the declination of the sun, found as described in Art. 180, in Equation (3) of Art. 175, and find the sun's azimuth, Z, referred to the meridian at the time of observation. Remember that the formula gives half Z.

Compare Z with the observed azimuth referred to the line of stakes and find the azimuth of the line of stakes referred to the meridian. The meridian may then be laid out.

The operation should be performed more than once, and the mean of the results taken. If the transit has a full vertical circle, the observations should be in pairs, one of each pair with the telescope direct, the other with the telescope reversed, the mean of each pair making a single determination.

To avoid correcting for the sun's semi-diameter and to eliminate instrumental error, the wires may be brought tangent to the lower and left limbs, the azimuth and vertical angle read, the transit reversed, the wires brought tangent to the upper and right limbs, and the azimuth and vertical angle read again. The mean of the two determinations should give the azimuth and vertical angle of the center of the sun for the instant midway between the times of the two measurements, if they are very close together. The work should be done as quickly as practicable. It is well to note the exact time of the observation. It may be used with the method of Art. 183 to find the error of the watch.

EXERCISE

Find the meridian by the method of this article and compare the result with meridians obtained by stellar observations.

183. Time and longitude. The angle ZPS, Fig. 133, is known as the hour angle of the sun, and when found and reduced to time gives the time interval between *apparent* solar noon and the instant of observation. Calling the angle t, by an equation of Spherical Trigonometry

$$\sin t = \frac{\sin Z \cos h}{\cos \delta}. \qquad (1)$$

When this is reduced to time at the rate of 15° to the hour and added to or subtracted from 12, it gives the apparent solar

instant of the observation. To this the equation of time, as found for the day and hour by interpolating in the tables in the Ephemeris, must be added or subtracted to get local clock time, and to this must be applied the difference in longitude, reduced to time, between the place of observation and the standard meridian controlling the watch time carried. If the place is west of the standard meridian, its time is slow, and the difference must be added to the calculated time to get the watch time; if east, the difference is subtracted, as local time will be fast of standard time. When this time is found, the difference between it and that recorded at the time of observation will be the error of the watch, which will be fast or slow according as the calculated time is earlier or later than the recorded time.

Time may be found also by noting the watch time of the sun's transit of the meridian and applying the equation of time. This will give the watch time of mean local noon.

When time is the object of the observation the longitude must generally be found from a map. It can be had to the nearest minute on a map on which the place can be located to the nearest mile from any given meridian. When longitude is the object of the observation, mean local time is found as above and the watch set to this time. The watch time is then compared with standard noon as ticked in to the nearest telegraph office. The difference between the watch noon and standard noon reduced to angular measurement, is the difference in longitude between the standard meridian and the place of observation.

Example

The value of t is found to be 18° 30′ in the morning of January 18, 1911, at a place whose longitude is 91° 32′ and at an instant when the watch reads 11 hr. 00 m. 00 s. a.m. Is the watch fast or slow, and by how much?

Suggestion. To find the equation of time refer to the page of Ephemeris reprint shown on page 251. Note that the equation of time for January 18 is 10 m. 19.09 s., that the increase for one day is the equation for the 19th less that for the 18th, or 10 m. 38.31 s. − 10 m. 19.09 s. Note that the place has $6\frac{1}{10}$ hrs. longitude and that the local time is about $1\frac{1}{5}$ hrs. before noon before correcting for equation of time, and 1 hr. after approximate correction, so that at the time of observation it is approximately

5 hrs. after noon at Greenwich or $\frac{5}{24}$ of a day toward January 19. Hence, $\frac{5}{24}$ of the daily change in equation of time should be applied to the equation of time for January 18 to get the proper quantity to use for the time of observation. The character of the observations which are not made with astronomical precision hardly warrants such close estimation, and the observer will do well if he determines the time to the nearest minute or half minute.

THE SOLAR TRANSIT

184. What it is. The solar transit consists of a transit with an attachment for determining the true meridian by an observation on the sun. The solar transit and the solar compass, essentially the same as the solar part of the solar transit, have been used extensively in laying out the public lands of the United States. The solar compass was invented by William A. Burt of Michigan, and has become known as Burt's solar compass. The United States Land Office has specified that the work of subdividing the public lands must be done with solar instruments or transits. The solar compass is not much used now. When a solar instrument is used, it is usually the solar transit.

The solar attachment consists essentially of an axis called a polar axis which may be adjusted to be perpendicular to the axis of the transit telescope tube; a line of sight, which in Fig. 136, page 258, consists of a lens in a block at A and a small silver disc ruled with lines at right angles forming a square in its center on a similar block at B; a second line of sight identical with the first, having its lens in the block at B and its ruled silver plate in that at A; and a so-called declination arc shown in the figure, about the center of which the lines of sight revolve as radii.

When the transit is set up with the telescope in the meridian plane and the telescope is turned on its horizontal axis until the telescopic line of sight lies in the equatorial plane as shown in the figure, the polar axis will be parallel to the earth's axis or, as is commonly said, will point to the pole, the radius of the earth being neglected.

If the line of sight of the attachment be then made perpendicular to the polar axis by bringing the vernier on the line of sight arm to read zero on the declination arc, it will also lie in

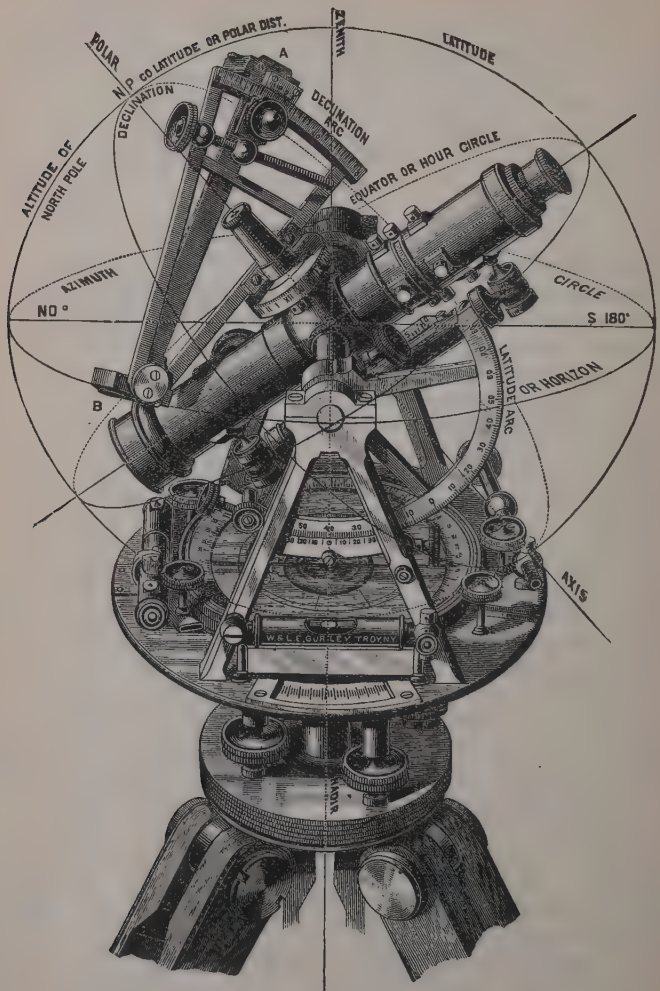

POLAR
N P CO LATITUDE OR POLAR DIST.
A
ZENITH
LATITUDE
DECLINATION
DECLINATION ARC
ALTITUDE OF NORTH POLE
EQUATOR OR HOUR CIRCLE
AZIMUTH
CIRCLE
N 0 °
S 180°
LATITUDE ARC
OR HORIZON
B
AXIS
W. & L. GURLEY TROY N.Y.
NADIR

Fig. 136.

the equatorial plane, and, if the attachment be revolved about its polar axis, the line of sight produced will describe the equatorial circumference on the celestial sphere.

Neglecting the daily change in the sun's declination, and supposing the sun to have north declination on a given day, it will traverse a path on the celestial sphere which will be the circumference of a circle that is parallel to the equatorial plane and might be called the parallel of declination, being analogous to a parallel of latitude on the earth's surface.

If the line of sight AB now be turned about the center of the declination arc by the angular amount of the declination for the day considered, as shown in the figure for instance, and if the attachment be revolved about its polar axis, the line of sight extended will traverse the path of the sun for the day. If the declination be south, the declination arc and its line of sight can be reversed and the lens B and silver plate A will constitute the line of sight, which, when revolved about the polar axis, will traverse a circumference parallel to and south of the equatorial circumference and which will be the path of the sun for the day.

The telescope is elevated to the plane of the equator, using the vertical circle or arc as a latitude arc on which, in the case shown in the figure, the colatitude has been set off.

It should be clear, from what has preceded, that with the transit set as in the figure the sun can be made to shine through one of the lenses of the lines of sight AB on to the opposite silver disk; that is to say, the line of sight can be brought to the sun by setting off the proper declination on the declination arc and revolving the attachment about its polar axis.

Now if the transit is revolved in azimuth so that the telescope is no longer in the meridian or equatorial plane and the polar axis no longer points to the pole, the line of sight even with the proper declination set will not trace the path of the sun on being revolved about the polar axis, nor can it in general be directed to the sun by simply so revolving it.

185. Meridian by the solar transit. The foregoing furnishes the clew to the operation of the solar transit in finding the meridian. If, with the proper colatitude set on the vertical circle and declination on the declination arc, the line of sight of the attachment can be brought to the sun, the telescope will

lie in the equatorial and meridional planes and may be tipped to range out a line which shall mark the meridian.

A carefully adjusted solar transit is set up over a point; the latitude of the place having been determined from a good map, a transit observation on Polaris or the sun, or by the solar transit as explained in Art. 188, the colatitude is set off on the vertical circle; the declination of the sun found as described in Art. 180 but affected by a refraction correction explained later, is set off on the declination arc; the transit is revolved in azimuth and the line of sight is revolved about the polar axis until an image of the sun is centered in the engraved square on the silver plate opposite the lens. At this instant the transit telescope lies in the meridian. The final azimuth setting is done by the slow-motion screw. A stake is set some distance away to the south (north in the southern hemisphere). The point under the instrument and the distant stake define the meridian.

At the base of the attachment is a small circle on which the apparent local time of the observation will be indicated. If this is reduced to mean time by applying the equation of time for the day, and this mean time again reduced to the standard watch time carried, a comparison should indicate any error of the watch.

Exercises

1. After reading the next two articles find the meridian through some fixed point and compare with a Polaris or transit and sun determination.
2. Note the time of the observation on the watch, and on the attachment, and find the watch error.

186. Refraction. Thus far nothing has been said of refraction. It must always be considered in setting off the declination. The effect of refraction has been discussed in the chapter on leveling. It appears there that the object is always seen higher than it really is. Hence, if the sun's declination is north, and the exact amount is set off on the declination arc, and the polar axis is brought into the meridian, and the line of sight pointed toward the sun, the sun's image would not be formed exactly in the little square, because the sun's rays would seem to come from a higher point than its true place. It is true that for small differences, like that of refraction or small errors in setting the latitude, the sun's image may be brought on the

square by slightly turning the instrument in azimuth, thereby destroying the correctness of the determination of meridian. Hence it is necessary to know and correctly set off the latitude and the declination *corrected for refraction.* The correction for refraction is greatest near the horizon, and is nothing at the zenith. Since it always makes a luminous body appear higher than it is, the correction must be added to north declinations and subtracted from south declinations to make the line of sight point higher than if the correction were not applied. The corrections to be used are as given in Table VII, page 480.

Comstock's method of finding the refraction correction will be found at p. 472.

EXERCISES

1. If the declination is north 5° 21′, what declination should be set on the declination arc at 9 o'clock in the morning in latitude 42° north?

2. Find the refraction correction by Comstock's method and compare with the tabular value of Table VII.

187. Limitations. Since the sun's motion has a horizontal azimuth, and a vertical component except just at noon when the motion is all horizontal, or in azimuth, the sun's image will appear to move diagonally out of the square on the silver plate if the instrument is left stationary for a little time, and if the polar axis is in the meridian the image can be kept in the square only by moving the line of sight about the polar axis and changing the declination setting; the latter never more than one and a half minutes an hour. Just at noon, however, the motion being all in azimuth, and the line of sight, polar axis, and main telescope all being in the same vertical plane, the image will move out of the square between the horizontal lines and can be kept in the square for some little time by turning the transit in azimuth. At this time, therefore, the meridian cannot be determined precisely. It cannot be done well within one or two hours either side of noon.

188. Latitude. For the same reason, since at noon the exact meridian is not needed to get the sun in the line of sight, this is the time to observe for latitude, as follows:

The instrument is set up a little before noon. The colatitude (as nearly as known) is set on the vertical circle, and the dec-

lination for noon, corrected for refraction, on the declination arc. The sun's image is brought on the silver disk between the horizontal lines by using the azimuth motion of the transit and the vertical circle tangent screw. The image will appear to get lower as the sun goes higher. The image is kept in the square on the disk till it appears to begin to move upward. The sun is then at its highest point; and if the declination has been properly set off and the plates carefully leveled, the vertical circle will read the colatitude of the place.

The solar transit may then be used at noon for finding latitude, and between 8 o'clock and 10.30 o'clock a.m. and 1.30 o'clock and 5 o'clock p.m. for determining a meridian. It could be used earlier and later but for refraction, which is of unknown and very irregular amount near the horizon.

Exercise

Find the latitude of the place occupied, and compare the result with a Polaris determination or a direct transit determination.

ADJUSTMENTS OF THE SOLAR TRANSIT

189. Named. The adjustments of the solar apparatus are simple. They consist in making the lines of sight parallel to each other and at right angles to the polar axis when the declination arc reads zero; and in making the polar axis perpendicular to the telescope. The transit is supposed to be adjusted.

190. Lines of sight. These are made parallel by making each line parallel to the edges of the blocks containing them. To do this, the bar carrying the line of sight is removed and replaced by a bar called an adjuster, which is simply a table upon which to rest the lines of sight while adjusting. The bar containing the lines of sight is placed on the adjuster, and, by any means, the sun is brought into one line of sight. The bar is turned over quickly (not end for end). If the image still falls in the square, the line of sight is parallel to the two edges of the blocks. If not, the silver disk is moved through one half the apparent error of position of the sun's image and tested again till correct. The bar is now turned end for end, and the other line of sight adjusted. The two now being parallel to the blocks are parallel

to each other. The adjuster is removed, and the bar replaced on the instrument.

191. Declination vernier. The declination vernier is made to read zero, and, by any means, the sun is brought into one line of sight. The bar is quickly and carefully revolved on the polar axis. If the sun appears to be in the other line of sight, the vernier is in adjustment. If not, the arm is moved by the slow-motion screw till the image is centered, and the reading of the vernier noted. The vernier is adjusted by loosening it and moving it half the apparent error. The test should be made again until the adjustment is complete.

192. Polar axis. To make the polar axis perpendicular to the telescope axis, the plates and the telescope are carefully leveled, and then the solar apparatus is leveled by the capstan-headed screws shown underneath the attachment. This is precisely the same as leveling up an ordinary leveling instrument, and is performed with the aid of an auxiliary level that rests on the blocks of the collimation bar. This bar is set so that the declination is zero, and is brought into the plane of the main telescope and leveled, then at right angles to this position and leveled again, and the operation repeated until the attachment is leveled in both directions. This is an important adjustment, and should not be omitted.

Exercise

Make all the adjustments of the solar attachment if one is available.

SAEGMULLER SOLAR ATTACHMENT

193. Description and use. Figure 137 shows another form of solar attachment, which consists simply of a small theodolite (so called because the telescope will not transit) attached to the top of the telescope of an engineer's transit. This is the invention of Mr. George N. Saegmuller, formerly of Washington, D.C., now of the Bausch and Lomb Optical Company.

In operation it is similar to the last-described attachment. The difference is that a telescopic line of sight is substituted

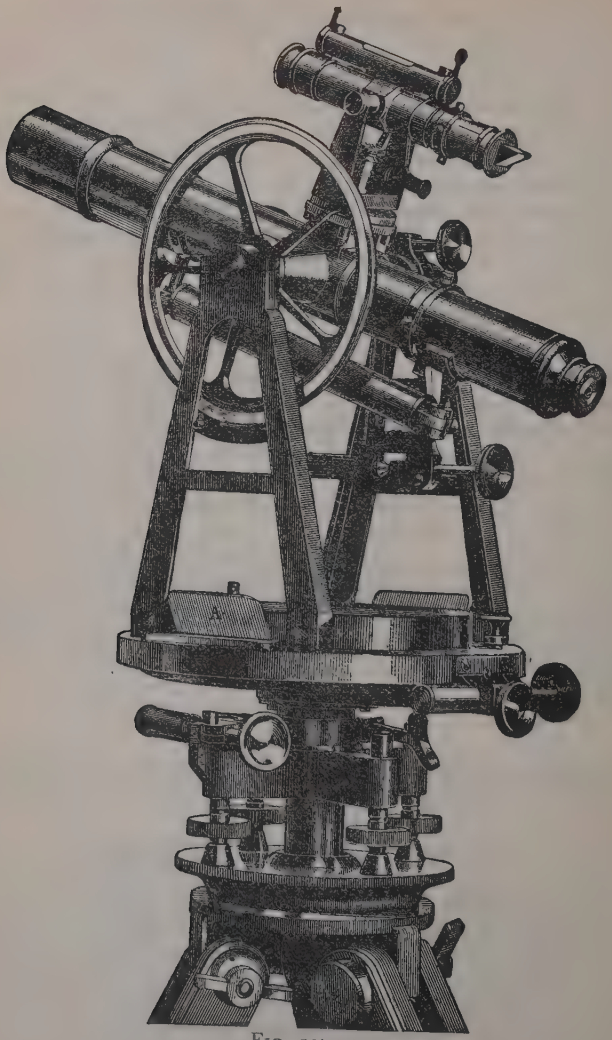


FIG. 137.

for the lens and disk, and the small level is used in conjunction with the vertical circle for a declination arc.

If the transit be set with the telescope level and the object end to the south, and if the declination is north, then if the object end of the telescope is turned down an amount equal to the corrected declination and the small bubble brought to the center of its tube, the angle between the main and small telescopes will equal the declination. If the object end is pointed upward by setting off the colatitude from zero (not from the declination reading), the instrument is ready for use.

The instrument is turned in azimuth and the small instrument about its polar axis till the sun's image is seen in the small telescope; then the large telescope and polar axis lie in the meridian. There is a diagonal eyepiece to the smaller telescope to facilitate observations. The instrument is approximately pointed by the small disk sights above the level tube. The objective is turned up in setting off south declination.

194. Adjustments. The adjustments of this instrument are two: the adjustment of the polar axis and that of the line of sight and small bubble. The first is performed, after the transit has been carefully adjusted, by making the main telescope level, then leveling the small telescope over one set of capstan-headed screws at the base of the attachment and adjusting, and then over the other precisely as in adjusting the plate bubbles of the transit, correcting first by screws and next by tipping the small telescope. The second adjustment is performed indirectly by making the two lines of sight parallel. The distance between the centers of the two horizontal axes is measured, and two parallel lines the same distance apart are drawn on a piece of paper; the paper is tacked up at some distance from the instrument and about on the same level, with the lines horizontal. The two bubbles are made parallel by making both level, with the telescopes pointing toward the paper; the line of sight in the large telescope is set on the lower line on the paper and the wires of the small telescope adjusted until the line of sight of the small telescope cuts the upper line.

GENERAL SURVEYING METHODS

CHAPTER X

LAND SURVEYING

195. General statement. A land survey is made for the purpose of:

1. Getting the necessary data for writing a description of a piece of land and finding its area.

2. Finding and marking the boundaries of a described tract.

The first constitutes an original survey, the boundaries being in evidence and shown to the surveyor who makes the survey. The second involves a resurvey to find the marks of an original survey from a written description or map. A location survey is distinguished from an original survey in that it is a survey of a piece of land whose boundaries are not known on the ground but are planned beforehand according to some system. Thus, the original surveys of the United States public lands were location surveys, since they served to locate certain corners whose positions were marked on a plan that actually existed on paper either as a drawn map, or as a necessary result of a formulated rule for subdivision. A survey to locate block corners or a lot in a tract of land, the subdivision of which has been made, is also a location survey, though it may involve a resurvey of the outer boundaries of the tract.

Thus there may be said to be three classes of land surveys: 1. Original surveys; 2. Resurveys; 3. Location surveys; though the last are a subclass of the first. Town lot surveying and subdivision of city property will be described under City Surveying, Chapter XIV.

In the older countries of the world and in the eastern parts of the United States all property was originally sold or granted in more or less irregular tracts described by "metes and bounds,"

that is, by giving the bearings and lengths of the several sides in order around the tract and describing the corners established or existing at points of change in direction. Often the larger grants containing thousands of acres were bounded, not by metes and bounds but by coast line, rivers, and watershed divides. On the western coast of the United States, and in some of the territory acquired from Mexico, the large grants to individuals or companies made before the United States came into possession are described in the same way; and, like the old Dutch and English grants of the eastern United States, have been subdivided into more or less regular tracts and sold to individuals who have still further subdivided their holdings or sold regular or irregular parcels from them, all of which are described by "metes and bounds" or, when finally reduced to city lots, by lot and block number. In the remainder of the United States, except Texas, and indeed in a large part of the Pacific coast, the lands were purchased or acquired by conquest by the United States, and these public lands have been divided and subdivided according to a definite plan, which will be described under following paragraphs, and sold to individuals by the government which has described them by simply noting their location in the general scheme of subdivision.

FARM SURVEYS IN THE OLDER PARTS OF THE UNITED STATES

196. An original farm survey. Mr. Brown may own a farm, a portion of which is timber land, which he is to sell to Mr. Black. The boundaries are sufficiently marked by the edges of the growth of timber. After the timber is gone, however, there will be nothing to mark the boundaries, unless the tract is fenced. There is no way in which Mr. Black may know how many acres he is to buy, unless the tract is surveyed. Neither can Mr. Brown give Mr. Black a deed to the property that would contain a definite description from which the tract could be laid out on the ground. A surveyor is called in and shown the tract and asked to make a survey of it, to compute the acreage, and to write a description of the plot.

At the corners that are shown him he sets monuments, preferably of stone, concrete, or iron. Too many surveyors set merely small stakes, which soon rot or are pulled out. These

monuments are "witnessed" by trees, or other natural objects near by, whose positions relative to the corner are observed and noted. The witness points are marked as indicated in Art. 197. The surveyor then runs a traverse around the field, determining the bearing or azimuth and length of each side. The bearing should be the true bearing and it should be so stated in the notes. He keeps notes of the work in a notebook in the form shown on page 138, or if the survey be a large one, in the form shown on page 293, or on a sketch, which is afterwards "written up" in one of the forms just mentioned, and from the notes he "tables" the survey and computes the area as explained in Chapter V.

He may, and usually does, make a map of the survey. This map should conform to the requirements stated in the Appendix, page 452. The map is made as the map of a traverse, as explained in Art. 75. It is best plotted by latitudes and longitudes. The bearings and lengths should be written so that they will read from left to right along the several lines in the direction in which they were run. The lettering for such a map may be in Roman (Plate VII) or in neat script, or a plain free hand letter as that in Fig. 138. The title should state whether the bearings or azimuths are true bearings or azimuths and, if they are not, should give the amount and direction of the magnetic declination. In modern land surveying all directions should be given with reference to the true meridian, which is the only reference line that can be reëstablished without doubt.

197. Description. Having made the map (Fig. 138), the surveyor writes a description, in the following form:

Beginning at a post marked B.1., at the S.E. corner of the land of Joseph Brown, from which post a hard maple tree, 8 inches in diameter, bears S. 10° W. 10 links, and a white ash, 12 inches in diameter, bears N. 70° W. 50 links, both of which trees are blazed and marked B.1.B.T. (Black 1 Bearing Tree), and running thence N. 10° 30′ W. along the easterly line of said Joseph Brown, six and forty-two one-hundredths ($6\frac{42}{100}$) chains to a post marked B.2., from which post a hickory tree, 10 inches in diameter and marked B.2.B.T. bears S. 68° W. 30 links; thence N. 84° W. seven and fifty one-hundredths ($7\frac{59}{100}$) chains to a stone about 12 inches long and 6 inches square, set flush with the ground and marked B.3., from which stone a

beech tree, 15 inches in diameter and marked B.3.B.T. bears, etc., ... thence, etc., to the point of beginning, containing ..... acres, more or less, the bearings in this description being all true bearings.

The surveyor usually does his tabling and writes his descriptions in a book which he keeps for the purpose, for future reference. Copies are made, and, with a tracing of the map, are furnished to the person for whom the survey is made. This is the simplest kind of land surveying.

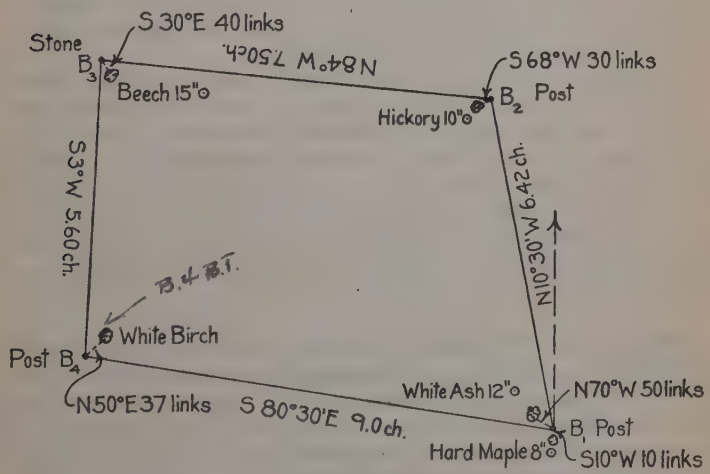


FIG. 138.

EXERCISE

Let the student complete the description in accordance with the data shown on the map.

198. Resurveys. These are far more difficult than original surveys. A resurvey consists in retracing an original survey on the ground, from a description similar to that just given. The difficulties arise from the destruction of monuments and from errors in original work, from change in declination of the needle (if the surveys were run by the magnetic compass and referred to the magnetic meridian), from insufficient data in the

original description, such as failure to state whether the survey is referred to the true or magnetic meridian, failure to state the declination on which the survey was made, lack of bearing trees or other reference points, etc., and from conflicting testimony of interested owners as to where the corners were. It is impossible to point out all the difficulties that will be encountered by a surveyor in his attempt to reëstablish the monuments of an old survey. The more of this work that he does, the more firmly will it be impressed upon him that the only kind of corners to establish are those that will be as nearly permanent as possible, and that minuteness of detail and accuracy in all descriptions are well worth the time they take.

Referring to the original survey previously described: After Mr. Black has owned the wood lot for a number of years, and perhaps has cleared it, and it has descended to his son and his son's son, and the corners are mostly obliterated, it is sold to Mr. Johnson who, desiring to fence it off, and, moreover, to see whether the land he is paying for is all there, employs a surveyor to " run it out," giving him the description written by the original surveyor as he finds it in his deed, with possibly some errors in copying.

199. Preliminary investigation. The surveyor first tables the survey from the description to see if it is practically correct. An error of closure not greater than one in three hundred in a very old compass survey would indicate the probability of correctness of description. The field may be plotted roughly to scale, and the angles between the sides may be computed. The ground is next examined to find traces of formerly established corners, witness trees or objects, or probably correct fence lines. Testimony of old settlers may be requested to confirm or dispute existing lines or objects, but too much reliance must not be placed on such testimony.

200. Survey. If the surveyor can recover a single line of the original survey, and the notes are correct, his work will be comparatively easy. (He simply has to establish a series of lines of given bearing and length.) Setting the instrument over the initial end of the recovered line he reads the bearing of it and compares his reading with the recorded reading. If they differ, as they probably will, he turns his declination vernier so that

when the instrument is set in the direction of the line, the compass shows the original bearing. (The difference between the recorded bearing and the present bearing, with allowance for possible instrumental differences, is the change in declination since the time of the original survey. With this the surveyor is not concerned immediately.) The surveyor then measures the discovered line and divides his measurement by the recorded measurement, if they differ, for a coefficient by which to multiply other recorded distances, to give probable present distances to be used. He then goes to the farther end of the line, turns the instrument in the direction of the next course and measures the properly multiplied recorded distance to the point where the corner may have been. He looks for evidences of the corner, stumps of witness trees, or the original corner stone or stake. Finding none, he carefully slices off the top soil with a shovel for some distance around the supposed corner, not spading it up, to try to find the hole where the original stake was placed, the remains of a fence post, or other evidence of the position of the corner. If nothing can be found, the point established by measurement may be assumed right for the time, and the next course laid out in similar manner and the corner searched for. In this manner the survey may proceed to the place of beginning. If preferred, the course preceding the discovered course may be ranged out after the failure to locate the original third corner. If no corners at all are found but the survey closes within reasonable limits on the starting point, it may be assumed to be fairly correct, may be balanced, the bearings and lengths corrected and rerun in the field, establishing the new corners as the true corners of the survey. If by chance the survey closes within the limits of precision the surveyor has set for his own work, he need not rerun the lines, but this is not likely to occur.

He should remember that *nothing will absolutely insure the correctness of the work but the finding of the old corners.*

201. Change of declination. If the compass has no declination vernier the lines must be run by new bearings which are found by affecting the original bearings by the change of declination.

The following consideration will explain this: Let it be assumed that the original survey was made with reference to the magnetic meridian and that the declination at that time was east

10°. Then, any line that is recorded due north will be 10° east of north. Any line recorded due south will be 10° west of south, etc. That is, all the points of the compass are turned 10° to the right. Let it be assumed that the declination at the time of the resurvey is 8° east. A line recorded as due north will be 8° east of north, and a line recorded as due south will be 8° west of south. The points of the compass have been turned back two degrees to the left. And hence, if a line were run out with the original bearing, it would lie two degrees to the left of the true place. It would be necessary to run the line with a bearing two degrees to the right of its originally recorded bearing. Thus, if its original bearing were north, it must be rerun with the needle reading N. 2° E.[1] Hence, the following rule is serviceable:

RULE: *To rerun lines recorded by their magnetic bearings, change the bearings by an amount equal to the change in declination and in a direction opposite to that change* — that is, *left or right.*

If the compass has a declination vernier and only a single corner of the survey is to be found, and, further, if the change in magnetic declination since the time of the original survey is known, the vernier may be turned so that lines may be run on the original bearings. If the change in declination has been 2 degrees to the left, the vernier must be turned so that magnetic north will show a bearing of N. 2° W. Hence, set the compass line of sight in the magnetic meridian and turn the compass box by the declination vernier 2 degrees to the left (counterclockwise) as shown by the change in needle reading. The following rule may be serviceable:

RULE: *Set the declination vernier so that the compass shall read magnetic bearings erroneously by an amount equal to and in the direction (left or right) of the change in declination.*

To find the change in declination, if it is not known, determine the bearing of a line of the original survey and compare this with the bearing recorded. The difference is the change in declination in amount and in opposite direction (left or right).

If only two nonadjacent corners can be found, the procedure may be as follows:

[1] The student should make a sketch of the lines and needle when studying this paragraph.

Run a random traverse from one corner to the other and compute the length and bearing of the closing line; compute the length and bearing of the same line by the original survey notes, using the courses on either side of it; the difference in bearings will be the change of declination or angle between meridians used, and the difference in length will show the difference in original and present measurements. It is best to run the random line as nearly as may be over the lines of the original survey so that approximately the same total length and number of courses may be used. If the length of the closing lines by the original notes is L and that by the new survey is l, each originally recorded distance should be multiplied by $\frac{l}{L}$ to get the probable length by the new measurements. If the bearing of the closing line was N. 36° 10′ E. and the present bearing is found to be N. 34° 32′ E, it may be assumed that the needle has moved to the right 36° 10′ minus 34° 32′, and the declination vernier may be set correspondingly or the original bearings changed as indicated in the preceding rules.

When a transit is used, the angles at the corners of the field may be determined from the recorded bearings and, when one side has been recovered, the angles and distances may be measured, to recover the other sides. This does away with all consideration of, or change in, declination. It is by far the better method to pursue with lands of any considerable value. The angles determined from compass readings cannot be depended on to minutes, and may be found to vary five or ten minutes. It would be well if, when a transit is first used on a resurvey, the corrected bearings and distances could be introduced in a correction deed which could be filed with the proper authority.

202. Report and map. Upon the completion of the resurvey, the surveyor should report to his employer just what he finds. It is not his business to decide controversies. He may advise, just as an attorney would do; but he has no authority to correct errors or to establish corrected corners as *the* corners.[1] His business is to make an examination, to reset lost corners in their original positions, if he can find them, and to report his method of procedure and the reasons for his action to his employer, who

[1] See "Judicial Functions of Surveyors," by Judge Cooley, Appendix, p. 440.

may then take such action as he chooses. A neat and explicit map should accompany the report. The surveyor may be assisted in his work by an understanding of the principles of Art. 218 deduced from many court decisions.[1]

The map will be made by latitudes and longitudes, the bearings and distances written along the courses always reading in the direction of the survey. Old corners found should be noted and new corners set, together with such witness objects as may have been observed. These should be plotted on the drawing, and bearings and distances to them indicated along dotted lines. The map should conform to the requirements of Appendix III, pages 452–455.

203. Application of coördinates. In all this work the coördinate system is very helpful. Usually the true corners of a tract of land cannot be occupied by the instrument, nor can the lines be seen throughout their length. If it is known where the corners are, and it is desired to make a survey for a map or a description, a random survey is made with corners as near as practicable to the true corners. This survey is balanced and the coördinates of its corners are determined. From such of the corners as are near true corners, angles or azimuths, and distances to the true corners are noted, and from these the coordinates of the true corners are determined. From these coordinates, lengths and bearings of true lines may be found.

If the survey is for the purpose of relocating lost corners, and the bearings of the lines and at least one point in each are known, the corners may be located, even though the lines cannot be occupied by the instrument, provided the corners and known points are accessible. This would be done by running a random survey as before with pointings to the known points (and such corners as are known), then finding the coördinates of the known points, and by Problem II, Art. 127, the coördinates of the wanting corners. Knowing now the coördinates of the points in the random survey and those of the true corners, by Problem I, Art. 127, find the bearing and distance from a corner of the random to the nearest wanting true corner, and locate the corner.[2]

[1] These and many other decisions may be found in a valuable "Manual of Land Surveying," by Hodgman and Bellows.

[2] For model example, applied to city property, see Appendix, page 433. The student may

UNITED STATES PUBLIC LAND SURVEYS

204. Value and character of work. The original surveys for the subdivision of the public lands of the United States are location surveys. The method adopted, imperfectly as the work has been done, has been of incalculable value in definitely describing each separate tract or parcel of land sold to individuals, and in providing that lines once established by the United States deputy surveyors shall remain as the lines they purport to be, even though found to be placed improperly. The latter provision has been of particular value in making the land lines permanent. The work of subdividing the public lands is almost completed, and hence the work of the surveyor of the future will be largely resurveying; that is, relocating corners that have been or are supposed to be lost, and dividing into smaller parcels the areas already located.

If the original surveys had been properly executed, the work would not be difficult. In some instances the work was most wretchedly done, either willfully or through ignorance; and the corners, when established, were placed far from their proper positions. The notes have been returned generally in a form indicating correct work, and hence has arisen more or less difficulty in relocating the corners.

205. General scheme of subdivision. Certain points have been selected in different parts of the country through which true north and south lines, called *principal meridians*, have been run and marked out on the ground. Intersecting these principal meridians at the initial point parallels of latitude, known as *base lines*, are run. On either side of these meridians the land is laid out in approximately square parcels, six miles on a side, called *townships*. A tier of these townships running north and south is called a *range*. The townships are described as being "Township No. —— south or north of a named base line and range No. —— east or west of a named principal meridian." This definitely locates every township.

The lines that divide the ranges are called *range lines*, and

tell how to proceed if the bearings and lengths are all given, with but two corners known, and lines so grown over or occupied by structures that they cannot be run out directly without great labor.

those that divide a southern from a northern township are called *township lines*. Each township is further divided into thirty-six *sections*, each approximately one mile square. These sections are numbered from one to thirty-six. Each section, then, is definitely located by its number, township, and range. The sections have been further divided into quarter sections, as the northeast quarter, the southwest quarter, etc. Sometimes the sections have been divided into halves, described as the north half or the east half, etc. The Government does not divide the land into smaller divisions than quarter sections, but it sells less areas than this, and, in such case, and when original purchasers sell a portion of their purchase, it is usual to sell a quarter of a quarter, or half of a quarter, or even a quarter of a quarter of a quarter section; and the method of describing these fractional portions is the same as that used to describe the quarter sections. The description would be written as follows for the piece described: The N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of Sec. 26, Tp. 8 N., R. 4 E., Mt. Diablo Meridian.

The positions of the meridians having been chosen more or less at random and at different times, as the necessity for surveys in different localities developed, it is to be expected that the surveys extending east from one meridian will not close with regular full sections or townships on the surveys extended west from the next easterly meridian. The same is true of tiers of townships extended north and south from adjacent base lines. The result is fractional townships and sections. Sometimes these are larger than the standard division, sometimes smaller. If very much larger, the surplus in a township is divided into lots which are numbered. These lots are made to contain as nearly as possible 160 acres. There are other circumstances that will appear, that cause a departure from the ordinary method of subdividing and describing.

The work in a given state is under the direction of a United States surveyor-general. The work was formerly done by contract, a surveyor taking a contract to perform a definite portion of work for a specified sum per mile. This has been the principal cause of much bad work.

The lands classified as public lands and subdivided according to the method outlined, include all land north of the Ohio River and west of the Mississippi River, except Texas, and including Mississippi, Alabama, and Florida, except in all the

above-named territory such lands as belonged to individuals at the time the territory became a part of the United States.

The public lands of Texas are, by a provision of the laws admitting Texas into the Union, the property of the state. No general scheme for their subdivision has been developed. They have been sold in parcels as nearly square as may be. The Spanish vara is the unit that has been adopted for measurement. The vara is, in Texas, $33\frac{1}{3}$ inches. There are 5645 square varas in an acre.

The following is very much condensed from the "Manual of Surveying Instructions," issued by the General Land Office in Washington.

206. Historical note. The first surveying of the public lands was done in Ohio under an act passed by Congress in 1785. The territory included in this early survey is now known as "The Seven Ranges." The townships were divided into thirty-six sections one mile square, numbered consecutively, as in Fig. 139.

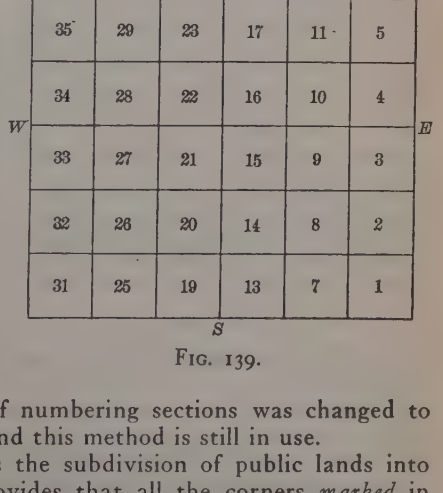


FIG. 139.

In 1796 the method of numbering sections was changed to that shown in Fig. 140, and this method is still in use.

An act of 1805 directs the subdivision of public lands into quarter sections, and provides that all the corners *marked* in the public surveys shall be established as the proper corners of sections which they were intended to designate, and that corners of half and quarter sections *not marked* shall be placed, as nearly as possible, "equidistant from those two corners which stand on the same line." This act provides further that "the boundary lines *actually run* and *marked* . . . *shall be established* as the *proper boundary lines* of the sections or subdivisions for which they were intended; and the *length* of such lines *as re-*

turned by . . . the surveyors shall be held and considered as the true length thereof. . . ."

An act of 1824 provides " that whenever, in the opinion of the President of the United States, a departure from the ordinary mode of surveying land on any river, lake, bayou, or water course would promote the public interest, he may direct the surveyor-general in whose district such land is situated . . . to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water course, and running back the depth of forty acres." [1]

N

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

W E

S

FIG. 140.

An act of 1820 provided for the sale of public lands in half-quarter sections, the quarters to be divided by lines running north and south; and an act of 1832 provided for the sale of the public lands in quarter-quarter sections, and that the half sections should be divided by lines running east and west. The latter act also provided that the Secretary of the Treasury should establish rules for the subdivision of fractional sections.

207. Legal requirements inconsistent. Existing law requires that the public lands be laid out in townships *six miles square* by lines running due north and south, and others east and west; also that the township shall be divided into thirty-six sections, by two sets of parallel lines, one governed by true meridians and the other by parallels of latitude, the latter intersecting the former at right angles, at intervals of one mile; and each of these sections must contain, as nearly as possible, six hundred

[1] Let the student determine the width and depth in chains of such a strip. This provision is carried out where the water front rather than area is the valuable item.

and forty acres. These requirements are manifestly impossible because of the convergency of the meridians, and the discrepancies will be the greater as the land divided is farther north. The law also provides that the work of subdivision shall be so performed as to throw all shortages or surplus into the northern and western tiers of sections in each township. To harmonize these various requirements as much as possible, the following methods have been adopted by the general land office:

208. Principal reference lines. Initial points are established astronomically under special instructions. From the initial point a "principal meridian" is run north and south. Through the initial point a "base line" is run as a parallel of latitude east and west.[1] On the principal meridian and base line the section and quarter corners, and meander corners at the intersection of the line with all streams, lakes, or bayous, prescribed to be meandered, will be established. These lines may be run by solar instruments, but methods involving the use of the transit with observations on Polaris at elongation are now preferred.[2] Two sets of chainmen are used. Each set measures forty chains, and if the discrepancy between their measurements is not more than 8 links on even ground or 25 links on mountainous surfaces, the proper corner is established at the mean of the two positions determined. If the discrepancy exceeds these limits, the line must be remeasured.

209. Standard parallels. Such parallels, called also correction lines, are run east and west from the principal meridian as parallels of latitude at intervals of twenty-four miles north and south of the base line. "Where standard parallels have been placed at intervals of thirty or thirty-six miles, regardless of existing instructions, and where gross irregularities require additional standard lines, from which to initiate new, or upon which to close old, surveys, an intermediate correction line should be established to which a *local* name may be given, *e.g.*, 'Cedar Creek Correction Line'; and the same will be run, in all respects, like the regular standard parallels."

[1] A list of base lines and principal meridians will be found in the Appendix, pages 455–459.

[2] See the "Manual of Surveying Instructions" for the detail of methods. The methods of Chapter IX may be used for the meridians.

210. Guide meridians. Guide meridians are extended *north* from the base line, or standard parallels, at intervals of twenty-four miles east and west from the principal meridian.

When existing conditions require the guide meridians to be run *south* from a standard parallel or a correction line, they are initiated at properly established closing corners on the given parallel. This means that they are begun from the point on the parallel at which they would have met it if they had been run north from the next southern parallel. The point is obtained by computation, and is less then twenty-four miles from the next eastern or western meridian by the convergence of the meridians in twenty-four miles.

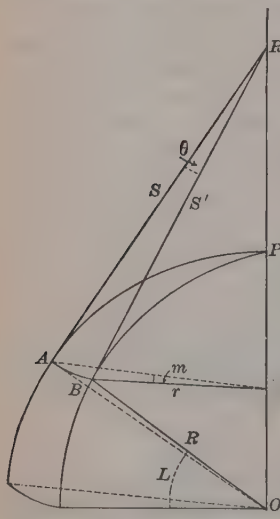


FIG. 141.

In case guide meridians have been improperly located too far apart, auxiliary meridians may be run from standard corners, and these may be designated by a local name, *e.g.*, "Grass Valley Guide Meridian."

211. Angular convergence of two meridians. This is given by the equation

$$\theta = m \sin L, \qquad (1)$$

where m is the angular difference in longitude of the meridians and L is the mean latitude of the north and south length under consideration. The linear convergence in a given length of meridian, l, is approximately

$$c = l \sin \theta. \qquad (2)$$

The derivation of equation (1) is as follows, assuming the earth to be a sphere, which will introduce no error of consequence in this work. In the figure (141) R is the mean radius, r is the radius of a parallel, S and S' are tangents to the two meridians. The angle θ between these is the angular convergence of the meridians in the latitude L. m is the difference in longitude.

$$r = R \cos L. \tag{3}$$

$$S = R \cot L. \tag{4}$$

S and S' may be considered as radii of the arc AB, which has also the radius r. Since a given length of arc subtends angles inversely proportional to the radii with which it may be drawn,

$$\frac{\theta}{m} = \frac{r}{S} = \frac{R \cos L}{R \cot L}, \tag{5}$$

whence $\theta = m \sin L$, which was to be found.

Since the distance between meridians is usually given in miles, this must be reduced to degrees. This is done by first finding the linear value of one degree of longitude for the mean latitude, using the value of r given in equation (3). To make (3) and (4) strictly correct, the normal at A should be used instead of R; but the mean radius will give results sufficiently close for land surveying.[1]

The following method for getting θ was given to the author by Professor Thomas U. Taylor of the University of Texas. Referring to page 280, equation (1), Fig. 141,

$$m = \frac{57.3^\circ \cdot AB}{r}, \text{ and } r = R \cos L.$$

If AB, given in miles, $= n$, and $R = 20890591.5$ (Clarke's spheroid, 1866, page 7),

$$m = \frac{57.3 \times 5280 \times n}{20890591.5 \cos L}.$$

Substituting this in equation (1), page 280, and multiplying by 3600 to reduce θ to seconds, gives

$$\theta'' = 52.13\ n \tan L. \tag{6}$$

Equation (2) may be demonstrated thus. Referring to Fig. 142, AB and CD may be considered meridians between parallels BD and AC a short distance apart, DC and AB may then be considered straight. The linear convergence is the difference between BD and AC or approximately JC. If EF is a meridian

[1] See Tables IX and X, pages 490, 491, for values of θ and length of 1′ of longitude in various latitudes.

through the midpoint of the parallels and BH and DG are parallel to it, then $AH + GC = 2\,l \tan \frac{1}{2}\theta = JC$ approximately $= l \tan \theta$ approximately $= l \sin \theta$ approximately, because for very small angles, as these always are, $\tan \theta = \sin \theta$ very nearly.

212. To run a parallel of latitude. 1. *With the solar transit or solar compass.* Having a point on the parallel, set the instrument over it, determine the meridian and run a line at right angles thereto. If the meridian is determined at each set-up of the instrument, the line thus run will approximate to a parallel of latitude.

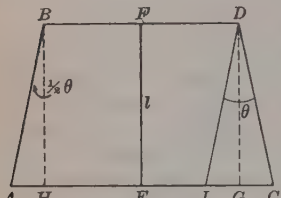


FIG. 142.

2. *Tangent method with a transit.* Having a point on the parallel, determine the meridian and run a line at right angles thereto. This line will be the arc of a great circle and will be tangent to the parallel at the point of beginning. At any desired distance, n miles from the starting point, offset north (south in the southern hemisphere) a distance given by the equation

$$O_{\text{feet}} = \tfrac{2}{3} n^2 \tan L, \qquad (1)$$

in which L is the latitude of the place and n is in miles. The end of the offset will be on the desired parallel.

The direction of the meridian in which the offset is to be run is found by deflecting from the great circle an angle

$$\delta = 90^\circ + \theta, \qquad (2)$$

θ being given in seconds by the equation of the preceding article. These equations will meet all the requirements of the United States Land Survey instructions.

The derivation of equation (1) is as follows:

Referring to Fig. 141, page 280, imagine the arc of the great circle to be run from B to A. It will fall below A at some point C, CA being the offset O of equation (1). Imagine also the chord AB drawn. Since the angle between a chord and tangent to any arc is one half the angle subtended at the center by the arc, the angle $ABC = \frac{1}{2}\theta$.

If AB be called n, given in miles, and O be first considered in miles, $\frac{1}{2}\theta = \frac{O}{n} 57.3^\circ$ and $O = \frac{\frac{1}{2}\theta n}{57.3}$.

Since $\theta = m \sin L$, $m = \frac{57.3n}{r}$, and $r = R \cos L$, $O_{\text{miles}} = \frac{n^2 \tan L}{2 R_{\text{miles}}}$.

Multiplying n and R by 5280 to reduce O to feet

$$O = \frac{5280^2 n^2 \tan L}{2 \times 20890591.5} = \tfrac{2}{3} n^2 \tan L.^1$$

The "Manual of Instructions" gives tabulated values for offsets from tangents in various latitudes, and the acute angles between the tangents and meridional offsets. Figure 143 illustrates the method; the table will be found at pages 285–286. The offsets are given for whole miles. For the half miles they may be found by remembering that offsets are as the squares of the distances from the tangent point. For the first 6 miles they may be had from the table, since the offset at a half mile is $\frac{1}{4}$ that at one mile; the offset at $1\frac{1}{2}$ miles is $\frac{1}{4}$ that at 3 miles, etc., and the table runs to 12 miles. For the angles of the half mile, offsets with the tangent interpolation may be made between the two adjacent mile offset angles, which will give results to the nearest minute, which is close enough.

3. *Secant method with a transit.* This consists in running a six-mile secant line cutting the parallel in two points, four miles apart, and measuring offsets from each half mile and mile point to the curved line according to a table of these quantities prepared beforehand. Such a table is found at page 287. To use it set up the transit on the township corner, and project the range line south a distance given as the offset for 0 miles in the second column of the table for the proper latitude. Establish a point and set the transit over it. From the meridian line just extended turn the acute angle for 0 miles found in the same second column and run a line 6 miles offsetting the quantities shown in the third, fourth, etc., columns at each half mile and mile in the directions shown in the same column. The ends of the offsets will lie on the parallel. The quantities are given

[1] This equation and discussion were suggested to the author by Professor Thomas U. Taylor of the University of Texas.

FIG. 143.

FIG. 144.

Offsets in True North Lines from Tangent to Parallel and Acute Angles from Tangents to Offsets

Latitude	1 Mile		2 Miles		3 Miles		4 Miles	
	Offset	Angle	Offset	Angle	Offset	Angle	Offset	Angle
°	*Chains*	° ′	*Chains*	° ′	*Chains*	° ′	*Chains*	° ′
30	0.006	89 59.50	0.023	89 59.00	0.053	89 58.50	0.09	89 58.00
31	0.006	89 59.48	0.024	89 58.96	0.055	89 58.44	0.10	89 57.92
32	0.006	89 59.46	0.025	89 58.92	0.057	89 58.37	0.10	89 57.83
33	0.007	89 59.44	0.026	89 58.87	0.059	89 58.31	0.10	89 57.75
34	0.007	89 59.42	0.027	89 58.83	0.061	89 58.25	0.11	89 57.66
35	0.007	89 59.39	0.028	89 58.79	0.064	89 58.18	0.11	89 57.57
36	0.007	89 59.37	0.029	89 58.74	0.066	89 58.11	0.12	89 57.48
37	0.008	89 59.35	0.031	89 58.69	0.068	89 58.04	0.12	89 57.39
38	0.008	89 59.32	0.032	89 58.65	0.071	89 57.97	0.13	89 57.29
39	0.008	89 59.30	0.033	89 58.60	0.074	89 57.90	0.13	89 57.19
40	0.008	89 59.27	0.034	89 58.55	0.076	89 57.82	0.13	89 57.09
41	0.009	89 59.25	0.035	89 58.49	0.079	89 57.74	0.14	89 56.99
42	0.009	89 59.22	0.036	89 58.44	0.082	89 57.66	0.14	89 56.88
43	0.009	89 59.19	0.038	89 58.39	0.085	89 57.58	0.15	89 56.77
44	0.010	89 59.16	0.039	89 58.33	0.088	89 57.49	0.16	89 56.66
45	0.010	89 59.13	0.040	89 58.27	0.091	89 57.40	0.16	89 56.53
46	0.010	89 59.10	0.042	89 58.21	0.094	89 57.31	0.17	89 56.41
47	0.011	89 59.07	0.044	89 58.14	0.097	89 57.21	0.17	89 56.28
48	0.011	89 59.04	0.045	89 58.08	0.101	89 57.12	0.18	89 56.15
49	0.012	89 59.00	0.046	89 58.01	0.104	89 57.01	0.19	89 56.02
50	0.012	89 58.97	0.048	89 57.94	0.108	89 56.90	0.19	89 55.88
	5 Miles		6 Miles		7 Miles		8 Miles	
30	0.14	89 57.50	0.21	89 57.00	0.29	89 56.50	0.37	89 56.00
31	0.15	89 57.40	0.22	89 56.88	0.30	89 56.36	0.39	89 55.83
32	0.16	89 57.29	0.23	89 56.75	0.31	89 56.21	0.40	89 55.67
33	0.16	89 57.19	0.24	89 56.62	0.32	89 56.06	0.42	89 55.50
34	0.17	89 57.08	0.25	89 56.49	0.33	89 55.91	0.43	89 55.32
35	0.18	89 56.97	0.25	89 56.36	0.35	89 55.75	0.45	89 55.15
36	0.18	89 56.85	0.26	89 56.22	0.36	89 55.59	0.47	89 54.96
37	0.19	89 56.74	0.27	89 56.08	0.37	89 55.43	0.48	89 54.78
38	0.20	89 56.61	0.28	89 55.94	0.38	89 55.26	0.50	89 54.59
39	0.20	89 56.49	0.29	89 55.79	0.40	89 55.09	0.52	89 54.39
40	0.21	89 56.37	0.30	89 55.64	0.41	89 54.91	0.54	89 54.18
41	0.22	89 56.24	0.32	89 55.48	0.43	89 54.73	0.56	89 53.97
42	0.23	89 56.10	0.33	89 55.32	0.44	89 54.54	0.58	89 53.76
43	0.24	89 55.96	0.34	89 55.15	0.46	89 54.35	0.60	89 53.54
44	0.24	89 55.82	0.35	89 54.98	0.48	89 54.14	0.62	89 53.31
45	0.25	89 55.67	0.36	89 54.80	0.49	89 53.94	0.64	89 53.07
46	0.26	89 55.52	0.37	89 54.62	0.51	89 53.72	0.66	89 52.83
47	0.27	89 55.36	0.39	89 54.43	0.53	89 53.50	0.68	89 52.57
48	0.28	89 55.19	0.40	89 54.23	0.55	89 53.27	0.71	89 52.31
49	0.29	89 55.02	0.42	89 54.02	0.57	89 53.03	0.74	89 52.03
50	0.30	89 54.84	0.43	89 53.81	0.59	89 52.78	0.77	89 51.75

Latitude	9 Miles		10 Miles		11 Miles		12 Miles	
	Offset	Angle	Offset	Angle	Offset	Angle	Offset	Angle
°	*Chains*	° ′	*Chains*	° ′	*Chains*	° ′	*Chains*	° ′
30	0.47	89 55.50	0.58	89 55.00	0.71	89 54.50	0.84	89 54.00
31	0.49	89 55.31	0.60	89 54.79	0.74	89 54.27	0.88	89 53.75
32	0.51	89 55.13	0.63	89 54.58	0.76	89 54.04	0.91	89 53.50
33	0.53	89 54.93	0.65	89 54.37	0.79	89 53.81	0.95	89 53.25
34	0.55	89 54.74	0.68	89 54.16	0.82	89 53.57	0.98	89 52.99
35	0.57	89 54.54	0.70	89 53.93	0.86	89 53.32	1.02	89 52.72
36	0.59	89 54.33	0.73	89 53.71	0.89	89 53.08	1.06	89 52.45
37	0.61	89 54.12	0.75	89 53.47	0.91	89 52.82	1.10	89 52.16
38	0.64	89 53.91	0.78	89 53.23	0.95	89 52.55	1.14	89 51.88
39	0.66	89 53.69	0.81	89 52.98	0.99	89 52.28	1.18	89 51.58
40	0.68	89 53.46	0.84	89 52.73	1.02	89 52.00	1.22	89 51.28
41	0.70	89 53.22	0.87	89 52.47	1.06	89 51.72	1.26	89 50.97
42	0.73	89 52.98	0.90	89 52.20	1.09	89 51.42	1.31	89 50.64
43	0.75	89 52.73	0.93	89 51.92	1.14	89 51.12	1.35	89 50.31
44	0.79	89 52.47	0.97	89 51.64	1.18	89 50.80	1.40	89 49.97
45	0.81	89 52.21	1.00	89 51.34	1.22	89 50.47	1.45	89 49.61
46	0.84	89 51.93	1.04	89 51.03	1.26	89 50.14	1.50	89 49.24
47	0.87	89 51.64	1.07	89 50.72	1.31	89 49.79	1.56	89 48.86
48	0.91	89 51.35	1.12	89 50.38	1.35	89 49.42	1.61	89 48.46
49	0.93	89 51.03	1.16	89 50.04	1.40	89 49.04	1.67	89 48.05
50	0.97	89 50.71	1.20	89 49.68	1.45	89 48.65	1.73	89 47.62

for 3 miles reading from the top of the table down. The offsets for the remaining 3 miles are the same as for the first three in reverse order and the proper column headings are at the bottom of the table. The acute angles with the meridians are also the same in reverse order. At the end of 6 miles produce the secant 1 chain and measure over northerly, at right angles, the distance found in the last column of the table. Through the point thus found produce the next secant from the end of the first and offset as before. Figure 144, which will make the process clear, and the Table are taken from the "Manual of Instructions." The offsets are so short that their direction may be estimated without appreciable resulting error in position. This is the better of the transit methods.

213. Township exteriors. Each twenty-four-mile "square" block is, when practicable, subdivided into townships at one time, the work being done as follows:

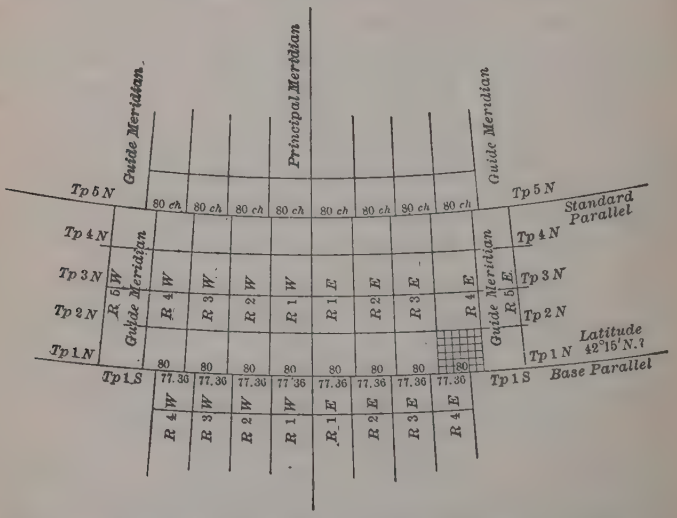
Beginning at the southeast corner of the southwest township of the block the meridian is run due north six miles, the section and quarter corners being set in their proper places. Then a

Azimuths of the Secant, and Offsets, in feet, to the Parallel

Arguments: latitude in left-hand column and distance from starting point at top or bottom of table. Right-hand column gives deflection angles and nat. tan.; to radius 66 ft.

Latitude	Azimuths and Offsets at—							Defl. Angle and Tan.
	0 miles	½ mile	1 mile	1½ miles	2 miles	2½ miles	3 miles	
°								
30	89° 58.5′ 1.93 N.	89° 58.7′ 0.87 N.	89° 59.0′ 0.00	89° 59.2′ 0.67 S.	89° 59.5′ 1.15 S.	89° 59.7′ 1.44 S.	90° (E. or W.) 1.54 S.	3′ 00.2″ 0.69 ins.
31	89° 58.4′ 2.01 N.	89° 58.6′ 0.91 N.	89° 58.6′ 0.00	89° 59.2′ 0.70 S.	89° 59.5′ 1.20 S.	89° 59.7′ 1.50 S.	90° (E. or W.) 1.60 S.	3′ 07.4″ 0.72 ins.
32	89° 58.4′ 2.09 N.	89° 58.6′ 0.94 N.	89° 58.9′ 0.00	89° 59.2′ 0.73 S.	89° 59.5′ 1.25 S.	89° 59.7′ 1.56 S.	90° (E. or W.) 1.67 S.	3′ 15.0″ 0.75 ins.
33	89° 58.3′ 2.17 N.	89° 58.5′ 0.97 N.	89° 58.8′ 0.00	89° 59.1′ 0.76 S.	89° 59.4′ 1.30 S.	89° 59.7′ 1.62 S.	90° (E. or W.) 1.73 S.	3′ 22.6″ 0.78 ins.
34	89° 58.2′ 2.25 N.	89° 58.5′ 1.01 N.	89° 58.8′ 0.00	89° 59.1′ 0.79 S.	89° 59.4′ 1.35 S.	89° 59.7′ 1.69 S.	90° (E. or W.) 1.80 S.	3′ 30.4″ 0.81 ins.
35	89° 58.2′ 2.33 N.	89° 58.5′ 1.05 N.	89° 58.8′ 0.00	89° 59.1′ 0.82 S.	89° 59.4′ 1.40 S.	89° 59.7′ 1.75 S.	90° (E. or W.) 1.87 S.	3′ 38.4″ 0.84 ins.
36	89° 58.1′ 2.42 N.	89° 58.4′ 1.09 N.	89° 58.7′ 0.00	89° 59.0′ 0.85 S.	89° 59.4′ 1.46 S.	89° 59.7′ 1.82 S.	90° (E. or W.) 1.94 S.	3′ 46.4″ 0.87 ins.
37	89° 58.0′ 2.51 N.	89° 58.3′ 1.13 N.	89° 58.6′ 0.00	89° 58.9′ 0.88 S.	89° 59.3′ 1.51 S.	89° 59.7′ 1.89 S.	90° (E. or W.) 2.01 S.	3′ 55.0″ 0.90 ins.
38	89° 58.0′ 2.61 N.	89° 58.3′ 1.17 N.	89° 58.6′ 0.00	89° 58.9′ 0.91 S.	89° 59.3′ 1.56 S.	89° 59.7′ 1.95 S.	90° (E. or W.) 2.08 S.	4′ 03.6″ 0.93 ins.
39	89° 57.9′ 2.70 N.	89° 58.2′ 1.21 N.	89° 58.6′ 0.00	89° 58.9′ 0.94 S.	89° 59.3′ 1.62 S.	89° 59.7′ 2.02 S.	90° (E. or W.) 2.16 S.	4′ 12.6″ 0.97 ins.
40	89° 57.8′ 2.79 N.	89° 58.1′ 1.25 N.	89° 58.5′ 0.00	89° 58.9′ 0.98 S.	89° 59.3′ 1.68 S.	89° 59.7′ 2.10 S.	90° (E. or W.) 2.24 S.	4′ 21.6″ 1.00 ins.
41	89° 57.7′ 2.89 N.	89° 58.0′ 1.30 N.	89° 58.4′ 0.00	89° 58.8′ 1.02 S.	89° 59.2′ 1.74 S.	89° 59.6′ 2.17 S.	90° (E. or W.) 2.32 S.	4′ 31.2″ 1.04 ins.
42	89° 57.7′ 3.00 N.	89° 58.0′ 1.35 N.	89° 58.4′ 0.00	89° 58.8′ 1.05 S.	89° 59.2′ 1.80 S.	89° 59.6′ 2.25 S.	90° (E. or W.) 2.40 S.	4′ 40.8″ 1.08 ins.
43	89° 57.6′ 3.11 N.	89° 58.0′ 1.40 N.	89° 58.4′ 0.00	89° 58.8′ 1.08 S.	89° 59.2′ 1.86 S.	89° 59.6′ 2.33 S.	90° (E. or W.) 2.48 S.	4′ 50.8″ 1.12 ins.
44	89° 57.5′ 3.22 N.	89° 57.9′ 1.45 N.	89° 58.3′ 0.00	89° 58.7′ 1.12 S.	89° 59.2′ 1.93 S.	89° 59.6′ 2.41 S.	90° (E. or W.) 2.57 S.	5′ 01.0″ 1.16 ins.
45	89° 57.4′ 3.33 N.	89° 57.8′ 1.50 N.	89° 58.3′ 0.00	89° 58.7′ 1.16 S.	89° 59.1′ 2.00 S.	89° 59.5′ 2.49 S.	90° (E. or W.) 2.66 S.	5′ 11.8″ 1.20 ins.
46	89° 57.3′ 3.44 N.	89° 57.7′ 1.55 N.	89° 58.2′ 0.00	89° 58.6′ 1.21 S.	89° 59.1′ 2.07 S.	89° 59.5′ 2.59 S.	90° (E. or W.) 2.76 S.	5′ 22.8″ 1.24 ins.
47	89° 57.2′ 3.57 N.	89° 57.6′ 1.61 N.	89° 58.1′ 0.00	89° 58.6′ 1.25 S.	89° 59.1′ 2.14 S.	89° 59.5′ 2.67 S.	90° (E. or W.) 2.86 S.	5′ 34.2″ 1.28 ins.
48	89° 57.1′ 3.70 N.	89° 57.5′ 1.66 N.	89° 58.0′ 0.00	89° 58.5′ 1.30 S.	89° 59.0′ 2.22 S.	89° 59.5′ 2.78 S.	90° (E. or W.) 2.96 S.	5′ 46.2″ 1.33 ins.
49	89° 57.0′ 3.82 N.	89° 57.5′ 1.72 N.	89° 58.0′ 0.00	89° 58.5′ 1.34 S.	89° 59.0′ 2.30 S.	89° 59.5′ 2.87 S.	90° (E. or W.) 3.06 S.	5′ 58.6″ 1.38 ins.
50	89° 56.9′ 3.96 N.	89° 57.4′ 1.78 N.	89° 57.9′ 0.00	89° 58.4′ 1.39 S.	89° 59.0′ 2.38 S.	89° 59.5′ 2.97 S.	90° (E. or W.) 3.17 S.	6′ 11.4″ 1.43 ins.
Latitude	6 miles	5½ miles	5 miles	4½ miles	4 miles	3½ miles	3 miles	Defl. Angle and Tan.
	Azimuths and Offsets at—							

random or trial line is run westward toward the corresponding township corner on the principal or guide meridian, and temporary section and quarter section corners are established on this line at full distances from the range line so as to throw the shortage in the western half mile. When the principal meridian is reached, the amount by which the trial line falls north or south of the township corner previously established is noted,



The figures "80" and "77.36" are the theoretic lengths of "miles" or section sides for the west-most "mile" of each township on opposite sides of the standard parallel shown. The difference is the convergence of range lines in 24 miles in latitude 42° 15′.

FIG. 145.

and with this distance and the known bearing of the trial line, a bearing for a true return course will be computed and the true line run from west to east, keeping the distances as on the first line and setting correct permanent section and quarter corners. If the trial line fails to meet the township corner by more than 3 chains, either north, south, east, or west, the trial line, and so much of the entire township boundary as is necessary, must be rerun to discover and correct the error. After the first town-

ship is run the others of the same range are established similarly. The several ranges are then surveyed in like manner, proceeding from west to east until the final range line before the guide or principal meridian is reached. This line is run due north six miles and a random line run westward for the north boundary of the township to the west of the range line, and eastward for the north boundary of the township to the east of the range line, and the final lines in the opposite directions, the measurements on both sides being full from the east side of the township so as to throw the discrepancy in the western half mile.

214. Subdivision of townships. Each township is next subdivided into sections as follows:

Beginning on the south line of the township, at the corner common to sections 35 and 36, run northerly *parallel* to the eastern *range line* of the range in question, one mile, setting a quarter corner at the half mile. Establish the corner common to sections 25, 26, 35, and 36, and run east on a random line to the corner common to sections 25 and 36 on the range line, setting at the half mile a temporary quarter corner. From the observed failure to meet the corner on the range line, compute the bearing of a true line, and run this from the corner west, setting the permanent quarter corner at the *middle* point of the line.

Proceed thus with each succeeding section to the north till section 1 is reached. From the corner common to sections 1, 2, 11, and 12, the meridional line is run to the section corner on the next township line, by a random line corrected back; but the quarter corner is set at forty chains from the south end so as to throw whatever discrepancy there may be into the north half mile. In case the township is the most northerly of a twenty-four mile block, the line between sections 1 and 2 is *not run to the corner on the correction line*, but is run *parallel to the range line*, to an intersection with the correction line where a closing corner is established and its distance from the section corner noted.

This process is repeated for each tier of sections until the fifth tier is reached. From the corners on the section line between the fifth and sixth tiers random or trial lines are run west and east respectively for the sections to the west and east of the north and south line, and corrected back, but the

quarter corner of the western mile is set just forty chains from the eastern boundary so as to throw the discrepancy into the westerly half mile.

The instructions require that the surveyor beginning a subdivision shall rerun the eastern range line and the southern township line for one mile each to get a comparison between his own meridian and chaining and those of the original survey of the exterior lines, and he should make his own work correspond with the previous work. The limit of closure allowed on each one mile random line is a half chain. If the error of closure exceeds this, the line must be rerun.

Figure 146 represents a township with perfect subdivisions. The directions written along the various lines indicate the directions in which they are run. Probably no townships that have been surveyed are like that in the figure, all of them being more or less distorted by inaccuracies, and some of them being very much distorted. When very bad work is discovered in time to correct it, before such alteration will interfere with acquired rights of individuals, the corrections may be made under rules prescribed by the General Land Office. When individuals have bought the land, the corners as actually set must remain the corners, however erroneously placed.

215. Meandering a stream. This consists in running lines along its bank to determine its direction and length. The left and right banks of a stream refer to the banks as they would be to one passing downstream.

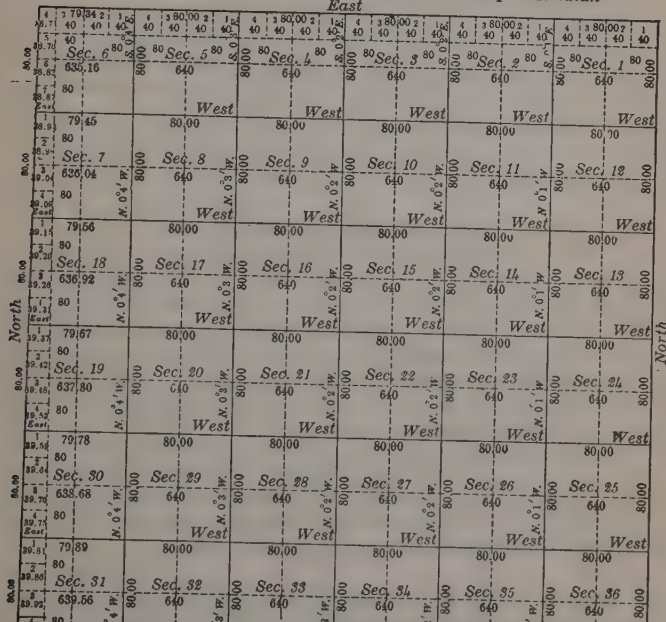
The " Instructions " provide for the meandering of navigable streams and those whose widths are three chains and upward. They are to be meandered on both banks. Lakes, deep ponds, bayous, etc., of twenty-five acres or more are also meandered. Corners called *meander corners* are established wherever a meander crosses standard base lines, township lines, or section lines.

A meander of tidal waters follows the high-water line.

216. Corners. Stones, posts, trees, and earth mounds are used to mark the corners. The kind of corner depends on the material afforded by the country. The " Instructions " designate the kinds of corners to be established and the method of marking them.

When it is impossible to establish a corner in its proper place,

auxiliary corners are established within twenty chains of the true place, on each of the lines approaching the corner. These corners are called *witness corners*. The center quarter corner of



The above plot represents a theoretical township with perfect subdivisions, contiguous to the north side of a Standard Parallel; in assumed Latitude 42° 15′ N., and Longitude 100° 00′ W. of Gr. Area 23024.16 A.

FIG. 146.

a section is located by connecting the opposite *north and south* quarter corners by a straight line, and placing the center corner at the *middle of this line*. Exceptions that will suggest themselves are necessary in the northern and western tiers of sections.

Any person having to do with surveys of lands that have

been subdivided as part of the public lands, should procure the "Instructions" and study them carefully. These instructions have varied from time to time and it is well to consult those in force at the time the original surveys in hand were made.[1]

217. Notes. The following sample page of the field notes prescribed in the instructions is given as an excellent form of notes for any land survey. For field purposes the author prefers a sketch on which all notes are made, and from which each night the field book may be written up in the form given. Variations in items noted and in nomenclature will suggest themselves to the maker of other kinds of surveys.

6TH GUIDE MERIDIAN EAST, THROUGH TPS. 13 N., BETWEEN RS. 24 AND 25 E.

CHAINS.	
	Survey commenced August 29, 1890, and executed with a W. & L. E. Gurley light mountain transit, No. —, the horizontal limb being provided with two opposite verniers reading to 30″ of arc.
	I begin at the Standard Corner of Township 13 North, Ranges 24 and 25 East, which I established August 29, 1890. Latitude 45° 34.5′ N., longitude 107° 24′ W.
	At this corner, at 8 h. 54 m. P.M., by my watch, which is 3 h. 49 s. fast of local mean time, I observe Polaris at *eastern elongation* in accordance with instructions in the manual, and mark the point in the line thus determined by a tack driven in a wooden plug set in the ground, 5.00 chs. north of my station.
	August 29, 1890.
	August 30: At 6 h. 30 m. A.M. I lay the azimuth of Polaris, 1°49.5′ to the *west*, and mark the TRUE MERIDIAN thus determined by a cross on a stone firmly set in the ground, west of the point established last night.
	The *magnetic bearing* of the true meridian is N. 18° 05′ W., which reduced by the table on page 100 of the Manual gives the *mean mag. decl. 18° 02′ E.*
	From the standard cor. I run
	North, bet. Secs. 31 and 36.
	Descend over ground sloping N. W.
2.60	Creek 10 lks. wide in ravine, 45 ft. below the Tp. cor., course N. 32° W.

[1] He will be much helped also by a book entitled "Manual of Land Surveying" by Hodgman and Bellows. This work contains the gist of many court decisions and instructions, resulting from many years' experience in this work. Dorr's "Surveyor's Guide" will also be found to contain many practical suggestions.

6TH GUIDE MERIDIAN EAST — *Continued*

CHAINS.	
7.50	To edge of table land, bears N. E. and S. W.; thence over level land.
17.40	Bluff bank, bears N. 58° W. and S. 58° E.; descend abruptly 40 ft.
19.00	Bottom of ravine, course S. 58° E.; ascend 50 feet to
22.00	Edge of table land, bears S. 58° E. and N. 58° W.; thence over level land.
	Difference between measurements of 40.00 chs., by two sets of chainmen, is 18 lks.; position of middle point
	By 1st set, 40.09 chs.
	By 2d set, 39.91 chs.; the mean of which is
40.00	Set a limestone, 16 × 7 × 5 ins., 11 ins. in the ground, for ¼ sec. cor., marked ¼ on W. face, and raise a mound of stone, 2 ft. base, 1½ ft. high, W. of cor.
42.60	Stream, 6 lks. wide, in ravine 15 ft. deep, course N. 60° W.
47.00	Enter heavy oak timber, bears E. and W.
53.00	An oak, 30 ins. diam., on line, 1 mark with 2 notches on E. and W. sides.
55.20	Creek, 20 lks. wide, 1 ft. deep, course N. 83° W.
55.40	Right bank of creek, begin very steep rocky ascent.
60.00	Top of ridge, 250 ft. above creek, bears N. 80° W. and S. 80° E.
64.00	Begin descent.
	Difference bet. measurements of 80.00 chs., by two chainmen, is 22 lks.; position of middle point
	By 1st set, 79.89 chs.
	By 2d set, 80.11 chs.; the mean of which is
80.00	The point for sec. cor., 150 ft. below top of ridge, falls on a flat rock in *place*, 10 ft. E. and W. by 6 ft. N. and S., on which I
	Cut a cross (×) at the exact cor. point, for cor. of secs. 25, 30, 31, and 36, marked with 5 grooves on N. and 1 groove on S. sides; from which
	An oak, 10 ins. diam., bears N. 22° E., 54 lks. dist., marked T. 13 N., R. 25 E., S. 30, B. T.
	A dogwood, 5 ins. diam., bears S. 64½° E., 42 lks. dist., marked T. 13 N., R. 25 E., S. 31, B. T.
	An ash, 13 ins. diam., bears S. 51° W., 37 lks. dist., marked T. 13 N., R. 34 E., S. 36, B. T.
	An oak, 9 ins. in diam., bears N. 34° W., 42 lks. dist., marked T. 13 N., R. 24 E., S. 25, B. T.
	Land level and mountainous.
	Soil, gravel and rock; 4th rate.
	Timber, oak.
	Mountainous or heavily timbered land, 33.00 chs.

218. Principles for guidance in resurveys. I. *Construing descriptions.* The following principles have been applied to the construing of descriptions that are inconsistent, obscure in meaning, or imperfect:

1. The description is to be construed favorably to the purchaser, unless the intent of both parties can be certainly ascertained. If that intent can be ascertained, the description will be construed accordingly.

2. The deed must be construed according to the conditions existing, and in the light of the facts known and in the minds of the parties, at the time the instrument was drawn.

3. Every requirement of a description must be met, if possible. Nothing is to be rejected if all requirements are mutually consistent.

4. If some parts evidently are impossible and, by rejecting such parts, the remainder forms a perfect description, such impossible parts may be rejected.

5. A deed is to be construed so as to make it effectual rather than void.

6. If land is described as that owned and occupied by an individual, the actual line of occupation is a requirement or call to be met in the location.

7. A line described as running a definite distance to a definite known line or object, will be construed as running to that object, whatever distance is required. If the known object is uncertain as to position, the written distance may be used.

8. The terms "northerly," "southerly," "easterly," and "westerly" are to be construed, in the absence of other information, as meaning due north, south, east, and west.

9. When a definite quantity of land is sold and nothing appears to indicate its form — as, for instance, ten acres in the northeast corner of B's land — the land will be laid out as a square, unless this is manifestly impossible.

10. A description by "metes and bounds" will convey all the land within the boundaries, be it more or less than the area mentioned in the deed.

11. Property described as bounded by a highway extends to the center of the highway, unless specifically noted otherwise. City lots are usually exceptions.

12. A description by metes and bounds, followed by a statement that the land described is a particular well-known

parcel, will be construed to convey the well-known parcel, though the metes and bounds do not fulfill the necessary conditions.

II. *Water boundaries.* 1. Local laws of different states give different constructions to the word " navigable " and the surveyor must examine the laws of the state in which he works. The United States statutes provide for the streams within the area known as the public lands as follows:

" All navigable rivers, within the territory occupied by the public lands, shall remain and be deemed public highways; and, in all cases where the opposite banks of any streams not navigable belong to different persons, the stream and the bed thereof shall become common to both."

2. Grants of land bordering on navigable streams carry only to high-water mark, while on non-navigable streams they carry to the center, or *filum aquæ.*

3. The common law holds those streams only to be navigable in which the tide ebbs and flows. The civil law considers a stream navigable that is capable of being used as a commercial highway.

The courts of Pennsylvania, North Carolina, South Carolina, and Alabama follow the civil law, while those of Maine, New Hampshire, Massachusetts, Connecticut, New York, Maryland, Virginia, Ohio, Illinois, Indiana, and Michigan follow the common law.

4. The bank is the continuous margin where vegetation ceases. The shore is the sandy space between it and low-water mark.

5. A description reading to " the bank " or " along the bank," is construed to mean " the bank," and to include no portion of the stream.

6. Islands in rivers fall under the same rule as the land under water, and belong to one adjoining proprietor or the other — unless previously lawfully appropriated or reserved — according as they are on one side of the center or the other. The *filum aquæ* is midway between lines of ordinary low-water mark, without regard to the position of the main channel.

7. Riparian rights, unless expressly limited, extend to the middle of the navigable channel.

8. In some states the tide lands are held to belong inalienably to the people of the state and may not be sold to individuals. In others a different policy has been pursued.

9. A boundary by the shore of a millpond carries to low-water mark.

10. Boundary lines of lots fronting on a river extend into the river at right angles to the thread of the stream, without regard to the form of the bank.

11. Land made by the drying up of a lake or the deposit of alluvium along a river accrues to the adjacent owners, and should be so distributed among them that each will receive such a portion of the made area as his former frontage on the water was of the entire former frontage. If, however, the water front is the valuable item, as it would be along a navigable river or lake in a city, the new frontage is to be distributed according to the old frontages.

12. Meander lines run along streams in connection with the United States Public Land Surveys are not boundaries but are used for computing areas. The rule of the General Land Office has been that all navigable streams and streams more than 3 chains wide should be meandered on both banks. Some state courts have held that the meander lines are boundaries; that the government sold only the area bounded by these lines and have judged property rights accordingly even on non-navigable streams. Such decisions when made by the highest court of jurisdiction must stand, but they are not in accordance with the intent of the original instructions which were given, for surveying purposes only, to secure a marking of the sinuosities of the streams, and not for the purpose of reserving the title to stream beds in the government or people; nor to change the old established law that the riparian owner has title to the land under water to the thread of the stream unless expressly stated to the contrary in his deed. In this matter the surveyor must learn the law of the state in which he is working.

III. *Special field rules.* 1. Monuments control courses and distances. That is, if the location of an original monument can be ascertained certainly and the recorded distance does not reach that monument, the line must, nevertheless, be run to the monument. In the absence of sufficient evidence to determine the monument, the description will govern. (The surveyor should use every effort to find evidence as to the location of the lost corner.)

2. Adverse possession of land for a definite period of time (varying in different states), even without color of title, con-

stitutes title in fee; but the possession must be adverse; that is, the true line must be known to the parties or the line of occupation must be acquiesced in by them. If the true boundary is unknown, and each claims to own only to the true line, no adverse possession can arise.

3. Boundaries and monuments may be proved by any evidence that is admissible in establishing any other facts.

4. A resurvey after original monuments have been lost is for the purpose of finding where they *were*, and not where they *should have been*.

5. Purchasers of town lots have a right to locate them according to the stakes which they find planted and recognized, and no subsequent survey can be allowed to unsettle them. The question afterwards is not where they should have been, but where they were planted with authority, and where lots were purchased and taken possession of in reliance on them.

6. Of two surveys that disagree, made many years apart — the monuments being lost — the original survey will be preferred, particularly if the line of the first survey has remained unquestioned for many years.

7. When streets have been opened and long acquiesced in, in supposed conformity to a plot, they should be accepted as fixed monuments in locating lots or blocks contiguous thereto.

8. A beginning corner is of no greater dignity or importance than any other corner.

9. A call for a lot by a name or number that it bears on a mentioned plot will prevail over courses and distances, and sometimes over monuments.

10. Any monuments or marks set in an original subdivision of a tract of land may be used to locate any other monuments of the same tract that may be lost, but no resurvey monuments or any other than monuments of the original survey can be so used with equal authority.

Illustrative Example

Dorr's "Surveyor's Guide" gives an interesting example of imperfect work and complications arising therefrom. Referring to Fig. 146*a*, the approximate rectangle inclosed in the heavy lines is Sec. 5, Tp. 40 N., R. 10 E. of the 4th Prin. Meridian. As

returned by the U. S. Surveyors it included also the dotted portion north of the correction line. The notes of the eastern line gave the following information: From *a*, the S. E. corner north about 80 rods to the Wisconsin River with meander trees on the south bank; about 40 chains north, the Wisconsin River again with the quarter post in the river; the northeast corner about $\frac{3}{8}$ mile farther north in a swamp and 30.24 chains east of the S. E. corner of Sec. 32, Tp. 41, North (*h*). Section 5 of Tp. 40 could not extend north of the correction line, which passes within a few feet of the point where the $\frac{1}{4}$ cor. *c* is said to be. The irregularities led to a lawsuit for the establishment of the east and west quarter section line. Witnesses testified to the existence of the S. E. cor. at *a*, the meander corner and trees at *b*, the west $\frac{1}{4}$ cor. at *e*, the N. W. cor. at *f* and the S. W. cor. at *d*. Four surveyors disagreed as to how the east and west $\frac{1}{4}$ line should be located, but none of them agreed with the court's decision. The figure shows how three of the surveyors would run the line. The fourth, *D*, would agree with *C* except that he would run from a point in the river at a proper "proportional distance" between the banks at *c* and *k* as returned in the original notes instead of just 40 chains from *a*. In spite of the marks found one of the attorneys argued that the section had never been actually surveyed and that therefore the section must be divided "proportionally" and the court so ordered.

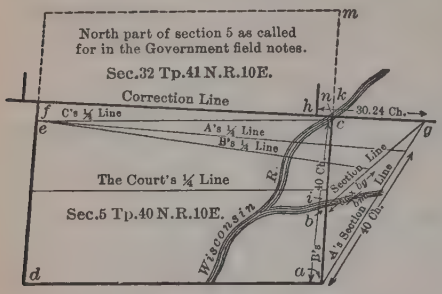


FIG. 146*a*.

By "proportional distance" is meant a proper proportional part of the original measurement of a given line when the new measurement differs from the original measurement. Thus if there were points on both banks of the river near *c* with recorded distances from *c* to *k* and from *c* to the south bank, not lettered in the figure, then *c* should be established by the proportion: *As the new measurement from bank to bank is to the recorded measurement so should the new measurement from either*

bank to c be to the recorded measurement between those points. *B* determined his starting point on the east section line somewhat in this way. He ran the section line from *b* to *g*, 30.24 chains east of *h* and then measured from *b* a distance determined by the proportion: *As bc is to bm so is the required distance according to present measurement to the presently measured distance bg.* This being a north section of a township, *A* adopted a different principle and said the east section line must run from *a* to *g* 30.24 chains east of *h* as recorded, and the quarter corner on this line must be 40 chains from *a*.

Discuss this case and determine according to the principles of this article and the rules of page 463 *et seq.* how the east and west $\frac{1}{4}$ line should have been run.

CHAPTER XI

CURVES

219. Use of curves. When a road, railroad, or canal is built, the center line is laid out on the ground. The center line is, in the case of a railroad, always a series of straight lines connected by arcs of circles to which the straight lines are tangent. In the case of a canal or wagon road, this is not always true, as the line is sometimes irregular, following more closely the conformation of the ground than does a railroad. It would be better if canals and wagon roads were always laid out as railroads are. In the case of wagon roads, the radii of connecting curves may, of course, be short. Many park roads and suburban streets are located as arcs of circles and straight line tangents. Park drive curves are often irregular and sometimes are successive arcs of varying radii, known as compound curves. Railroad curves are frequently compound curves.

220. Principles. Only the fundamental principles of the laying out of these curves will be given here. The straight lines are called technically *tangents*. It is customary now in railroad work to introduce a short arc of easement curve between the tangent and the circular arc. The easement curve is one whose radius of curvature begins with infinity and decreases uniformly to that of the circular curve it joins.

The curves are known by the angle subtended at the center by a chord of one hundred feet. Thus, if such a chord subtends 4°, the curve is known as a 4° curve. Any two tangents not parallel intersect at some point, and the angle at the point, between one tangent produced and the other tangent, measured in the direction in which the line is to bend, is known as the intersection angle. Curves are measured in chords of one hundred feet. This distance is a *station*, whether used on curve or tangent.

A curve four hundred feet long means a curve of four stations.

It does not mean that the arc is just four hundred feet long, but that there are four chords, each of one hundred feet. In Fig. 147, two tangents, AV and VB, are connected with a curve of radius R. AB is known as the *long chord*, C. DE is the *middle ordinate*; M. $AV = VB$ is the *tangent distance*, T. VE is the *external distance*, E. From the figure it is evident that

$$T = R \tan \tfrac{1}{2}\Delta, \quad (1) \qquad M = R \operatorname{vers} \tfrac{1}{2}\Delta, \quad (3)$$

$$C = 2R \sin \tfrac{1}{2}\Delta, \quad (2) \qquad E = R \operatorname{ex\,sec} \tfrac{1}{2}\Delta. \quad (4)$$

External secant is a trigonometrical function not commonly mentioned in Trigonometry, but very useful in handling curves. It is the "secant minus one." In the above equations it will be seen that the radius R is used, while it has been said that curves are known by their "degrees." If AB is one hundred feet, Δ becomes D, the degree of the curve, and from (2)

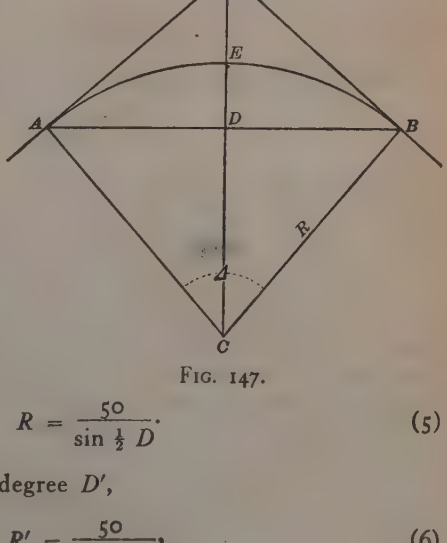


FIG. 147.

$$R = \frac{50}{\sin \frac{1}{2} D}. \quad (5)$$

For any other curve of degree D',

$$R' = \frac{50}{\sin \frac{1}{2} D'}, \quad (6)$$

$$\therefore \frac{R}{R'} = \frac{\sin \frac{1}{2} D'}{\sin \frac{1}{2} D}.$$

For small angles it may be assumed that the sines vary as the angles; whence

$$\frac{R}{R'} = \frac{\frac{1}{2} D' \sin 1^\circ}{\frac{1}{2} D \sin 1^\circ} = \frac{D'}{D}. \quad (7)$$

The radius of a 1° curve is 5729.65 feet. Except in close land surveys or surveys of city streets, the radius may be taken as 5730.00 feet. In using curves in land surveys, the term "degree" should be abandoned and all circles designated by their radii. Since the values of all the functions given in equations (1), (2), (3), (4), vary directly as R, Δ remaining constant, these functions will all vary inversely as D, assuming the correctness of equation (7). They would vary exactly in this way if the one hundred feet that measures D were measured on the arc instead of as a chord. This will be clear from the principles of Geometry. The length of a curve for a given value of Δ and D is in stations,

$$N = \frac{\Delta}{D}, \tag{8}$$

and in feet measured in chords of one hundred feet,

$$L = 100\frac{\Delta}{D}. \tag{9}$$

Frequently it is required in practice to know the values T and E for given or assumed values of Δ and D. If a table is made of values of these quantities for a great number of values of Δ and for $D = 1°$, the value of T or E for any other value of D would be obtained at once by dividing the value found in the table opposite the given Δ, by the given value of D. In practical railroad work this method may be used for curves of less than 8° or 10° without serious error. For sharper curves than these, the quantities should be computed by equations (1), (2), (3), and (4). If, however, the sharper curves are measured in fifty-foot chords, up to say 14° to 16° curves, and in twenty-five-foot chords thereafter, the approximate method may be used up to at least 20° curves or 24° curves without serious error. This means that a 14° curve will be defined as one in which a chord of fifty feet subtends at the center an angle of 7°. A 20° curve is one in which a twenty-five-foot chord subtends an angle of 5°; etc.[1]

221. Laying out the curve. The point at which the curve is to begin is known when the point V and the tangent distance

[1] The author thinks the "degree" should be the angle subtended by an arc of 100 feet. While chords would be of unequal lengths all functions of a D° curve would be corresponding functions of a 1° curve divided by D.

T are known. For the latter, Δ and D must be known. Let the point of beginning, called the P. C. (point of curve), be A. A transit is set at A, Fig. 148, an angle $= \frac{1}{2} D$ turned from the tangent AV to C. The chain or tape is stretched from A and the farther end made to coincide with the line of sight at C.[1]

From the line CA an angle equal to $\frac{1}{2} D$ is laid off, and the chain stretched from C and the farther end put in the line of sight at D. Thus the points one hundred feet apart are located. The final deflection angle VAB is $\frac{1}{2}\Delta$. If the curve is not of a length expressed by a whole number of stations, there must be turned off for the *subchord* a deflection usually made proportional to the length of the subchord; thus in the figure, if EB is fifty feet, the deflection EAB is one half of $\frac{1}{2} D$, etc. This again assumes that the curve is measured on the arc, and the method is practically correct for curves of less than 10°. For sharper curves than this a computation should be made. Thus, since Δ is known, there remains to turn off from AE an angle equal to $\frac{1}{2}\Delta - VAE$. This being half the angle subtended by the chord EB, that chord may be computed. If the curves of higher degree are laid out with short chords as before described, the proportionality of chord to deflection angle may still be assumed.

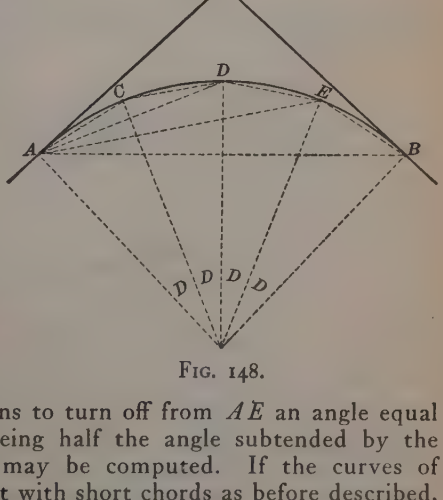


FIG. 148.

If the whole of the curve cannot be seen from the beginning, the transit may be moved to a point on the curve already located and the work continued as follows: Let it be assumed in Fig. 148 that there is an obstruction in the line AE so that E cannot be seen from A. Move the transit to D, and with the verniers set at zero, with the lower motion, turn the line of sight on A. Now lay off to the right an angle equal to D (the deflection from

[1] Let the student show that this procedure locates the point C on the curve.

A to *D*) and the line of sight will lie in the tangent to the curve at *D*. Transit the telescope and deflect to the right for the station *E*, etc., as usual. Not much more than 90° of an arc should be located from one point.[1]

Points occupied by the transit are usually solid *hubs* driven flush with the ground with a small tack to mark the exact point. The other points are, in railroad and common road surveys, merely marked with stakes. In close land surveys and city work, each point should be marked with hub and tack.

Illustrative Example

Two tangents intersect at station 746 + 60 with a deflection angle of 36°. It is required to connect them with a 3° curve. (*a*) What will be the station of the P. C.? (*b*) If the stationing is changed to read around the curve as is usual, what will be the station of the P. T.? (*c*) Tabulate the deflection angles to each station on the curve. From Table VIII find *R* for a 3° curve and its logarithm. Then

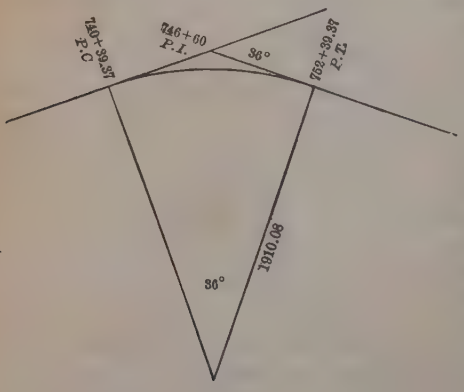


Fig. 149.

(*a*) $T = R \tan \frac{1}{2} I$
$= 1910.08 \tan 18°.$

Log R	3.28105
Log tg 18°	9.51178
	2.79283 = log 620.63

$$746 + 60.00 - (6 + 20.63) = 740 + 39.37 \text{ P. C.} \quad \text{Ans.}$$

(*b*) The curve has $36° \div 3° = 12$ stations of length. Therefore

$$740 + 39.37 + 12 = 752 + 39.37 \text{ P. T.} \quad \text{Ans.}$$

[1] Let the student draw a figure of a curve carrying it to the half circle and show why the 180° point could not be definitely located from the beginning.

(*c*) The first full station on the curve will be sta. 741, which is 60.63 feet from the P. C. The deflection for 60.63 feet is $\frac{60.63}{100} \times \frac{3^\circ}{2}$ or 0° 54.6′. The deflection for the final chord of 39.37 feet is $\frac{39.37}{100} \times \frac{3^\circ}{2}$ or, since the curve is a whole number of stations long, it is the remainder of a full chord deflection after deducting the first subchord deflection or 1° 30′ − 0° 54.6′ = 0° 35.4′. Therefore, the full tabulation of deflection angles from the P. C. is as follows, reading up:

Sta.		Defl. Angle
752 + 39.37 P. T.	+ 0° 35.4′	18° 00′
752		17° 24.6′
etc.		to
45	+ 1° 30′	6° 34.6′
44	+ 1° 30′	5° 24.6′
43	+ 1° 30′	3° 54.6′
42	+ 1° 30′	2° 24.6′
41	+ 0° 54.6′	0° 54.6′
740 + 39.37 P. C.	0° 00′	0° 00.0′

In a notebook all the figures would be given.

Examples

1. A 4° curve begins at station 36 + 50, and the central angle is 16° 00′. Tabulate the deflection angles from the P. C. to the several stations and determine the station of the P. T.

2. In the above example what is the tangent distance? The external distance? The long chord? The middle ordinate?

3. In the above example what is the true length of the circular arc?

4. Two tangents intersect at station 182 + 75, with a deflection angle of 26° 30′. It is required to connect them with a 3° 30′ curve. What will be the station of the P. C. and, if the station numbering is changed to read around the curve instead of along the tangents, what will be the station of the P. T.? (The stationing usually reads along the curve.)

5. Tabulate the deflections to each station on the curve.

6. It is required to connect the tangents named in Example 4 by a curve that shall pass approximately 39 feet inside the vertex. What degree curve should be used, and what will be the station of P. C. and P. T.?

7. A 24° curve is to be located by 25-foot chords. What is the correct deflection for each chord?

8. If the 24° curve is to be so located that four chord lengths shall cover the arc subtended by a 100-foot chord, what is the length of each chord and what the deflection?

222. Location by measurement only. A method for locating arcs approximately by the use of the chain or tape is as follows, and is known as the method by *chord offsets*.

In Fig. 150, AV is a tangent to which a curve of degree D is to be joined at A. If AB is one station,

$$Bb = 100 \sin \tfrac{1}{2} D.$$

Imagine the curve continued back one station to E. $eE = Bb$, and f, in EA produced, is distant from b by an equal amount, therefore

$$fB = 2\ Bb.$$

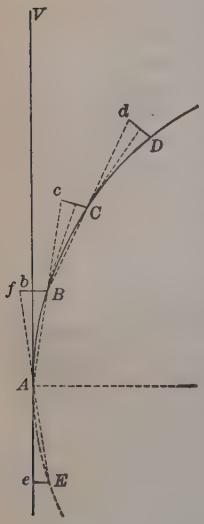


FIG. 150.

Bb is known as the *tangent offset* t, and fB is the chord offset $= 2\ t$. To locate the curve, swing the tape from A and place the farther end of it a distance t from the tangent. Stretch the tape from B in line with AB and make a mark at C. Swing the tape about B till cC equals $2\ t$. Proceed in the same manner for the remainder of the curve. If the curve begins with a fractional station, the tangent offset for the fractional station may be computed and laid off; and for the remainder of a full station back of A, the same thing may be done. The direction of the chord of a full station on the curve is now determined, and the work may proceed as usual.

EXAMPLE

Find all the offsets necessary to lay out a 3° curve, with the tape alone, by offsets from the chord produced; the curve beginning at station 367 + 30 and ending at station 372 + 60.

223. Location by ordinates from the chord. Frequently it is convenient to locate several points on a curve by measuring

ordinates or offsets from the chord. This occurs in laying out curved property lines in curved suburban subdivisions and in

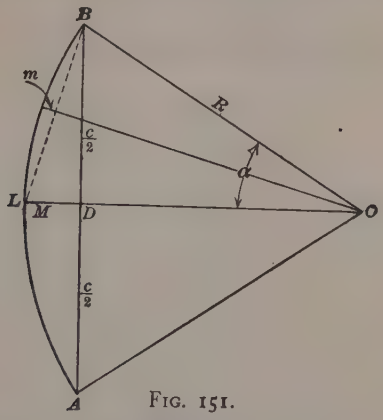


FIG. 151.

parkings. In Fig. 151 the chord and radius are supposed to be known. The angle BOA is then known and may or may not be used. Using it,

$$\sin \tfrac{1}{2} AOB = \sin \alpha = \frac{C}{2R}$$

$$\left.\begin{aligned} M &= R \text{ vers } \alpha \\ m &= R \text{ vers } \frac{\alpha}{2} \end{aligned}\right\}, \qquad (1)$$

etc.

or, from triangle BDO

$$(R - M)^2 = R^2 - \left(\frac{C}{2}\right)^2,$$

$$\pm (R - M) = \pm \sqrt{R^2 - \left(\frac{C}{2}\right)^2},$$

$$M = R - \sqrt{R^2 - \left(\frac{C}{2}\right)^2}, \qquad (2)$$

$$= R - \sqrt{\left(R + \frac{C}{2}\right)\left(R - \frac{C}{2}\right)}, \qquad (3)$$

in which R and C are required to be known.

It may also be shown from Fig. 152 that the ordinate at any point L, on the chord distant a from the middle point F, is

$$K = \sqrt{(R+a)(R-a)} - \sqrt{\left(R+\frac{C}{2}\right)\left(R-\frac{C}{2}\right)}. \qquad (4)$$

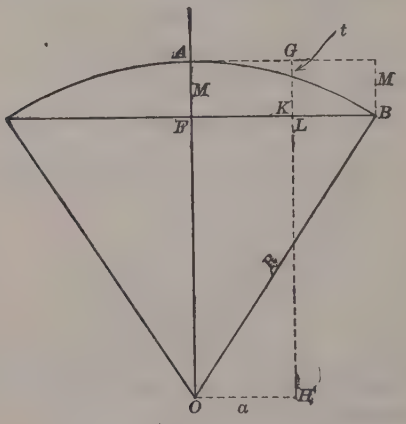


FIG. 152.

By expanding the radical in Equation 2 by the binomial formula and discarding all terms after the second, it may be shown that

$$M = \frac{C^2}{8R} \text{ approx.} \qquad (5)$$

If the arc be considered a parabola and a tangent be drawn from A, offsets t, at distances a, etc., from A will be as the squares of those distances, so that

$$\frac{t}{M} = \frac{a^2}{\left(\frac{C}{2}\right)^2},$$

$$t = 4M\frac{a^2}{C^2},$$

$$K = M - t = M\left(1 - 4\frac{a^2}{C^2}\right) \text{ approx.} \qquad (6)$$

If a be successively at the quarter and eighth points of the chord

$$\left.\begin{aligned} a &= \tfrac{1}{8}C, \qquad K = \tfrac{15}{16}M \\ a &= \tfrac{1}{4}C, \qquad K = \tfrac{3}{4}M \\ a &= \tfrac{3}{8}C, \qquad K = \tfrac{7}{16}M \end{aligned}\right\} \text{ approx.} \qquad (7)$$

EXAMPLES

1. For R, 50, 100, 500, 1000, and 5000 feet respectively find M for chords of 50 and 100 feet by equations 1, 3, and 5. Note when equation (5) gives sufficiently precise results.

2. For the same radii and chords find K for $a = \frac{1}{8}C$, $\frac{1}{4}C$, and $\frac{3}{8}C$ by Equations (4) and (7) and note when (7) gives sufficiently precise results.

224. Compound curves. These are used in railroad and other locations to bring the line of the work to conform more nearly to the shape of the surface along which it lies. Figure 153 shows such a curve of two arcs only. Compound curves have frequently several arcs of different radii. In compound curves the tangent distances are always unequal, and in the case of two arcs the shorter tangent will be adjacent to the arc of shorter radius. The two arcs must have a common tangent at their junction P. The sum of the central angles will be equal to the angle Δ. The arcs are laid out as are simple curves, one at a time. The arc AP is laid out from A to P, and the transit is then moved to P, and the arc PB is laid out. The problems in simple and compound curves are merely exercises in the Geometry of the circle, often solved by Trigonometry.

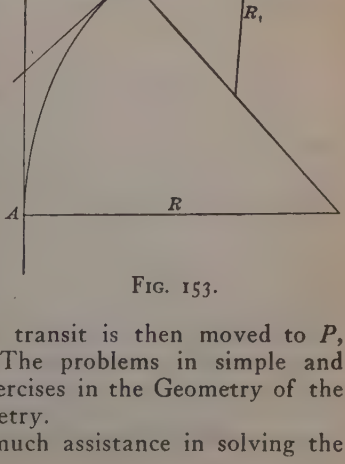


FIG. 153.

One element of curves is of much assistance in solving the

problems that arise in laying them out and changing them to better fit the ground where they are located. This element is:

PROPOSITION: *If two circular arcs begin at a common tangent point and end in parallel tangents, their long chords are coincident in direction and vary only in length.*

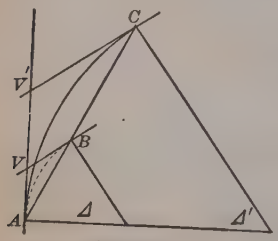


FIG. 154.

In Fig. 154 let AV and VB be the tangents to the curve AB; and AV' and $V'C$, the tangents to the curve AC. A is a common tangent point. Since AV and AV' are coincident, and VB and $V'C$ are parallel, $\Delta = \Delta'$. The deflection angles from the common tangent to the points B and C will each equal $\frac{1}{2}\Delta$. Therefore, AB is coincident with AC.

EXAMPLE

A 4° curve begins at sta. 381, is compounded at sta. 385 to a 6° curve which ends at sta. 389 + 25. Find the central angle of each arc, the total central angle, and the two tangent distances P. C. to intersection and intersection to P. T.

CHAPTER XII

TOPOGRAPHIC SURVEYING

TOPOGRAPHY

225. General statement. In this chapter will be given the most common methods for making topographic surveys of areas not so large as to require geodetic methods. The methods of the U. S. Coast and Geodetic Survey and of the U. S. Geological Survey differ one from the other, and from the methods here given. The Coast Survey methods for detail topography are fairly precise, while those of the Geological Survey are for the most part only approximate. They secure remarkably good results for maps that are to be made to a scale of approximately a mile to the inch. The student who wishes to study these methods is referred to the publications of the respective bureaus, to Gore's "Geodesy," Wright's "Adjustment of Observations," Ingraham's "Geodetic Surveying," Wilson's "Topographic Surveying," Crandall's "Geodesy," etc.

226. Methods for representing surface form. In Figs. 155 and 156 are shown two common methods for representing the configuration of the surface of the ground. Figure 155 shows the method by hachures, or hill shading, used only to give a general idea of the hills and hollows. Figure 156 shows the same ground by the method of contours or lines of equal elevation.

The word *contour* as used in surveying means a line all points of which have the same elevation above or below some assumed level surface, as, for example, sea level. Let the student imagine some section of country with which he is familiar covered with water so deep that the tops of the highest hills are just in the surface of the water. Let it be imagined that the water flows away till the surface has dropped five feet vertically. The intersection of the water surface and the land surface would

be a five-foot contour line. Let the surface drop five feet more, and there would result another five-foot contour line. The contour line may then be said to be a level line lying wholly in the surface of the earth.

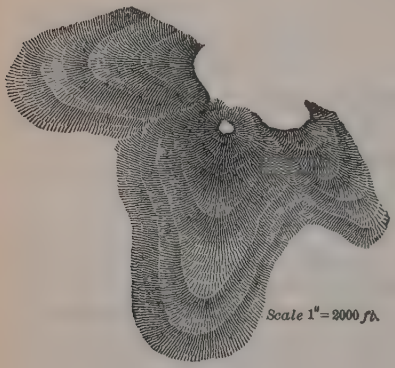


FIG. 155.

The contours of a theoretically perfect conical hill would be a series of circular circumferences having their centers in the axis of the cone. A contour map of such a hill would be a series of circumferences of concentric circles. This is seen from Fig. 157. If it is assumed that the base of the hill has an elevation of one hundred feet above some datum, then the contour map showing "ten-foot contours" would be as in the lower part of the figure. The horizontal distance between any two contours shown will indicate the rate of slope of the surface. The direction of steepest slope is always in the direction of the common perpendicular to two consecutive contours. If two contours of the same elevation were to be found running together on a map, and continuing as a single line and then separating again, there would be indicated a perfect knife-edge ridge or hollow. Since such a thing does not exist in nature, no such representation can be found on a correct contour map. Moreover, except in the case of an overhanging bluff or a cave, contours of the same or of different elevations could not cross, and hence with the exceptions noted a correct contour map will not show contour lines crossing. A contour line

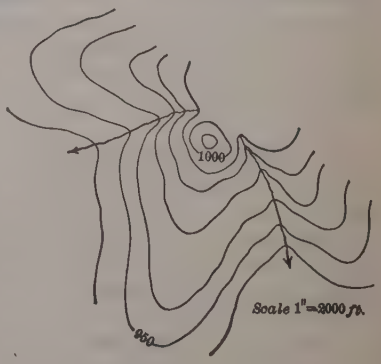


FIG. 156.

that closes upon itself indicates a peak higher than the surrounding ground or a hole without an outlet. Since any contour line, except one that closes on itself as above described, must continue indefinitely, closing on itself somewhere, to be sure, no contour line can begin or end within the border of a map unless it closes on itself. With the exception noted, all contours must appear first in the border of the map and disappear in the same or another edge. When contours cross a stream, it is customary in mapping to run them up to one bank and stop them, beginning again on the opposite bank. This merely means that the portions that run up the banks of the stream and unite in the bed are omitted. In some cases, however, the contours are carried across the stream.

A contour map is thus seen to show with definiteness the conformation of the surface, while hachuring gives only a general idea of the surface.

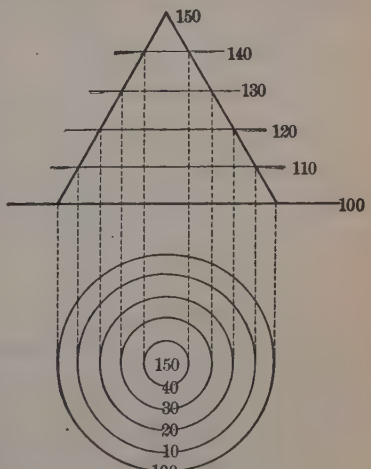


FIG. 157.

227. Field methods for small area. There are various ways of making topographical surveys, according to the use to which the information obtained is to be put, or the extent of territory to be mapped. A more or less precise and extensive topographical survey is required when a large and important building is to be erected, when a park is to be laid out, or any landscape gardening done; when a railroad, canal, or road is to be built, or when a system of irrigation or drainage is to be planned.

When the area to be mapped is small, as, for instance, a city lot, or block, or a very small farm, the following method is used:

The area is divided into squares or rectangles of from twenty feet to one hundred feet on a side, stakes being driven at each corner. The stakes along one edge will usually be lettered and along another edge they will be numbered. Any interior stake would then be known by its number in its lettered row: thus,

stake X (Fig. 158) would be known as stake D_3. Levels are now taken at all corners and any points of abrupt change of slope not at a corner. The contours are obtained from the level notes, as described below. Contours are usually named with their elevations above the assumed datum. In one map the interval between contours will be uniform.

228. Contour map. To draw the contour map from the notes that have been taken, lay out in pencil on paper, to scale, a map of the squares that have been used in the field. Note lightly in pencil the elevations of the points where levels have been taken. Assume that the slope along any line is uniform from one known elevation to the next. If the contours are being drawn with an interval of two feet, for instance, and if either of the known elevations is an even two feet, as 112 feet or 114 feet, then a contour line passes through the known elevation point. If not, and this usually will be the case, the point where the contour crosses the line between the two points of known elevation must be found by interpolation. When found, the point is numbered with the elevation of the line passing through it, and other points are found. This is done over the whole map, and, finally, points of equal elevation are connected, giving contour lines.

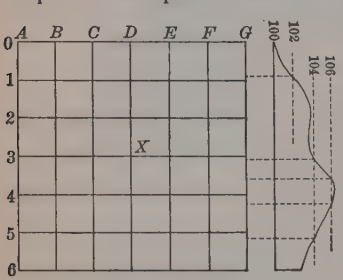


FIG. 158.

The lines will not usually be drawn straight between points, but will be curved, either from a knowledge of the ground or, in the absence of such information, so as to form no angles, but to make smooth, curved lines. Sometimes a profile of each line is drawn in pencil, as indicated for line G, and on this profile are noted the points having elevations of required contours. These are projected down on to the line, giving the points on the line intersected by the given contours. The contour lines are inked in in black or brown or red, according to the purpose of the map. The outside lines of the square are inked in black. The contour lines are numbered with their elevations usually at every fifth contour or at the even tens.

Example

By leveling, the elevations given in the following table were found at the points indicated. The tract is in the form of a square six hundred feet on a side and divided into one-hundred-foot squares. Draw the map to a scale of 100 feet per inch, locating five-foot contours.

Station	Elevation	Station	Elevation
A_0	153.0	B_0	154.5
A_1	155.5	B_1	159.6
A_2	158.0	B_2	163.7
A_3	158.5	B_3	166.5
A_4	156.4	B_4	165.5
A_5	152.8	B_5	156.0
A_6	149.5	B_6	155.5
C_0	155.4	D_0	155.3
C_1	162.1	$D_0 + 80$	160.5
C_2	169.0	D_1	160.0
C_3	173.7	D_2	170.2
$C_3 + 50$	175.5	D_3	180.1
C_4	170.4	$D_3 + 50$	192.2
$C_4 + 20$	166.0	D_4	178.5
$C_4 + 60$	167.0	D_5	168.7
C_5	164.8	D_6	161.5
C_6	159.4		
E_0	158.2	F_0	157.5
E_1	164.2	F_1	162.3
E_2	168.8	F_2	165.5
E_3	175.0	$F_2 + 50$	166.0
$E_3 + 20$	176.0	F_3	165.5
$E_3 + 60$	172.5	$F_3 + 50$	164.0
E_4	172.5	F_4	164.9
E_5	166.0	F_5	163.1
E_6	161.5	F_6	158.5
G_0	154.5	$G_3 + 40$	155.5
G_1	159.6	G_4	159.0
G_2	161.0	G_5	157.5
G_3	159.5	G_6	155.0
$0_C + 50$	154.0	$3_D + 15$	181.8
$2_D + 40$	167.5	$4_B + 75$	170.8
$3_C + 80$	182.0	$4_C + 10$	170.0

229. General field methods for moderate area. When the area is large, as a farm of twenty or more acres, or even less, or a city or county, or a portion of a given valley, the following method is adopted:

Points are located over the area to be mapped more or less at random, and their elevations are obtained. The points are then mapped. Assuming that the slope is uniform between those that are adjacent, the positions of contours between points are determined by interpolation, as was done with the squares. Equal elevations are then connected by curved lines, which are the contour lines. Sketches are made in the field to assist in shaping the contours.

FIG. 159.

Referring to Fig. 159 the points would be chosen along the dotted lines drawn approximately normal to the contour lines. Interpolations would be made between points along these lines and not between points on lines approximately parallel with the contours. In the field the points are located by *polar coördinates;* that is, by direction and distance from a known point. This work, as well as the determination of elevations, is best done with a transit and stadia. The plate bubbles of the transit must be fairly sensitive. If they are not, vertical angles will not be read well, and therefore elevation will be inaccurate, and forward and back vertical angles will not agree as they should.

The field work is as follows: A starting point, as the intersection of two streets, the corner of a farm, a triangulation station, or simply an arbitrary spot, is chosen, and if not otherwise marked is established by driving a firm stake that will not be easily disturbed. The transit is set over this point with vernier reading zero, and the instrument is pointed, by the lower motion, in the direction of the meridian. This may be the true meridian previously determined, the magnetic meridian as

shown by the needle, or an arbitrary meridian assumed for the purpose of the survey. If either of the latter, a stake is driven some distance away in the meridian, so that the direction may be accurately found again. It is assumed that the true meridian will have been so marked as to be found again.

If a triangulation station is used, the transit is set to read the azimuth to a visible distant station and pointed there by the lower motion; that is to say, is oriented on the distant station. The elevation of the point over which the instrument is set, if not known, is assumed and written in the notebook. A traverse line is now run, noting the azimuths of all courses, as described in Chapter VIII.

The distances are measured with the stadia and recorded in the book. The vertical angles are also noted. Vertical angles and distances are read from both ends of any course as a check. The traverse line is chosen with a view to obtaining from each station the largest possible number of pointings to salient features of the area to be surveyed, and, while the instrument is set at any station and before the traverse line is completed, these are taken. These additional pointings are usually called *side shots*. For each pointing the distance, azimuth, and vertical angle are read. These readings will locate the point and determine its elevation.

230. Details of transit-stadia method. After the instrument has been oriented at the initial station which may be marked $\boxdot_0$ (Stadia Station zero), a rod is held alongside and the height of the axis of the telescope is noted in the book as a heading to the notes taken from that station. While this is being done, another rodman (there are usually two or more) selects, by the direction of the person in charge of the survey or by his own judgment, a second point to be occupied by the transit. He drives a stake and marks it $\boxdot_1$ (Stadia Station 1). The rodmen are provided with hatchets for the purpose of making and driving stakes and clearing away trifling obstructions to sight. If there is much clearing to do, axmen are employed.

The rodman holds his rod vertical with its center just behind the stake from the instrument and the graduated side toward the observer. The observer turns the middle horizontal wire to a point on the rod corresponding to the height of the instrument, using the clamp and slow-motion screw of the vertical motion.

He notes then the lower wire to see whether it is near a fifty or hundred graduation, makes it coincide with such graduation, and reads the distance. He brings the horizontal wire again to the proper height, carefully centers the vertical wire on the center of the rod, and signals that he is through. Then he reads the azimuth and vertical angle, and calls them to the recorder, who enters them in the notebook. Then the observer reads the needle to check his azimuth reading and has the needle reading recorded for future reference.

While this is being done, the rodman who was at the instrument selects by direction or judgment a point for a side shot, and stands there with the graduated side of the rod toward the observer. The observer unclamps the upper motion now, and does not, as a rule, clamp it again till he leaves the station. He turns the line of sight on the rod, and after reading the distance he reads the azimuth — the needle is neglected on side shots — and vertical angle as already described for the stadia station, and signals that he is through. The azimuth need not be read closer than the nearest 5 minutes, and this may be done without the use of the vernier. The rodman who set the station now has found a point, and his rod is observed as last described, and the work continues in this way, observing first on one rod and then on another, according to which is first ready.

The points selected for side shots should be chosen as indicated in paragraph 229, Fig. 159. Points should be taken frequently along a stream to locate its course. Usually it is required to obtain information for the mapping of artificial structures, as roads, fences, houses, etc. Therefore, pointings should be taken to all fence corners and angles, to enough corners of all buildings to enable the building to be mapped, and to a sufficient number of points in all roads. If it is desired to indicate wooded lands, then pointings must be taken around the edges of timber. While all this is being done, the recorder is recording the notes and endeavoring to keep a sketch on the right-hand page of the book to assist the memory in mapping.

After all the side shots from $\boxdot_0$ have been taken, the instrument is moved to $\boxdot_1$, and a rodman goes to $\boxdot_0$. The instrument is oriented on $\boxdot_0$. The needle is read and recorded, as well as the distance and vertical angle. After one of the rodmen, who has come up to the station, has noted the height of the instrument, he goes ahead to select a new stadia station,

which is established and located as was ⊡1, and the work proceeds. Instead of requiring a rodman to be at the instrument each time a new point is occupied, the recorder or observer may carry a light graduated rod with which to determine the height of the transit or it may be done with a pocket tape. It is convenient to have the five-foot point of the stadia painted red to expedite the setting of the wire on the proper point for the determination of the vertical angle.

231. Notes. The form of notes used in stadia surveying is given below. The left page of the notebook is represented. The right page is used for sketching and for additional notes descriptive of the stations occupied or observed. The needle readings may be entered on the right page, or put in parenthesis over or under the azimuth record of the same pointing. The sketch should be as full as possible and neatly drawn.[1]

RENS. POLY. INST. PRACTICE SURVEY. JUNE, 1894.

SATURDAY, 14.

ADEY, OBSERVER.

INST. ON ⊡0. ELEV. 830.0. H. I. 4.00. DAVIS, RECORDER.

STATION	AZIMUTH	DISTANCE	VERT. ANGLE	DIFF. ELEV.	ELEV.
⊡1	331° 39′	556	− 1° 41′	− 16.3	
W. side road	34 30	61	− 0 50	− 0.9	829.1
Beginning slope	243 08	80	− 0 22	− 0.5	829.5
C. P. (Contour point)	31 21	119	+ 1 05	+ 2.2	832.2
C. P.	286 48	110	− 2 00	− 3.8	826.2
C. P.	19 39	141	+ 1 10	+ 2.9	832.9
Top slope	313 26	45	− 1 20	− 1.0	829.0
C. P. west road	356 20	123	− 3 00	− 6.4	823.6
C. P. on hill	347 32	94	− 3 47	− 6.2	823.8
C. P.	331 24	103	− 5 20	− 9.4	820.6
C. P.	334 05	120	− 5 40	− 11.7	818.3

[1] A field book devised by Mr. D. F. Turner is an excellent book for this work. On the right-hand page is a series of concentric circles with radial lines which enable the observer to sketch observed points quickly in their proper relative positions.

INST. ON ⊡1. ELEV. 813.7. H. I. 5.0.

⊡0	151 39	556	+ 1 41	+ 16.3	
□2 (described on rt. p.)	352 05	69	+ 2 55	+ 3.5	
C. P. on hill	152 45	436	+ 0 31	+ 3.9	817.6
C. P.	151 56	289	− 1 00	− 5.1	808.6
C. P., W. side road . .	141 11	191	+ 0 35	+ 2.0	815.7
C. P.	151 16	198	− 1 26	− 4.9	808.8
C. P., E. side road . .	141 33	305	+ 0 35	+ 3.6	817.3
Beginning of cross rd. W.	145 47	97	− 0 55	− 1.6	812.1

INST. ON □2. ELEV. 817.2. H. I. 4.9.

⊡1	172 05	69	− 2 54	− 3.5	
□3	348 09	156	− 2 11	− 6.1	
etc.			etc.		

The elevations of the stadia stations are not written in the sixth column, but in each heading. It happens sometimes that the vertical angles do not check so well as in the example given. When this is the case, the mean value of the difference of elevation is used for determining the elevation.

If it is necessary to correct the horizontal distance, it is done by the use of the table in Appendix, page 499. The differences in elevation are obtained from the same table, a diagram, or some form of slide rule. For mapping, see Art. 241 *et seq.*

232. Method for smooth ground. When the area is not too large and the slopes are gentle and the hills low, the work may be done by locating the contours on the ground. The instrument height is found with respect to a given contour, and with the telescope level, the rodmen are placed on the contour at the various positions selected by moving them up or down till the rod reading of the middle wire indicates the proper level. The distance and azimuth are then read and no vertical angle work is required. For the leveling work the instrument is used as a level. Thus, if the elevation of the initial station is found by leveling to be 496.2 and the height of the instrument above it is 4.8, the height of the instrument above the 500-foot contour is one foot and above 495-foot contour is 6 feet. To locate the

500-foot contour the rodmen are directed to points where the rod reads 1 foot, and to locate the 495-foot contour, if this comes within the area, the rodmen take points where the reading is 6 feet. A marker of paper, or a rubber band, may be put on the rod at the proper reading, so that it may be found quickly.

233. Stadia interval method for difference of elevation. When the vertical angle is small, so that it is not required to be read for horizontal distance reduction, some time can be saved and no accuracy sacrificed by using the stadia interval method for getting difference in elevation. This is done as follows:

After the distance and azimuth have been read and called, the telescope is leveled, note is made of some object or point in the field of view covered by the mid-horizontal wire; the lower wire, if an upward angle, or upper wire, if a downward angle, is then brought to the observed point; a point or object covered by the upper (or lower) wire is noted and the lower (or upper) wire brought to this point; this process is continued until the upper (or lower) wire is on the rod; then if the vertical angle is upward, the number of wire intervals that have been covered multiplied by the wire interval read for distance, is the elevation of the point on the rod covered by the upper wire, above the instrument (not the stake over which the instrument is set but the axis of the telescope); this elevation minus the rod reading of the upper wire, and plus the height of instrument above the ground is the difference in level of the two points. The notes can be kept in the same columns and right along with vertical angle work. Thus, instead of recording a vertical angle on the line of a given shot there will be recorded, for instance: $2 \times 4.22 - 3.62 + 4.84 = 9.66$; where 2 intervals of 4.22 feet each have been covered, the final reading of the upper wire is 3.62 and the height of instrument is 4.84. If the angle were downward, the lower wire would reach the rod first and the rod reading should be *added*. If the sight is so nearly level that the middle wire is on the rod when the telescope is level, the reading of that wire may be used directly with the height of instrument.

234. Field method for large areas. If the area to be surveyed covers several square miles, or is an elongated strip as a narrow valley, it is best to hang the topography on a system of triangles. By this is meant laying out a system of triangles by marking

their apexes with stakes or other form of station markers, and using these stations as beginning and closing stations for stadia traverse lines, and the sides of the triangles for azimuth reference lines. Unless the area is extensive, as a large county or a state, these triangles may be considered plane triangles, the curvature of the earth being neglected. In case the triangles are to be considered as plane, it will be best to assume for the meridian of the survey a true meridian near the middle of the area, east and west. The process of laying out such a system of triangles is as follows:

235. Measuring the base line. A fairly level stretch of ground is selected, if possible, near the middle of the area to be mapped, and a line of from one thousand feet to a half mile long, or longer, is measured very carefully. This line is called the *base line*. Its ends are marked with substantial monuments of stone or with solid stakes. If the survey is of sufficient importance to warrant the permanent preservation of the system of triangles for future reference, the monuments that mark the base line, and those that mark the apexes of triangles, should be stones not less than six inches square in cross section and two feet long. In the top face, which should be dressed, a hole is drilled large enough to take a half-inch copper bolt, which is leaded in. The head of the bolt is marked with a cross to determine the exact point, and the stone is set deep enough to be beyond the disturbing action of frost. The point may be carried to the surface by "plumbing," and a temporary point set for use. The location of each monument should be fully described with reference to surrounding objects of a permanent character, so that it may be found readily at a future date. If no permanent objects that can be used are to be found, then constructions of stone, brick, or timber are built near by on the surface of the ground, and the point is described with reference to these. Such precautions are usually necessary only for extensive geodetic surveys.

The measurement of the base line for the surveys of smaller areas, which are the only surveys to be described here, should be made with a precision of from one in ten thousand to one in fifty thousand, according to the scale of the map that is to be made, the area surveyed, and the importance of the work.

The two ends of the base line being determined and marked, the transit is set over one end, and a line of stakes ranged out

between the two ends. The alignment is made as perfect as possible, and the stakes are set as nearly as practicable at single tape lengths apart, center to center, or at half or quarter tape lengths. These stakes are made quite firm in the ground, and should not be less than two inches square in section. For high-class work the stakes should have on their tops tin or zinc strips on which to mark the measurements with fine scratches.

The measurements are then made, the temperature, pull, and grade elements being noted, and the distance between supports. The requisites for securing the required degree of precision are mentioned in Chapter I. If the absolute length of the tape at some standard temperature and pull is unknown, of course the absolute length of the line remains undetermined. For a precision of one in one million, which is attempted in the measurement of base lines for extensive geodetic surveys, as of a state or continent, much more precise methods than those described are necessary. These methods properly belong to the subject of geodetic surveying.

236. Measuring the angles. After the base line is established and measured, or even while it is being measured, prominent points in the area are chosen as *triangulation stations;* that is, apexes of triangles. These are so chosen that no angle shall be less than 30 degrees, nor more than 120 degrees. The first point is chosen, as nearly as possible, in a line at right angles to the base line at its middle point. A second point may be chosen opposite the first one, and the others are then located with reference to these and to one another. When these stations are established, signals are erected over them. For work of the magnitude here considered, these may be simply wooden poles, about one and one half inches in diameter, to which a flag of some white and dark cloth is tacked to make the signal more easily found. Such signals are usually called flags. A transit is then set over one extremity of the base line, and the angles formed at that point by lines to the adjacent stations are measured. When all of these angles have been measured, the transit is moved to the other extremity of the base line, and similar measurements are made there, and subsequently at each triangulation station. There are two methods for measuring angles, called respectively the method by repetition and the method by continuous reading. The first is explained in Exer-

cise 1, Art. 89, the angle being measured as many times as may be necessary to use practically the whole circle of 360°. The measurements are made to include the whole circle to eliminate errors of graduation. Usually both verniers are read to eliminate the effect of eccentricity of verniers. This is the method to use with an engineer's transit, because the pointing can be done with greater precision than the least reading of the circle.

The second method, by continuous reading, differs from the first in that each angle is not measured independently of the others at the point occupied. The telescope is pointed at each of the distant stations consecutively, and the vernier is read for each pointing, — the difference between the consecutive readings being the angle between the corresponding points. Thus, in Fig. 160 the instrument being at *I*, the telescope is first pointed to *A* and the vernier is read, then to *B* and the vernier is read, then to *C*, *D*, *E*, *F*, and *A* in succession. The reading on *A* subtracted from the reading on *B* gives the angle *AIB*. In this method it is necessary to read both to the right and left, and with the telescope both direct and inverted. Moreover, since each angle is measured on only one part of the limb, it is necessary, after completing the readings once around and back, to shift the vernier to another part of the limb and repeat the readings both forward and back, with telescope both direct and inverted. This is done as many times as there are sets of readings. Each set of complete readings to right and left, with telescope direct and inverted, gives one value for each angle. If it is desired to measure each angle three or five times, then three or five sets of readings must be taken. The amount that the vernier is shifted between each two sets is $\frac{180°}{n}$, where n is the number of sets.[1]

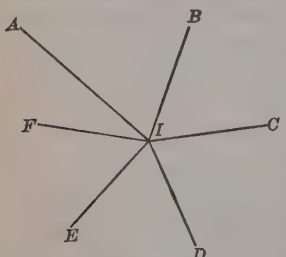


FIG. 160.

This method is used only with large instruments reading to single seconds.

A third method of measuring angles, which may be called a

[1] The student should make a program for observations by this method.

combination of the first two, saves some time, and gives results as good as the class of work warrants. It consists in measuring each angle a stated number of times, reading, measuring the succeeding angle the same number of times, reading, and continuing. After all the angles are read successively to the right, the telescope is inverted and pointed on the right-hand station, and the operation is repeated to the left. Three measurements of each angle each way are considered sufficient. This provides for all errors except those of graduation, which are usually too small to affect work done to the degree of precision required.

237. Notes. A good form for keeping the record of the notes of the triangulation, assuming that it is done by the repeating method and with an engineer's transit reading to thirty seconds, is shown below.

SATURDAY, JUNE 24, 1894. A.M.

INSTRUMENT ON △[1] A

STATION OBSERVED	VERNIER A	B	MEAN READING	COMPUTATION AND MEAN ANGLE
△ B	0° 00′ 00″	− 30″	359° 59′ 45″	15 01 45
1st reading .	62° 30′			360
	√ √ √ √ √ √			360
Inst. direct . .	360			735 01 45
				359 59 45
				6)375 02 00
△ C	15° 02′ 00″	01′ 30″	15° 01′ 45″	62° 30′ 20″
△ C	15° 02′ 00″	01′ 30″	15° 01′ 45″	15 01 45
				360
	√ √ √ √ √ √			360
Inst. reversed .	360			735 01 45
	360			359 58 15
				6)375 03 30
△ B	359° 58′ 30″	58′ 00″	359° 58′ 15″	62° 30′ 35″

[1] Triangulation station.

The notes of one set of readings are given. As many sets as may be desired are taken. For the purposes of a small topographical survey, one, or at most two, sets will be sufficient. The check marks indicate the number of times the angle is measured, a check being made after each measurement. Three hundred and sixty degrees must be added to the reading to the right each time the vernier passes the zero of the limb, or must be subtracted from the readings to the left. The latter process brings the same results as adding to the initial reading.

The form of notes to be taken with the third method is:

SATURDAY, JUNE 24, 1894.

INSTRUMENTS ON △ A

STATION	NO. MEAS.	READINGS			THREE TIMES MEAN ANGLE WITH AB	MEAN ANGLE
		VERNIER A	B	MEAN		
B	√ √ √	000° 00′ 00″	+00′ 30″	15″	000° 00′ 30″	
	√ √ √	000 00 30	01 00	000 00 30 45		
C	√ √ √	187 31 00	31 30	15		3)187° 31′ 00″
	√ √ √	187 31 30	32 00	187 31 30 45	187 31 30	62 30 20
D	√ √ √	398 51 00	51 30	15		3)211 20 00
	√ √ √	398 51 30	52 00	398 51 30 45	398 51 30	70 26 40
E	√ √ √	567 59 30	00 00	45		3)169 08 15
	√ √ √	567 59 30	00 00	567 59 45 45	567 59 45	56 22 45

238. Adjusting the triangles. All the angles of a given triangle are measured. If but two were measured and the third computed, the entire error of measurement of the two angles would be thrown into the third angle. It will be found

almost invariably, on adding the measured angles of a triangle, that the sum of the three angles is less or more than 180°. The error should be less than one minute, using the engineer's transit. If there is no reason to suppose that one angle is measured more carefully than another, this error is divided equally among the three angles of the triangle, and it is the corrected angles that are used in computing the azimuths and the lengths of the sides. This distribution of the error is called *adjusting* the triangle.

The base line is rarely as long as one of the sides of the main triangles. These are reached or developed from the base line as shown in Fig. 161. The base line *AB* is measured and its measurement is assumed correct. The angles at *A*, *C*, *B*, and *D* are measured and triangles *ACB* and *ADB* calculated for sides *AC*, *CB*, *BD*, and *DA*. *CD* may then be calculated from either triangle *ACD* or *CDB*, and in general two results will be obtained, since it cannot be expected that the angles have been so well measured and adjusted that they will be wholly consistent with rigid geometric figures. The measured angle *DBC* will probably not equal the sum of the adjusted angles *ABD* and *ABC*; neither will the sum of the measured angles *BCD* and *ACD* equal the adjusted angle *ACB*, etc.

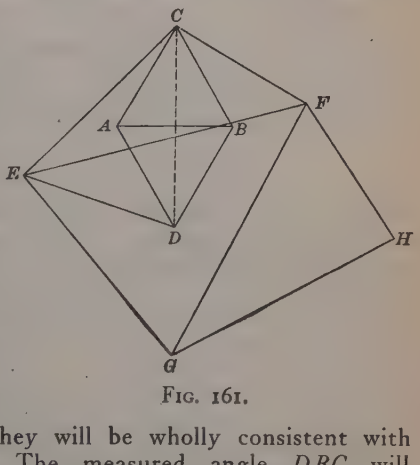


FIG. 161.

For the work contemplated here it will be sufficient to adjust each triangle separately, and use the mean of the two obtained values for *CD*. Assuming this mean value to be correct, the same procedure gives a value for *EF* which may be long enough for the side of a main triangle *EFG*, from which the system may be continued.

With the large systems of extensive geodetic surveys, very much more elaborate methods are employed, since a large number of triangles must be adjusted simultaneously so that they

will all be geometrically consistent, not only each by itself, but one with another.

239. Computing the triangles. When all the angles are measured and adjusted, and the azimuth of the base line, or any other line of the system, is determined, the azimuth of each of the sides may be found by computation. It is to be remembered that the azimuth of any line xy is the azimuth of the line $yx + 180°$. From the measured base line and angles, the lengths of each side of the system may be computed by the ordinary sine formula.

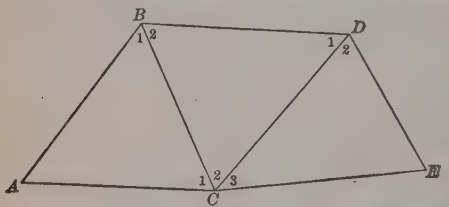


FIG. 162.

The computations can be much facilitated if systematically arranged. The following arrangement of the computations of the system of triangles shown in Fig. 162 is suggested. AB is supposed to be known. The adjusted angles are used. AC is determined before BC, because BC is to be used in the triangle BCD.

log AB	
	log AB − log sin C_1 = log $\frac{AB}{\sin C_1}$.
log sin C_1 . . .	
log $\frac{AB}{\sin C_1}$ . . .	
	log $\frac{AB}{\sin C_1}$ + log sin B_1 = log AC.
log sin B_1 . . .	
log sin A . . .	
	log $\frac{AB}{\sin C_1}$ + log sin A = log BC, etc.
log AC (value) .	
log BC ( ) .	
log sin D_1 . . .	Cologs can be used if preferred. Write the values found for AC, BC, etc., in the brackets.
log $\frac{BC}{\sin D_1}$ . . .	
log sin C_2 . . .	log $\frac{AB}{\sin C_1}$ may be written between log sin B_1 and log sin A if preferred.
log sin B_2 . . .	
Etc.	

240. Use of triangles. The elevation of each triangulation station is determined by leveling from some bench mark and

the elevations are adjusted as described in Art. 40. Any station may now serve as a beginning point for the topographical survey. The transit being set over Station X is oriented on Station Y by bringing the vernier to read the azimuth of the line XY, and setting the line of sight on Y by the lower motion. The stadia traverse and work then begin. The azimuth work may always be checked in the field by occupying each triangulation station as it is reached, and determining the azimuth of one of the triangle sides radiating from the station occupied. If this agrees with the known azimuth, the work is correct. If it does not agree within one or two minutes, the work should be rerun.

The elevations also serve to check the vertical angles, since the elevation of the closing triangulation station of the stadia traverse should be the same by leveling and stadia work within allowable error. The triangles and leveling are spoken of as giving horizontal and vertical control to the survey.

MAPPING

241. Scale. The scale of the map will depend on the territory covered and the use to which the map is to be put. A survey of a city lot may be mapped on a scale of, say, fifty feet per inch. The topographical maps of the United States Geological Survey are on a scale of about one mile per inch. A railroad preliminary survey may be mapped on a scale of from one thousand feet per inch, for ordinary purposes, to one hundred feet per inch, for close detailing. The maps of the United States Coast and Geodetic Survey and of the United States Geological Survey are mapped to a "natural scale"; that is, a given distance on the map is some round fraction of the distance it represents, as $\frac{1}{60000}$ or $\frac{1}{10000}$, etc. Maps of land surveys, or others made to show to non-technical persons, should be made at so many feet, or chains, or miles, to the inch; and the top of the map should be north. Maps of large territories, mainly for the use of those who are conversant with surveying methods, may be made with advantage to the natural scale.

242. The triangles. If the topographical survey has been based on a system of triangles, these triangles are drawn first.

To draw them, assume one station as the origin of the survey, and, knowing the lengths and azimuths of all the sides, compute the latitude and longitude of each station with reference to the assumed origin. Proceed then to plot the stations by latitudes and longitudes. It will be usual to assume one end of the base line as the origin, and the direction of the base line as that of the reference meridian. If, however, the work is referred to another meridian, that should be chosen as the meridian of reference for the computing and plotting.

The lines may be drawn also by means of a protractor; but this, while quicker and sufficiently precise if a large protractor is used, for much work, is of course not as accurate as latitudes and longitudes.

Another method is to use the dividers and the lengths of the triangle sides, describing arcs whose intersections locate the triangulation stations.

243. Outline of method for topography. A stadia line is first plotted to see that it closes properly. When this has been done satisfactorily, the side shots are plotted. Then the contours are drawn and all other objects that it is desired to map. If preferred, of course, the detail work can begin before the completion of the mapping of the side shots, and it is better perhaps to draw in buildings and other structures as soon as the points locating them are plotted.

244. The stadia line. The best method for plotting the stadia lines is by latitudes and longitudes, the computations for which are made with sufficient exactness by a diagram or by the trigonometer shown in Fig. 163. The arm is set to the proper azimuth and the length of sight is noted on the arm. Then the latitude and longitude differences are read on the cross-section scale.[1]

If the latitudes and longitudes of the triangulation stations have been determined, it will be known on the completion of computations for any one stadia line whether or not that line is correct. If it does not close on the proper triangulation station, and the error is small, that error may be distributed

[1] The degree numbering should be continued on this figure to 360°, and the signs for latitude and longitude differences marked on each quadrant of numbers. Let the student show how this numbering should be placed, and write the proper signs.

among the various courses, as in a land survey. If the error is great, it must be found. If the azimuths were checked in the field, the error must be in distance. Note the direction of the closing line and see whether it corresponds to any course. If it does, the error is probably in that course, and it should be re-measured. If it does not correspond to any one course, there is probably more than one error. In case of error of geographi-

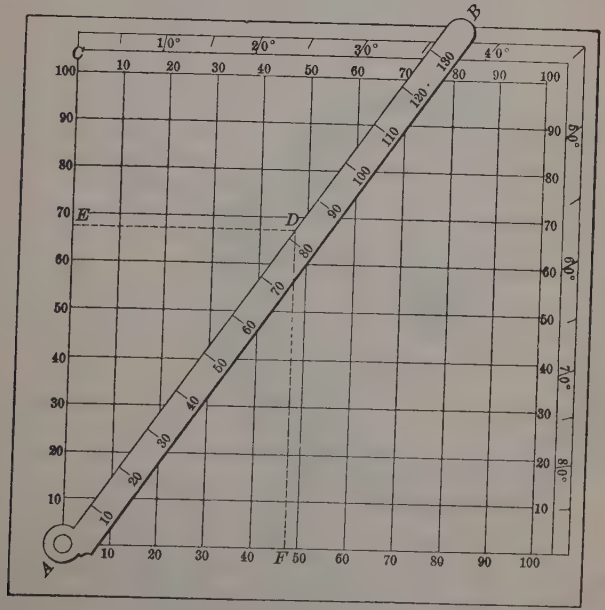


FIG. 163.

cal position due to mistakes in measurement, the elevations will not check unless the error is made on practically level ground; hence, if no field check of azimuths has been possible and the elevations check while geographical positions do not, the error is in azimuth.

The stadia line may also be drawn by the use of a protractor. The protractor should be large, either one of the expensive vernier protractors, like that shown in Fig. 164, or a paper protractor,

such as may be had from any dealer in surveyors' supplies. The paper protractor should be about fourteen inches in diameter, graduated to quarter degrees and printed on very light Bristol boards.

245. Side shots. The protractor may be prepared for use in mapping side shots as follows:

Number the graduations from zero to the right 360°. Draw a straight line through the zero and the 180° points and the center,

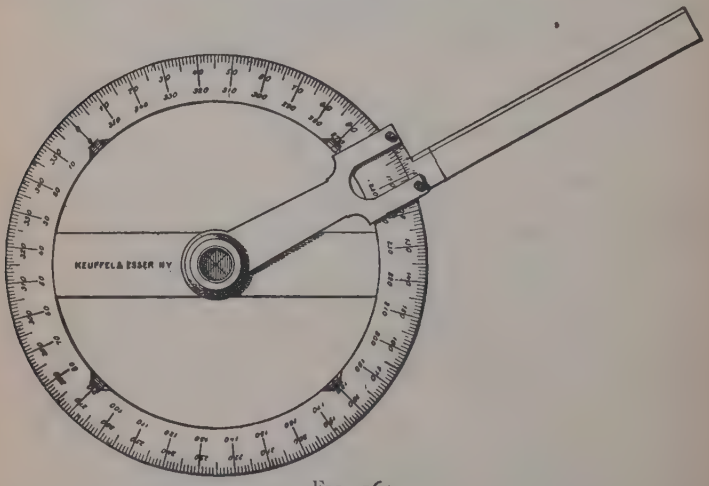


FIG. 164.

and continue the line to the end of the sheet. Do the same through the center and 270° and 90° points. The edges are trimmed true and square, and not less than an inch of paper is left outside the printed circle. With a pair of dividers set to the length of the longest average shots, when measured to the scale of the map, describe a circumference with the center of the protractor as a center, and with a sharp knife cut out the circle drawn.

To use the protractor, draw a meridian and east and west line through each stadia station. Center and orient the protractor by matching the 0° – 180° and 90° – 270° lines drawn on the protractor with the north-south and east-west lines drawn

through the point. After one stadia point has been occupied by the protractor the other orientations may be checked by testing the azimuth to the last station occupied or any other station whose azimuth from that occupied is known. Paste under the zero of the scale that is to be used a small piece of drawing paper, or other firm paper, through which, at the zero graduation, a fine needle is passed. The needle is stuck into the drawing at the station, and the rule may then be revolved about it as a center. The edge of the scale is brought to each azimuth in succession, and the distance is scaled at once and marked on the paper. If the point is taken merely for the purpose of obtaining elevations, its elevation is noted on the map when it is plotted. Otherwise, the name of the object is marked or the object is sketched in. In case the distance is so great that the point falls under the paper of the protractor, a mark is made with the name or elevation of the point and, when all the other points are plotted, the protractor is lifted and this point put in.

Fig. 165.

If a field map is to be made from which a finished map is to be traced, it may be made on the protractor sheet, or larger sheets may be had with a protractor printed on them. The azimuths may be transferred over the sheet with the parallel ruler.

With the aid of the sketches and the assumption that the slope is regular between adjacent plotted points, the contour points will be interpolated between plotted points, the contour lines sketched in and other objects drawn in. The triangulation stations, if there are any, should be inked in before any stadia plotting is done. The stadia lines are not drawn nor are the stadia stations inked. Buildings, fence lines, etc., should be inked before the contours are drawn, to prevent confusion. It is not customary, except in maps of small scale, to run contours across a road, or stream, or through a building. Contours are inked with the contour pen shown in Fig. 165.

In interpolating for contour points one should interpolate in a direction approximately perpendicular to the direction of the contour lines, not approximately parallel. A point interpolated between two points cannot be proved by interpolating between two other different points.

To interpolate between two points, use a piece of cross-section paper, or, better, profile paper, and a small straight-edged card. Place the edge of the profile paper on the two points; let their elevations be represented by the cross-ruled lines of the section paper and lay the edge of the card through the points that represent the elevations. Where the card edge crosses lines of contour elevations these may be projected down to the drawing. Thus in Fig. 166 let the bottom (top as it is held on the drawing) edge of the paper be elevation 125 for this particular case (it would vary in a different case, being the nearest round five feet below the low point); place the edge of the card through the two points *A* and *B* that may represent the elevations of the plotted

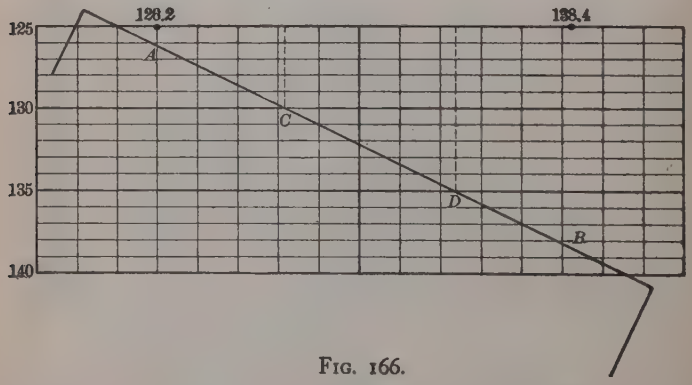


Fig. 166.

points and project *C* and *D*, points where the edge of the card crosses the 130-foot and 135-foot elevation lines, on the drawing as indicated by the dotted lines. If one-foot, two-foot, or ten-foot contours are required, the method is the same. No measurements are made, no calculations are made, no unusual apparatus is required. There are many devices for this kind of work.

246. Other protractors. A very convenient but expensive form of protractor for this work is Colby's, shown in Fig. 167. It is made in three pieces. The largest piece is the limb *L*, *L*, *L*, graduated from 0° to 360°, with fifteen-minute divisions; *E*, *E*, are projections, upon which weights are placed. The limb has

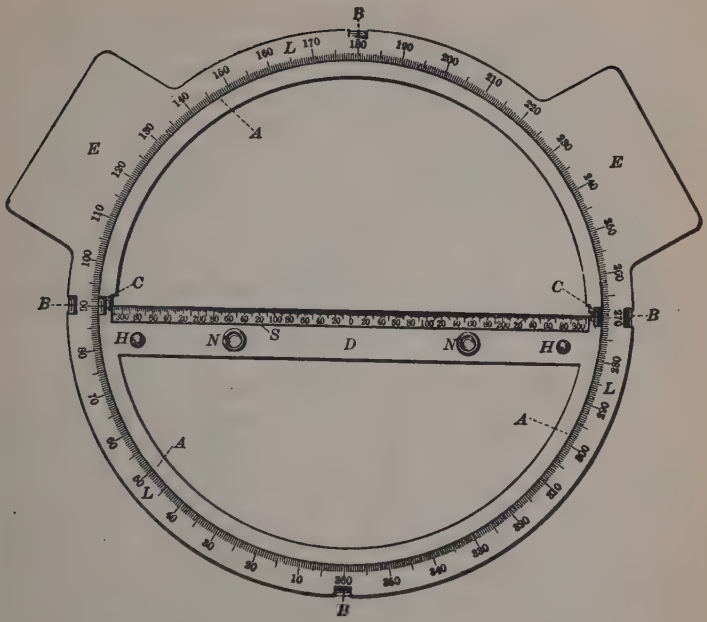


FIG. 167.

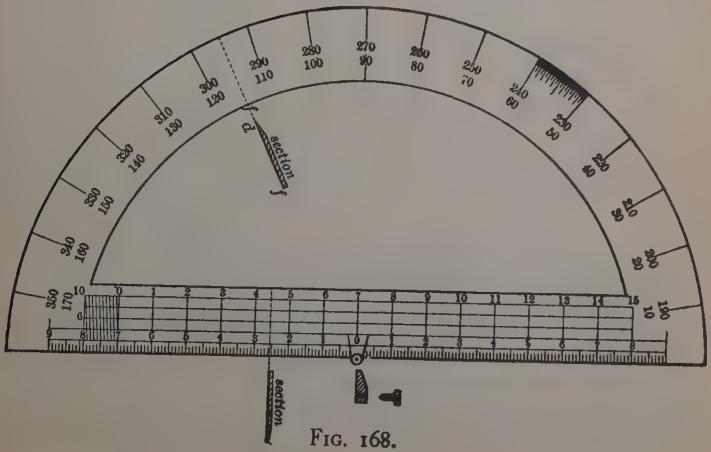


FIG. 168.

four indexes *B*, *B*, *B*, *B*, 90° apart, by which the protractor is oriented. *A*, *A*, *A*, is the alidade, fitted to revolve inside the limb. *C*, *C*, are indexes carried by the alidade, 180° apart. To the bar *D* of the alidade is attached the scale *S*, by the small screws and nuts, *N*, *N*. The scale has its zero mark in the middle, and is graduated both ways. It can be taken off by unscrewing the nuts, *N*, *N*, and a scale of different denomination may be substituted when desired.

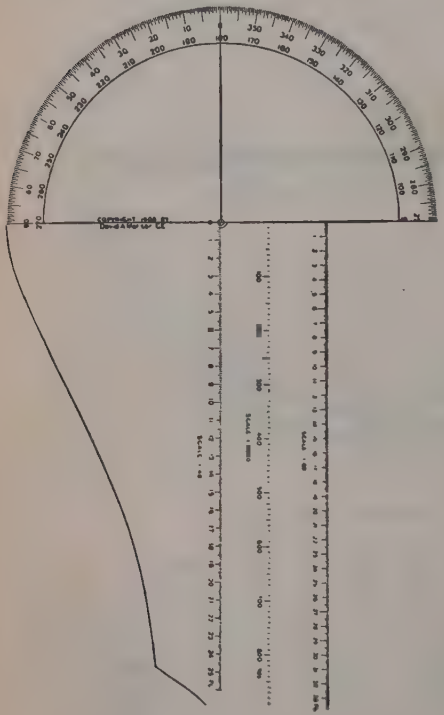


Fig. 169.

Another cheaper form is shown in Fig. 168. It is centered with a needle point, about which it will revolve. A meridian is drawn through the point over which the protractor is to be set, and the graduations are such that, if a given azimuth is brought to the meridian line, the diameter, on which is a scale for plotting the points, is in the given azimuth. This protractor was designed by Mr. J. A. Ockerson for use in the Mississippi River surveys.

Perhaps the best of all forms of cheap protractors is Molitor's. It is a modification of Ockerson's, and is shown in Fig. 169. It is printed on cardboard. A fine needle fixes it to the drawing at its center, about which it can be turned. A meridian is drawn through the plotted stadia station, the protractor centered at the station and swung until the meridian line is under the recorded azimuth on the circle. The graduated line then lies in

the recorded azimuth, the distance may be scaled and the point plotted. Other flat scales may be pasted or otherwise attached to the arm if the printed scale is not the one desired. It will be plain that if the plotted point has azimuths in the two northern quadrants, 0° – 90°, 270° – 360°, the circle must be on the meridian south of the point, if the azimuth is in the southern half of the circle, 90° – 270°, the circle will be on the meridian north of the point.

Exercise

Plot a contour map with five-foot contours at a scale of 300 feet to the inch from the following notes of a stadia survey, all pointings being taken from one position. Check the reductions.

Inst. at $\boxdot_0$ on Summit of Brown's Hill

Zero Az. = Mag. North. Elev. $\boxdot_0$ assumed 100.00

Station	Azimuth	Dist.	Vert. ∠	Diff. Elev.	Elev.
1.	357°00′	1025	– 2°12′	40.0	60.0
2. In ravine running northerly.	335°10′	885	– 2°28′	38.0	62.0
3.	3°15′	860	– 2°20′	34.8	65.2
4. In ravine running northerly.	1°25′	720	– 2°47′	35.0	65.0
5.	15°40′	635	– 2°42′	30.0	70.0
6.	26°50′	985	– 1°58′	33.8	66.2
7.	30°00′	1175	– 1°48′	37.0	63.0
8.	42°00′	1370	– 1°42′	40.5	59.5
9.	48°30′	1120	– 1°45′	34.2	65.7
10.	35°20′	730	– 2°15′	28.7	71.3
11.	52°50′	825	– 2°08′	30.7	69.3
12. In ravine running northerly.	17°10′	460	– 3°26′	27.5	72.5
13.	26°55′	420	– 3°25′	25.0	75.0
14.	50°05′	470	– 2°54′	23.7	76.3
15.	71°55′	960	– 2°03′	34.2	65.8
16.	72°30′	715	– 2°24′	30.0	70.0
17.	59°35′	340	– 3°22′	20.0	80.0
18.	30°35′	230	– 4°47′	19.0	81.0
19.	5°00′	135	– 6°26′	15.0	85.0
20.	48°15′	140	– 4°52′	11.8	88.2

Station	Azimuth	Dist.	Vert. ∠	Diff. Elev.	Elev.
21.	86°35′	85	− 6°48′	10.0	90.0
22. Head of ravine running easterly.	94°30′	235	− 4°56′	20.0	80.0
23.	82°25′	290	− 3°45′	19.0	81.0
24.	81°15′	545	− 2°49′	26.8	73.2
25. In ravine.	95°05′	490	− 3°16′	28.0	72.0
26. In ravine.	90°35′	655	− 2°50′	32.5	67.5
27. In ravine.	89°05′	835	− 2°32′	37.0	63.0
28. In ravine.	91°10′	905	− 2°27′	38.8	61.2
29.	102°55′	860	− 2°18′	34.5	65.5
30.	118°10′	1020	− 2°07′	37.5	62.5
31.	130°50′	1185	− 1°57′	40.0	60.0
32.	147°15′	860	− 2°12′	33.0	67.0
33.	147°15′	580	− 2°43′	27.5	72.5
34.	124°15′	690	− 2°32′	30.5	69.5
35.	143°30′	365	− 3°33′	22.6	77.4
36.	118°45′	420	− 3°14′	23.7	76.3
37.	130°30′	125	− 5°33′	12.0	88.0
38.	194°00′	140	− 7°30′	18.0	82.0
39.	168°50′	755	− 2°28′	32.5	67.5
40.	191°55′	615	− 2°49′	30.4	69.6
41.	191°55′	315	− 3°58′	21.8	78.2
42.	219°45′	340	− 4°00′	23.6	76.4
43. In ravine running southwesterly.	229°15′	425	− 4°05′	30.0	70.0
44. In ravine running southwesterly.	232°30′	710	− 3°01′	37.5	62.5
45.	223°25′	640	− 3°00′	33.7	66.3
46.	203°35′	835	− 2°25′	35.2	64.8
47. In ravine.	229°35′	945	− 2°31′	41.5	58.5
48.	224°15′	1025	− 2°21′	42.0	58.0
49. In ravine.	229°15′	1185	− 2°13′	46.0	54.0
50.	343°00′	960	− 2°12′	37.0	63.0
51.	331°00′	1180	− 1°59′	40.5	59.5
52.	320°05′	1350	− 1°52′	44.0	56.0
53.	315°30′	1115	− 2°01′	38.9	61.1
54.	326°30′	780	− 2°22′	32.2	67.8
55.	343°20′	440	− 3°16′	25.2	74.8
56.	300°05′	795	− 2°20′	32.5	67.5
57.	287°15′	930	− 1°58′	31.8	68.2
58.	2°30′	315	− 4°01′	22.0	78.0

Station	Azimuth	Dist.	Vert. ∠	Diff. Elev.	Elev.
59.	300°30′	300	− 3°55′	20.4	79.6
60.	274°45′	770	− 2°27′	33.0	67.0
61.	276°55′	470	− 2°59′	24.5	75.5
62.	340°45′	140	− 5°43′	13.8	86.2
63.	285°30′	175	− 4°37′	14.0	86.0
64.	262°55′	275	− 3°51′	18.5	81.5
65. Head of ravine.	240°20′	225	− 5°10′	20.0	80.0
66.	253°45′	570	− 2°49′	28.0	72.0
67.	243°00′	715	− 2°42′	33.8	66.2
68.	255°55′	900	− 2°25′	38.0	62.0

247. Finishing the map. If the map is to be finished in black, it may be finished as shown on Plate I, at the end of the book. If it is to be finished in color, a very effective result will be obtained by following the example given in Plate II, which shows the scheme in use by the United States Geological Survey. Plate III shows a more effective and more elaborate scheme. The contours are brown, while the parks and vegetation of every description are in green. The red is vermilion. Streams and all water will be blue. Roads may be black and dotted or full lines.[1]

248. Requirements for maps. It must be remembered that no map is complete without the following items:

1. A neat, explicit title, preferably in Roman letters.
2. Scale, both drawn on the map and given by figures.
3. The date of the survey.
4. Name of surveyor and draughtsman.
5. Direction of meridian.
6. A key to any topographic symbols that are used.
7. A neat line border.[2]

If contours are drawn, they should be numbered to make the map intelligible. It is necessary to number only every fifth or tenth line, which should also be drawn a little heavier than the others.

[1] An excellent work on "Topographical Drawing and Surveying" is that by Lieutenant Henry S. Reed, U. S. A.

[2] Other requirements for land maps will be found in Appendix, page 453, and Art. III.

THE PLANE TABLE

249. Description. Detail topographical surveys are sometimes made with the plane table; indeed, this is the standard instrument of the United States Coast Survey for such work, and is also largely used by the United States Geological Survey.

The plane table consists essentially of a drawing board with a suitable leveling device, mounted on a tripod, and a ruler for

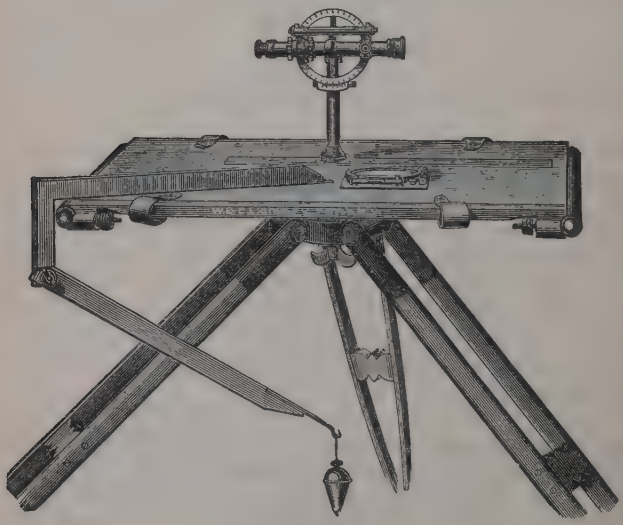


FIG. 170.

drawing. The ruler is attached to a line of sight, usually telescopic, but sometimes merely open sights, like those of the compass. The combined ruler and telescope is called the *alidade*. Fig. 170 is a complete plane table. Clips for holding the paper are shown, as well as the alidade, compass box, and levels, and a device for setting a point on the paper over a point on the ground. The plane table is, in its ordinary form, a very awkward instrument, and is used very little outside the two surveys mentioned. A simple form of leveling head is one invented by Mr. W. D. Johnson, and this device has been approved by the

topographers of the United States Geological Survey. It is shown in Fig. 171. By loosening the wing nut *d*, the table may be leveled, and when leveled, the nut *d* is tightened.

When the nut *g* is loosened, the table may be turned in azimuth without disturbing its horizontality. The whole arrangement is very light. It does not permit of as close leveling as does the ordinary form with leveling screws, and should not be used where contours are to be carefully determined by vertical angles. In making a map of a park showing the location of each tree, bush, etc., for planning new work, and when work may be, without loss, performed on pleasant days and omitted on wet or damp days, the plane table may be used with advantage, and, aside from the United States Coast and Geological Surveys, it is for this class of work that it is most used. For contour work the author prefers to use the transit and stadia. If the advantage of mapping at once in the field is desired, it may be obtained by having an assistant and a light drawing board. It is believed that a lack of notes is rather a disadvantage than an advantage, as is generally assumed. For large areas mapped to small scale with little detail mostly sketched the plane table is an excellent instrument.

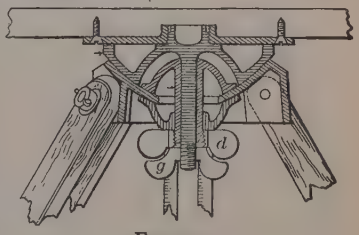


FIG. 171.

250. Use. The plane table is used for the immediate mapping of a survey made with it, no notes of angles being taken, but the lines being plotted at once on the paper. The simplest case is the location of a number of places from one point by azimuth and distance. The table is set up so that some convenient point on the paper is over a selected spot on the ground, and clamped in azimuth. Then the ruler is brought to the point on the paper and swung about it till the line of sight which is parallel in azimuth to the ruler[1] is directed toward a point that is to be located. A scale for the drawing is determined, and a short line is drawn along the ruler at about the

[1] It is only necessary that the two shall have a fixed angle in azimuth.

proper scaled distance, determined by measurement or by the stadia. Then the point is definitely located by scaling. A similar procedure locates other points. This is called the method of radiation. If a plane table is set up in the interior of a field all of whose corners are visible from the position of the table, the corners may be located thus and connected and a map of the field is at once obtained. There is evidently nothing by which

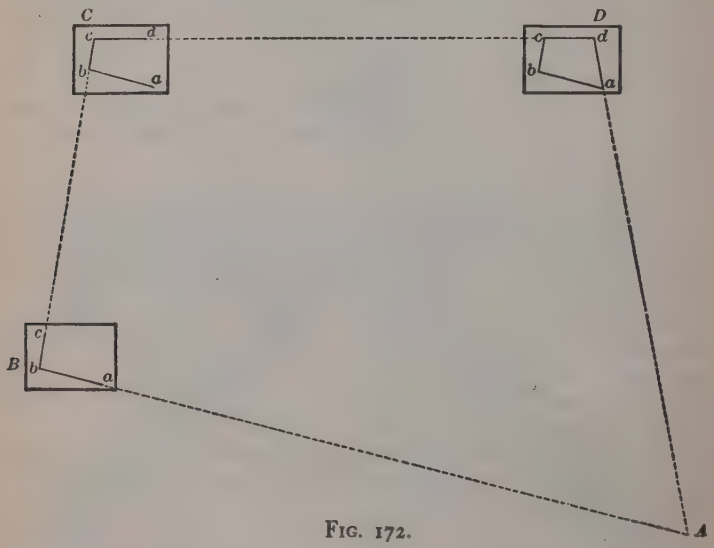


FIG. 172.

to determine the area except to scale the map for additional data or to use a planimeter.

Traversing may be performed with the plane table as follows: If it is a field that is to be "run out," set the table over one corner, choosing a point on the paper to represent that corner in such position that the drawing of the field to the scale selected will come on the board. In the four-sided field shown in Fig. 172, it would be necessary to occupy but two corners to map the field, though it would be better for a check to occupy the third. The figure shows the field and, to a very exaggerated scale, the plane table.

The table is first set over *B* and a point *b* marked on the

paper directly over *B*. The instrument being clamped, the alidade is brought to bear on *A*, and a line is drawn to scale equal to *BA*, which is found by measurement or by stadia. Then the alidade is directed to *C*, and the line *bc* is drawn to scale. The table is removed then to *C*, and so set up that the point *c* shall be over the point *C* and the line *cb* in the direction *CB*. This is hard to do with the plane table. If the scale is large, it must be done; but if the scale is very small, the following is a sufficiently close approximation: Set the table level, with *c* over *C* and with *cb* approximately in line with *CB*.

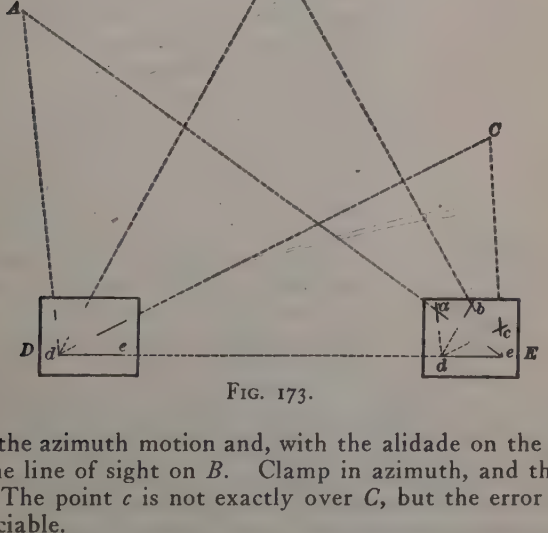


FIG. 173.

Loosen the azimuth motion and, with the alidade on the line *cb*, bring the line of sight on *B*. Clamp in azimuth, and the table is set. The point *c* is not exactly over *C*, but the error will be inappreciable.

The table being set over *C* and oriented on *CB*, turn the alidade toward *D* and draw *cd*. *da* may now be connected, giving the entire field. It will be better to occupy *D* and see whether the line *DA* will pass through *a* on the paper, and whether the length *da* equals to scale the length *DA*.

Another method very much used with the plane table is known as the method of intersections. Let it be required to locate the points *A*, *B*, *C*, Fig. 173, from the points *D* and *E*.

Measure DE, and lay off to scale on the paper a line equal to DE, in proper position so that the points desired will fall on the paper. Set the table over D and orient on E. Swinging the alidade about d, draw lines toward C, B, and A. Set the table over E and orient on D. Swinging the alidade about e, draw lines toward C, B, and A, and note the intersections of these lines with those drawn from d to corresponding points. These intersections locate the points.

251. The three-point and two-point problems. In filling in topography that is hung on a system of triangles, it is common to complete the triangulation and map it. Usually there will be drawn on a single sheet on the plane table only the topography adjacent to one or two triangulation stations, the work being carried on precisely as in the use of the transit and stadia, except that it is mapped at once. Ordinarily the first sheet will have at least one triangulation station plotted on it and a line toward another. These, indeed, might be drawn at random, so that they be located conveniently on the sheet for the work to be done. Usually there will be two or more triangulation stations, or previously occupied plane table stations, mapped on a new sheet. The table may be set over one and oriented on another. It happens not infrequently that the table cannot conveniently be set over any one of the stations mapped on the sheet. Then, if the table is set over any point at random and properly oriented (assuming that the latter could be done by the needle or otherwise) and the alidade is then revolved separately about each plotted point and directed toward the place in the field thereby represented, a line being drawn on the paper in this direction, the lines so drawn from the various points should all intersect at one point, which would be the properly mapped point over which the instrument is set. In Fig. 174, if points A, B, C, have been properly mapped in a, b, and c, and if the table is set up so that ab is parallel to AB, and consequently bc is parallel to BC, and lines are drawn through a, b, and c in

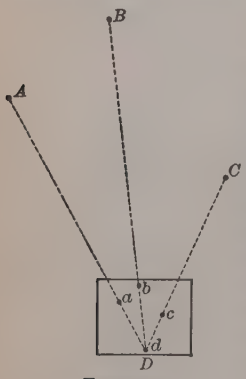


FIG. 174.

the directions *Aa*, *Bb*, and *Cc*, they should intersect in the point *d* which is over *D* over which the table is set. The difficulty is to orient the table. This is done (when three points are mapped) by what is called the *three-point problem* and, when two points are mapped, by the *two-point problem*, or *location by resection on the known points*. There are several solutions of these problems. The following are perhaps the simplest and most satisfactory.

1. *Three-point problem.* If three points are mapped and may be seen from the position of the table, place a piece of tracing paper on the table, and, assuming any convenient point on the tracing paper, draw lines from this toward the three field points. Remove the alidade and shift the tracing paper till the three lines pass through the three mapped points. Prick through the intersection of the three lines. This is the point required. Now orient the table on any one of the points.[1]

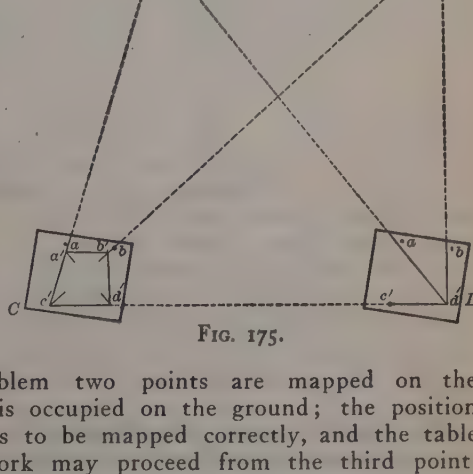


FIG. 175.

2. *Two-point problem.* In this problem two points are mapped on the paper, and a third is occupied on the ground; the position of this third point is to be mapped correctly, and the table oriented so that work may proceed from the third point. To do this, a fourth point is occupied temporarily. In Fig. 175 the field points have been mapped and are shown on the table in *a* and *b*. The point *C* is to be occupied. Place a piece of tracing paper on the table and set the table over a fourth point *D*, conveniently chosen. Through the point on the tracing paper that is over *D* draw lines toward *A*, *B*, and *C*. Estimate the distance *DC*, and lay it off to scale to *c'*. Set *c'*

[1] For Coast Survey method for solving the three-point problem see Appendix, p. 467 *et seq.* Students preparing for civil service examinations should study this method.

over C and orient on D. Draw lines toward A and B, and note the intersection of these lines with those already drawn from d'. The figure $a'b'c'd'$ will be similar to the figure $ABCD$. It will, however, not be drawn to the proper scale since the distance DC was only estimated. Therefore, $a'b'$ will be longer or shorter than it should be to represent properly the line AB to the scale that it is proposed to use. Let it be supposed to be shorter. Remove the alidade and shift the tracing paper till a' is over a, and the line $a'b'$ is coincident with ab. Construct a quadrilateral on ab similar to $a'b'c'd'$. This will give the quadrilateral $abcd$ (Fig. 176), and c is properly located relatively to a and b. The point is pricked through into the drawing, and the table is set up with c over C and oriented on any one of the other points, and the work proceeds. The tracing paper may be dispensed with, the drawing work being thereby somewhat increased. This method is seen to be merely a modification of the three-point problem.

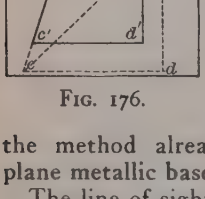


FIG. 176.

252. Adjustments. The bubbles for leveling the table are adjusted to be parallel to the base of the alidade. This is done by the method already given for adjusting a level having a plane metallic base. See Art. 56.

The line of sight is adjusted as in the transit and the axis of the telescope bubble is made parallel by the "peg" adjustment, the line of sight being reversed in azimuth by reversing the alidade on a line drawn on the table.

The horizontal axis is adjusted like that of the transit.

253. Further comparison of methods. From a paper[1] by Mr. D. L. Reaburn, M. Am. Soc. C. E., the following comparison of methods is taken. The comparison there given agrees with the author's ideas of the relative fields and merits of the plane table and transit stadia methods.

"The plane table is indispensable for geographical surveys, especially in mountainous regions of large relief. When provided with a micrometer eyepiece to the alidade, it can be used to great advantage on reconnaissance and exploratory surveys. Many thousand miles of such work have been conducted by the U. S. Geological Survey in Alaska.

[1] Eng. News, Vol. 66, p. 186, Aug. 10, 1911.

When used with stadia rods the plane table is adapted to mapping on scales up to about 500 ft. to the inch; on larger scales than this the problem becomes a mechanical one of locating points rather than sketching, and compared to the more rapid transit stadia method, the plane table is decidedly at a disadvantage.

The three-point method cannot be used to advantage on large scale work with the plane table. A chained traverse line is generally required for control. In rough and brushy country this traverse work is slow, laborious and expensive. . . . The transit stadia method, on the other hand, is not adapted to small scale mapping of extended areas.

The writer has, however, used the transit to advantage in connection with the plane table on 2000 ft. to the inch work, where it was desirable to do the sketching in the field. The points were located by transit and plotted on the plane-table sheet with a small protractor.

The transit stadia is well adapted to large scale surveys. On such surveys the location and plotting of the detail points constitutes the major portion of the work. The transit in the hands of a skilled observer is capable of locating these points with more ease and rapidity than any other instrument. The transit is better adapted to work in brush than the plane table, and there is not so much lost time from adverse weather conditions."

CHAPTER XIII

EARTHWORK COMPUTATIONS

ORDINARY METHODS

254. Occurrence of problem. The surveyor is called upon frequently to measure the volume of earth moved or to be moved in connection with various grading operations. These may consist of the grading of a city block to conform to the bounding street grades, the simple excavation of a foundation pit or cellar, the grading of a street, the building of a reservoir, the dredging of material from the bottom of a river, lake, or bay, etc.

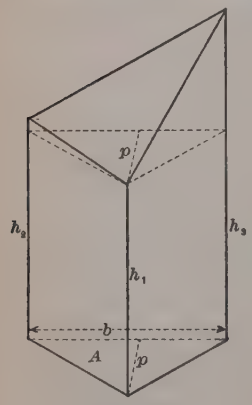


FIG. 177.

Payment for earthwork is usually by the cubic yard; sometimes, for small quantities, by the cubic foot. The measurement of earthwork is based on the geometrical solids, the prism, wedge, pyramid, and prismoid.

255. Prisms. The volume of any doubly truncated prism is $V = Ah$, in which A is the area of a right section, and h is the element of length through the center of gravity of the right section or is the length of the line joining the centers of gravity of the two ends.

In any prism having a symmetrical right base, the center of gravity of that base is at its center of form. This is true of all the regular polygons and of all parallelograms. In such prisms the length of the element through the center of gravity is the mean of all the edges. This is further true of a triangular prism.

The truncated triangular prism of Fig. 177 may be considered

to be a right prism of altitude h, and base A, surmounted by a pyramid of altitude p. $A = \frac{1}{2} bp$, and $p = \frac{2A}{b}$. The volume is $V = \frac{(h_2 - h_1) + (h_3 - h_1)}{2} b \frac{p}{3} + \frac{3h_1 A}{3}$. Substituting for p its value $\frac{2A}{b}$, $V = \frac{h_1 + h_2 + h_3}{3} A$.[1]

A truncated parallelopiped may be divided into two truncated triangular prisms in two ways, as shown in Fig. 178, and each set may be treated as above and the resulting volumes added. This gives twice the volume of the solid, which divided by two gives

$$V = \frac{h_1 + h_2 + h_3 + h_4}{4} A.$$

A prism having for its right base a regular pentagon may be divided into three triangular truncated prisms in five ways and treated as the parallelopiped, when the result

$$V = \frac{h_1 + h_2 + h_3 + h_4 + h_5}{5} A$$

is obtained.

256. Prismoids. Many of the forms dealt with in earthwork are more nearly prismoids than either prisms, wedges, or pyramids, and when this is the case the prismoidal formula gives more correct volumes.

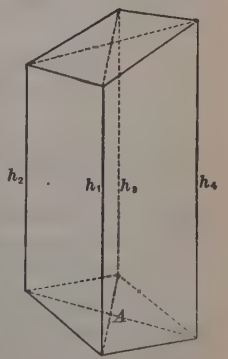


FIG. 178.

A prismoid is a solid having two parallel polygonal bases connected by triangular faces. This is equivalent, if the faces are conceived small enough, to the following, which somewhat better defines the forms dealt with in earthwork:

A prismoid is a solid having parallel plane ends with sides formed by moving a ruled line around the perimeters of the ends as directrices. Such a solid may be

[1] The student should show that $\frac{h_1 + h_2 + h_3}{3}$ is the length of the element through the center of gravity of the bases.

conceived as made up of a combination of prisms, pyramids, and wedges.

257. Prismoidal formula. If a single expression can be found giving the volume of a prism, pyramid, and wedge, it will give the volume of a prismoid, since such a solid is made up of the above three elementary solids, all having, in any given case, equal heights.

In the right

$$\left.\begin{array}{l}\text{prism}\\ \text{pyramid}\\ \text{wedge}\end{array}\right\} \quad V = \left\{\begin{array}{l} Ah \\ \dfrac{Ah}{3} \\ \dfrac{Ah}{2}\end{array}\right. \quad \begin{array}{c}\text{which may be}\\ \text{written}\end{array} \quad \left\{\begin{array}{l} \dfrac{h}{6}(A_1 + A_2 + 4\,A_m) \\ \dfrac{h}{6}(A_1 + A_2 + 4\,A_m) \\ \dfrac{h}{6}(A_1 + A_2 + 4\,A_m)\end{array}\right.$$

in which A_1 is one base, A_2 is the other base, and A_m is a section midway between the two end bases and parallel to them.

The single expression thus found is called the prismoidal formula, and is the correct formula for finding the volumes of prismoids.

A_m is not a mean of A_1 and A_2, but each of its linear dimensions is a mean of the corresponding dimensions of A_1 and A_2.

This formula is applicable to the sphere and all the regular solids of revolution.[1]

258. Approximations. To avoid the labor involved in computing the volume of prismoids by the prismoidal formula, frequently one of two assumptions is made. These are:

(1) The volume of a prismoid is $\dfrac{A_1 + A_2}{2}h$, called the method of average end areas.

(2) The volume of a prismoid is hA_m, called the method of mean areas.

The error of the first assumption is twice that of the second and of the opposite sign. To show this, let a_1, a_2, and a_m be homologous sides of the similar end and middle sections of a

[1] Let the student show the truth of this statement for the sphere and both spheroids of revolution.

given prismoid.[1] Then since the areas of similar figures are as the squares of their homologous sides, there may be written the following expressions for the volume of the prismoid, based on the true and approximate formulas. In these expressions K is any constant ratio by which a_2 is to be multiplied to give area.

For instance if the areas are circles, a is the radius, r, and k is π, remembering that $a_m = \frac{a_1 + a_2}{2}$.

By prismoidal formula

$$V_p = 2K\frac{h}{6}(a_1^2 + a_1a_2 + a_2^2) = \frac{4}{12}Kh(a_1^2 + a_1a_2 + a_2^2). \quad (1)$$

By average end areas

$$V_{ea} = \frac{Kh}{2}(a_1^2 + a_2^2) = \frac{6}{12}Kh(a_1^2 + a_2^2). \quad (2)$$

By mean areas

$$V_m = K\frac{h}{4}(a_1^2 + 2\,a_1a_2 + a_2^2) = \frac{3}{12}Kh(a_1^2 + 2\,a_1a_2 + a_2^2). \quad (3)$$

Subtracting (1) from (2),

$$V_{ea} - V_p = \frac{2\,Kh}{12}(a_1 - a_2)^2. \quad (4)$$

Subtracting (1) from (3),

$$V_m - V_p = -\frac{Kh}{12}(a_1 - a_2)^2. \quad (5)$$

It is seen that equation (5) is half of equation (4) and of opposite sign. From equations (4) and (5) it is seen also that the average end area method gives results too great, while the mean area method gives results too small. There are certain peculiar cases when the end areas are not similar figures in which the average end area method gives results too small.

By the prismoidal formula, or one of the approximations mentioned, the volume of all masonry work or earthwork may be computed.

[1] Prismoid end areas are not necessarily similar. They are assumed to be so here for simplicity.

It is seen from equations (4) and (5) that the errors of the assumptions given vary with the square of the difference in dimensions of the two end areas, and, therefore, it may be concluded that in cheap work, such as earthwork, if successive end areas are nearly alike, the simpler method of average end areas may be used, while if the areas are quite different, the prismoidal formula should be used; and in costly work, such as masonry, the prismoidal formula should always be used.

259. Area grading. When a large area, as a city block, is to be filled or excavated, the area may be divided into rectangles of such size that the four corners of each may be assumed to be in one plane. The lines of division will be so referenced that their intersections can be found again after the grading is done. Elevations of the corners of the rectangles will be found before grading, and after the grading is done. The difference in elevation at any one corner is the depth of cut or fill at that corner. The depth of cut or fill is marked on the stakes set at the corners to guide the workmen. The total volume is thus divided into a number of prisms of equal bases and known altitudes. The volume in cubic yards of one prism is, letting A be its base and a, b, c, d, its four corner heights,

$$V = \frac{A}{27} \cdot \frac{a + b + c + d}{4}.$$

The student may show that the volume of all the prisms is given by

$$V = \frac{A}{4 \times 27} (\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4), \qquad (1)$$

in which Σ is the sign for "sum of" and $h_1 h_2$, etc., are the corner heights of the several prisms, the subscript numeral indicating the number of prisms of which the h to which it is affixed is one corner. Thus c of Fig. 179 would be an h_4, b an h_2, etc.

The rectangles may be made larger by conceiving a diagonal as drawn in each rectangle. This would be drawn in the correct direction by inspection in the field, so that the assumed plane tops of the triangular prisms thus formed would most nearly correspond with the ground surface.

Assuming that A is still the area of a rectangle, the student should show that the volume of the series of prisms is given by

$$V = \frac{A}{6 \times 27} (\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4 + 5\,\Sigma h_5, \text{ etc.}), \quad (2)$$

and that the highest subscript may be 8.

The sides of such rectangles as have been mentioned may vary from twenty feet to one hundred feet, according to the character of the ground surface. If they are 27×40 feet for equation (1) or 27×60 feet for equation (2), the multiplier of the parenthesis will be 10; if 54×40 or 60, the multiplier will be 20; and if 27×20 or 30, the multiplier will be 5.

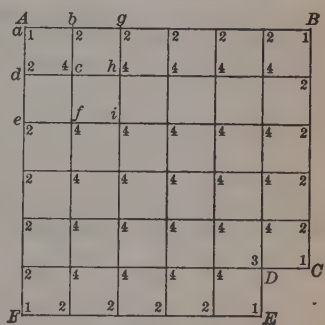


FIG. 179.

260. When the area is irregular. The general case is covered by the preceding article since a city lot or block is usually either filled or cut down. But sometimes it is high over a part of its area and low over the rest, when an irregular grade line will appear involving fractional squares. Thus in Fig. 180 it may be supposed that the lower right-hand portion is high and the rest low. The broken grade line *AB–H* is found by interpolation on each side that it crosses. Thus, the notes will show that *Q* is in cut and *R* in fill and the depth of each. Assuming the slope to be straight from *Q* to *R* and that *Q* is cut 2 feet while *R* is fill 1 foot, *A* would be $\frac{2}{3}$ the distance from *Q* to *R*. Similarly, the points *B* between *Q* and *T*, *C* between *T* and

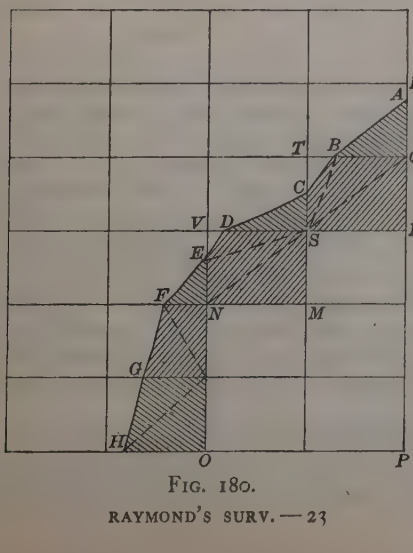


FIG. 180.

S, etc., are found. The fractional portions are computed separately as truncated prisms and added to the portion that can be lumped together, which in the figure is *LSMNOP*, containing five rectangles. The prisms *AQB*, *CSD*, etc., that have triangular bases may be computed according to the rules given, but we have no rule for a prism with an irregular four or more sided figure for a base. The best that can be done with *QBCSL*, *SDENM*, *FGN* and *GOH* is to divide them into triangles, as indicated by the dotted lines. These lines could be drawn from *B* and *C* to *L*, or *D* and *E* to *M* in the first two, or could be the opposite diagonals to those shown for the second two if such lines would conform more nearly with the surface conditions. The irregular areas on the fill side of the grade line are handled the same way if it is required to calculate both cut and fill.

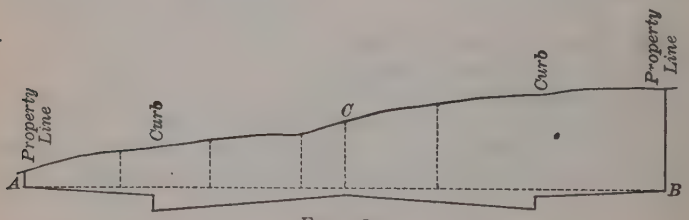


FIG. 181.

261. Street grading. In grading streets the sides are usually cut down or filled up vertical where possible, the principle being that the adjacent property must look out for itself. The best way to get the volume in street grading is to make cross profiles of the section to be graded, at such intervals as may be necessitated by the character of the ground, say from fifty to one hundred feet apart. On these cross profiles will be drawn the cross profile of the finished street, and the area between the two profiles at any section will be the area in cut or fill at that section. The volume between any two adjacent sections is computed as a prismoid. The areas may be drawn to scale on cross-section paper, and measured with a planimeter, or they may be computed.

Some engineers work with the grade of the center of the street, some with the grade of the curb lines, and some with the grade of the property lines. Stakes are usually set at the center

and sides and on them the depths of cut or fill are marked as a guide to the workmen. Where this is not possible, a list of cuttings may be furnished the contractor. Assuming the grade of the property line to be used in Fig. 181, the depths of cut to the line AB will be determined at such points in the cross section as may be necessary to give a correct area. The area above the line AB is computed from the field notes and used in getting the volume. The volume of the cutting below the line AB is a constant per unit of length of street, and may be readily computed for any given stretch. The slide rule is very useful in saving time in all volumetric computations.

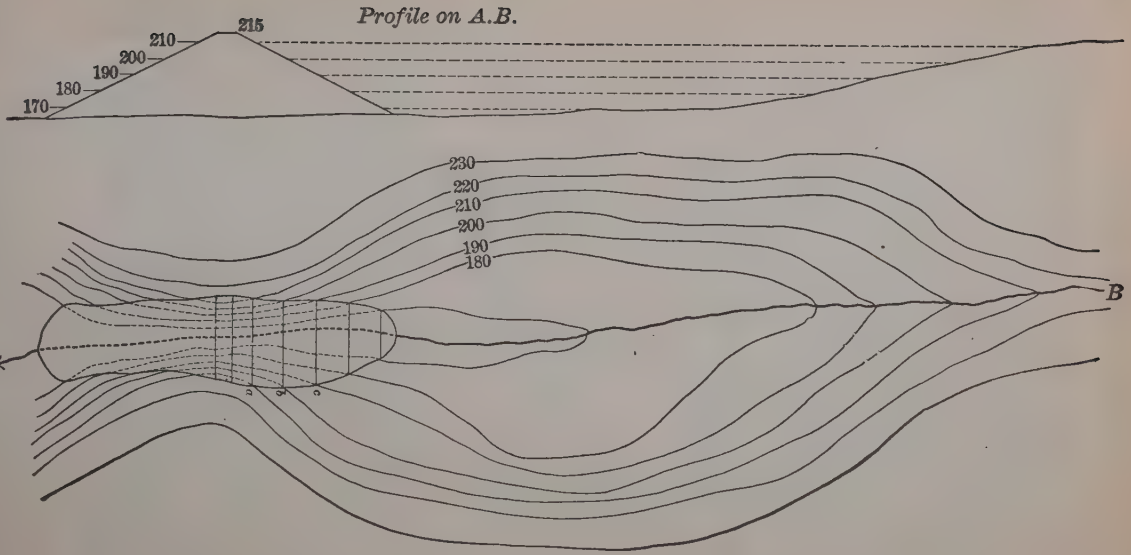
262. Excavation under water. When excavation under water is to be measured, it may be done by sounding on known lines, both before and after the excavating is done, and the volume is computed as in surface grading. This is not always the best method. The material is usually measured after removal. If it is placed in scows, the displacement of the scow for each load is obtained, and this, divided by the specific gravity of the material, gives the volume in the scow. It is necessary to observe the displacement of the scow for no load, and subtract this from the observed displacement for a given load.

The displacement is obtained by measuring the model lines of the vessel at varying depths, and computing the volume for these depths by the prismoidal formula. These volumes may be plotted on cross-section paper by laying off the depths to scale on one side of the paper, and at these points laying off the computed volumes to scale perpendicular to this axis of depths. A curve may now be drawn through the points thus plotted. Ordinates to this curve, at any scaled depth, will be to scale the volume for that depth.

To observe the displacement correctly, the water in the hold must be at the level obtaining when the displacement of the empty vessel was noted. The depth of the vessel in the water is best noted at four symmetrically situated points, and the mean is taken.

EXAMPLE

The measured load displacement of a scow is 3240 cu. ft. The specific gravity of the load material is 1.2. How many cubic yards in the load?



Plan

FIG. 182.

ESTIMATING VOLUMES FROM A MAP

263. A reservoir. Figure 182 is a contour map of a portion of a valley or ravine that it is proposed to convert into a storage reservoir by the construction of a dam across the narrow part shown. It is required to determine the capacity of the reservoir. At the location of the dam, its top width and side slopes being determined, the lines representing the top, and what will be contour lines of the finished surface, may be drawn on the map as indicated. The intersection of any contour with the corresponding contour of the finished dam will be a point where the dam will join the side of the valley, and a line, *abc*, connecting such points of intersection will indicate the edge of the proposed structure, or will be a line of no cut or fill, that is, a "grade line." The storage space may be conceived to be divided into a series of horizontal layers whose top and bottom areas are the closed figures formed by corresponding contours of surface and dam. These areas may be measured with a planimeter. The volume of any layer is obtained by averaging its two end areas and multiplying by its height, which is the contour interval. If h is that interval, the volume of a series of layers would be given by

$$V_{cy} = \frac{h}{27}\left(\frac{A_1}{2} + A_2 + A_3 + \text{etc.} \frac{A_n}{2}\right).$$

If there is a small volume below the lowest contour it may be omitted, if very small, or computed by averaging the lowest contour area with zero and using the scaled or measured height of the volume for the thickness. In a reservoir this small volume of water would probably be unavailable for use.

If it is preferred to use the prismoidal formula, the height of each prismoid will be twice the contour interval, and every other area will be a middle area, and the volume of the whole, assuming an even number of layers, will be

$$V_{cy} = \frac{h}{3 \times 27}(A_1 + 4\,A_2 + 2\,A_3 + 4\,A_4 + 2\,A_5 + 4\,A_6 + \text{etc.} \ldots A_n).$$

If there is an odd number of layers, the final layer may be computed separately, either by average end areas or by interpolating a middle area on the map and measuring it with the planimeter.

This prismoidal method is really not defensible unless the slopes are uniform from the first to the third areas, the third to the fifth, etc., which is rarely true. The solid is not a prismoid, and it is doubtful if anything is gained by considering it to be one.

The volume of material in the dam may be obtained in the same way.[1]

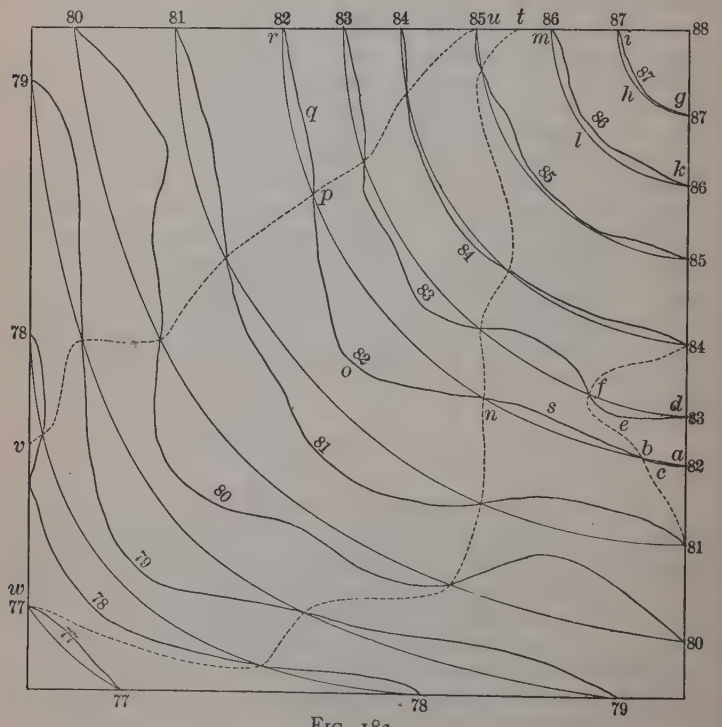


FIG. 183.

264. Application to surface grading. This method is applicable also to the measurement of irregular earthwork in general.

Let Fig. 183 represent a rectangular area that it is proposed to surface as indicated. The full irregular lines represent

[1] The student should show how to do this, using Fig. 182.

contour lines of the original surface. The more regular curved full lines represent contours of the surface as it is proposed to have it. The figures represent elevations above some datum surface. Conceive the original and graded surface to exist at the same time, and imagine the contour planes passed into the hill at their respective elevations, and consider particularly the 82-foot plane. The areas *a*, *c*, *b* and *n*, *o*, *p* will be in excavation, while the areas *b*, *s*, *n* and *p*, *q*, *r* will be in embankment. The points *a*, *b*, *n*, *p*, and *r* are " at grade," being neither in cut nor fill.

Connecting corresponding grade points, there result the dotted grade lines 84, *f*, *b*, 81; *t*, *n*, *w*; *u*, *p*, *v*, which are bounding lines of bodies of cut and fill.

Thus, the whole central portion of the figure is in excavation, while the upper left corner is in embankment.

The horizontal area in excavation or embankment at any level is the area included between original and finished surface contours at that level.

To get the volume of any single body of cut or fill, measure successive areas with the planimeter, and assume these to be end areas of figures that are as nearly prismoids as anything else, with altitudes equal to the contour interval. If the prismoidal formula is to be used for computing the volume, the altitude of a single prismoid is taken as twice the contour interval; that is, two layers are taken to make one prismoid. The volumes may be computed by the formulas of the last article. Assuming average end areas, the small body of cut on the right is found to have areas:

At elevation 81, 00.
At elevation 82, *acb*.
At elevation 83, *def*.
At elevation 84, 00.

$$\text{Volume} = \frac{1}{27}\left(\frac{00}{2} + acb + def + \frac{00}{2}\right) = \frac{1}{27}(acb + def).$$

The portion of fill between this cut and the large central body of cut has areas:

At 88-foot level, oo.
At 87-foot level, *ghi*.
At 86-foot level, *klm*.
etc. etc.

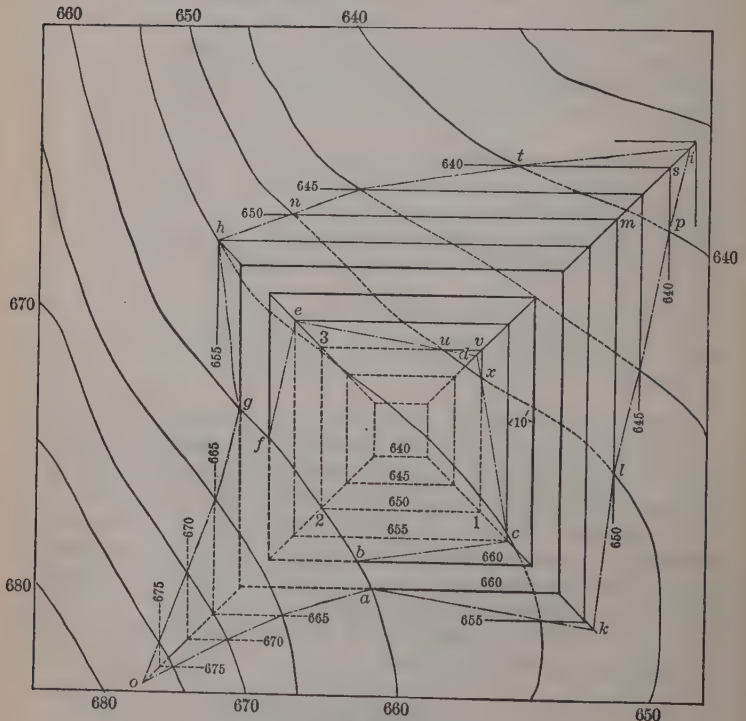


FIG. 184.

265. Application to structures. One example of the application of this method of estimating volumes to regular structures built on irregular ground will be given. The form of the structure is not presented as an example of good engineering, but merely to show the method of estimating volumes.

A small reservoir is to be built on a hillside, and will be partly in excavation and partly in embankment. Figure 184 shows

such a case. The contours, for the sake of simplicity, are spaced five feet apart. The top of the reservoir (shown by the heavy lines making the square) is 10 feet wide, and at an elevation of 660 feet. The reservoir is 20 feet deep, with side slopes — both inside and outside — of two to one, making the bottom 20 feet square, and elevation 640 feet, the top being 100 feet square on the inside. The dotted lines are contours that would be invisible if both original surface and completed reservoir were supposed to exist at the same time. The areas of fill all fall within the broken line marked *abcdefghik*, and the cut areas all fall within the broken line marked *abcdefgo*. These broken lines are grade lines. The areas of fill and cut are readily traced by following the closed figures formed by contours of equal elevation:

At 640-foot level area in fill is *pst*.
At 650-foot level area in fill is *lmnuvxl*.
At 650-foot level area in cut is 1 2 3 *ux* 1.

The other areas are easily traced. In the figures given, the lines have all been drawn in black for printing. In practice they should be drawn in different colors to avoid confusion.

SIMPLE RAILROAD EARTHWORK

266. Cross-sectional forms. When a road, railroad, or canal is to be built with regular roadbed widths and slopes, the methods followed in railroad grading are used, varied as may be necessary to suit the particular form of structure.

The form of cross section of an ordinary railroad embankment is shown in Fig. 185. The original ground line is *GG*, the finished subgrade is the width W at AB, above which is the ballast in which the cross ties are placed that carry the rails. The part of the earthwork or roadbed considered here is that below the subgrade. The center fill C, called the center height, the roadbed AB of width W, the side slopes, S, given as the ratio of the horizontal to the vertical, the side heights h_r and h_l, and the "distance out" d_r and d_l are quantities found or known and used in laying out the work or computing the section area for use in calculating the volume of earth moved to make the bank.

When the section is in excavation or "cut" it is generally taken out in the form shown in Fig. 186. The roadbed width

used in staking out and computing is W, which includes the subgrade width and the top width of the ditches DD which being of uniform quantity per unit length are computed separately.

In some cases a berme ditch, BD, is dug on the upper side of the cut to carry off the water that would otherwise wash over

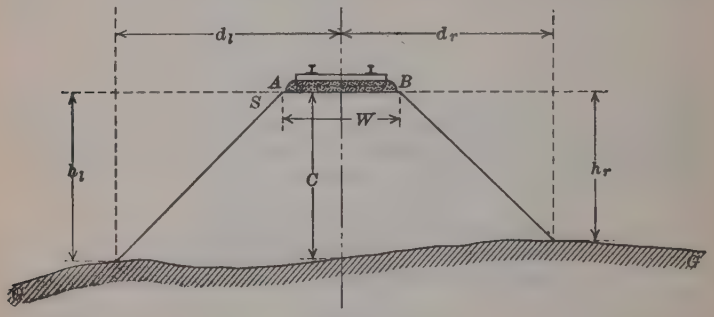


FIG. 185.

the side of the excavation. So far as staking out and computing is concerned the work is the same for cut or fill, but the side slopes may be different. For earth fills these are usually 1½ to 1, while for new earth cuttings they are frequently 1 to 1, and in rock they may be vertical or ¼ to 1.[1]

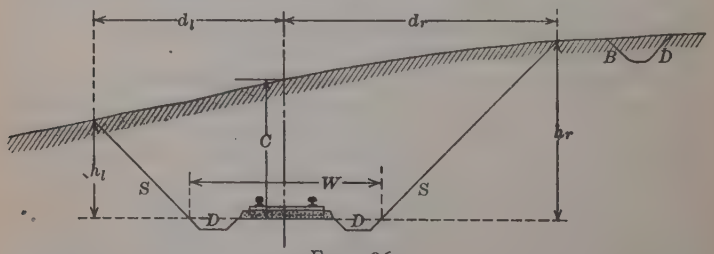


FIG. 186.

267. Staking out. This consists of marking the depth of cut or fill on the center stake previously placed, finding the points where the side slopes intersect the ground surface, placing

[1] For a full discussion of this subject see the author's "Elements of Railroad Engineering."

stakes there marked with the station number and the side height, and noting in the notebook the center height, side heights, and distances out, together with intermediate heights and distances, should the ground be irregular enough to require them. A profile will have been made of the center line, and a grade line will have been drawn upon it showing the center cut or fill to the scale of the drawing at all center points. In railroad surveying a station is 100 feet in length, is marked at each end by a stake, the first stake being numbered zero, the next 1, the next 2, etc., and the stake is called a station interchangeably with the 100 feet whose end it marks. The context in any statement must tell which is meant. The profile is generally drawn at a scale of 400 feet (4 stations) to the inch, horizontal scale, and 20 to 25 feet to the inch, vertical

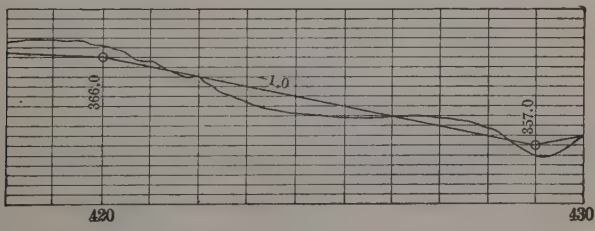


FIG. 187.

scale; hence only approximate values of cut and fill can be obtained from the drawing.

The actual values to the nearest 10th of a foot are obtained by noting the difference between field determined elevations of the ground at given stations and the grade elevations of those stations found from the data concerning the grade line as shown on the profile. Thus (Fig. 187) the grade line will show an elevation of 366.0 at station 420, a descending grade in the direction of survey of 1.0 per cent (1 foot per hundred) for nine stations where the elevation is 357.0, and an ascending grade from there on.

The level is set up to command some bench mark, and levels are run to the neighborhood of the station to be "cross-sectioned." Then the elevation of the line of sight above the grade at, say, station 420, is found to be 7.26 feet (7.3 for the purpose

of staking out). A rod is held at station 420 and is read, say, 3.6. If it read 7.3, the station would be "at grade"; as it reads 3.6 the cut is 3.7 and is marked on the stake − 3.7, or *C* 3.7.

Sta.	*Elev.*	*Grade*	*L*	*C*	*R*	
420	369.7	366.0	$\frac{3.0}{13.0}$	−3.7	$\frac{4.6}{14.6}$	
			$\frac{9.0}{21.5}$ $\frac{5.0}{6.0}$	+6.6	$\frac{7.2}{4.0}$ $\frac{6.8}{9.0}$ $\frac{8.4}{20.6}$	
			$\frac{-2.1}{12.1}$	0.0	$\frac{+2.4}{11.6}$	
			$\frac{-1.0}{11.0}$ $\frac{00}{6.0}$	+0.6	$\frac{+3.6}{13.4}$	

FIG. 188.

Some engineers use + for cut and − for fill. The cut of 3.7 is recorded in the notebook and the position of one side stake is then sought. This is found by trial. The distance out is $\frac{w}{2} + sh$, which may be proved by reference to Fig. 185 or Fig. 186. If the center cut is 3.7 the rodman estimates what the height

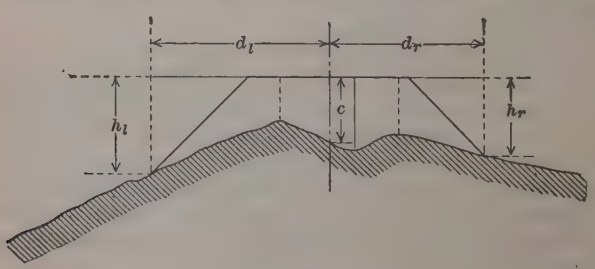


FIG. 189.

will be on the right, and using this estimated value computes d_r, and, using a metallic tape, measures d_r from the center and holds his rod. If the reading proves his guess right, he has the proper

place for the stake, which is marked and driven with the side height showing toward the center line. If his guess has not been right, he tries again and, if necessary, again, until the proper place is found. An expert rodman in ordinary ground will often guess right at first and will rarely need more than a second trial. After setting the right stake the left is set in the same way. If the ground is very irregular, it may be necessary to take elevations at fixed distances out from the center. No stakes are set, but the discovered heights and distances are noted.

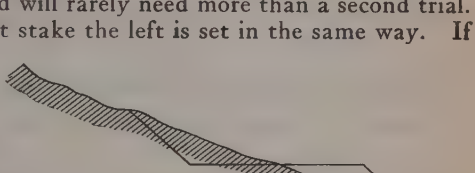


Fig. 190.

The form of notebook is shown in Fig. 188. The side heights and distances are written as a fraction. Either may be above the line so long as the practice is uniform. The first line shows the notes for station 420, using a 20-foot roadbed and slopes of 1 to 1. The second line shows the notes for the section work of the irregular station shown in Fig. 189, in which the roadbed is 16 feet and the slopes $1\frac{1}{2}$ to 1. The third line shows the notes of a side hill section (Fig. 190) in which the center is "at grade," and the fourth line shows the notes for a side hill section with the grade point to one side of the center. These grade points are found by trial.

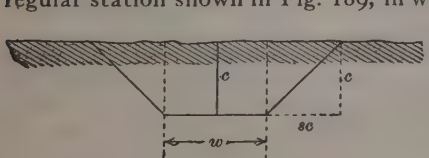


Fig. 191.

268. Computing the quantities. 1. *Areas.* The volume of a piece of railroad earthwork is calculated by getting the areas at each cross section and treating the solid between two consecutive sections as a prism, wedge, or prismoid (prismatoid). If the ground is level across (Fig. 191), the cross-section area is given by

$$A = wc + sc^2. \qquad (1)$$

If the ground slopes, so that what is called a 3-level section results, its area may be derived in one of two ways. Referring to

Fig. 192 the area is the sum of four triangles, two having common base c, and heights d_r and d_l and two having equal bases $\frac{w}{2}$ and heights h_l and h_r.

Therefore $$A = \tfrac{1}{2} cD + \frac{w}{2}(h_r + h_l). \qquad (2)$$

If the two sides be produced downward, they will meet below the center a distance $\frac{w}{2s}$ and the area may be considered to be the sum of two triangles having the common base $c + \frac{w}{2s}$, less the triangle of base w and height $\frac{w}{2s}$.

Therefore $$A = \frac{D}{2}\left(c + \frac{w}{2s}\right) - \frac{w^2}{4s}. \qquad (3)$$

As $\frac{w}{2s}$ and $\frac{w^2}{4s}$ are constant for any given piece of road this is a simple formula to work from. Referring to Fig. 193, the area is the sum of a series of trapezoids less the two triangles ABC and DEF, and is so computed.

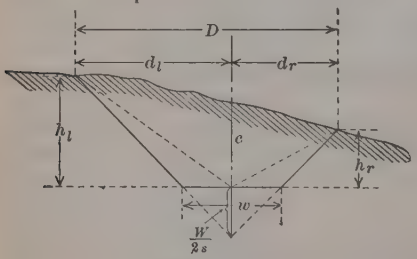


Fig. 192.

The several h's and d's will be given in the notes.

Each side may be considered a field whose coordinates are known from the notes and are as follows with the origin at the center of the base.

Cut ordinates	0	c	h_1	h_2	h_r	0	
Distance ordinates	0	0	d_1	d_2	d_r	$w/2$	

If these are multiplied together as indicated by the diagonal lines, the difference of the sums of the partial products will be the area. All but the first term, not used, and the last term will be in the notes in form ready for multiplying. The left side may be treated similarly. To the sum of the partial products

indicated by lines downward to the right that actually appear in the notes must always be added the extreme side height times half the roadbed. A side hill section with grade at the center will consist simply of a single triangle of cut and one of fill unless the section is irregular. Unless it is an irregular section, a side hill section with grade on one side (Fig. 194) will consist of half of a 3-level section of cut or fill plus a small triangle of the same kind and a triangle of the other kind of work.

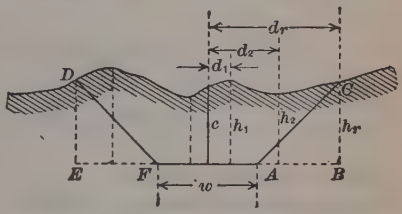


FIG. 193.

2. *Volumes.* If the solid between two sections is considered to be a prism or wedge or combination of these two, its volume is found by averaging the two sections and multiplying by the length between and dividing by 27 to reduce to cubic yards. If it is considered to be a prismoid, the middle area must be obtained either in the field, or by finding an area whose linear dimensions are means of the corresponding linear dimensions of the end areas. The end area method is used ordinarily. The work is best done by the use of tables or diagrams. When the prismoidal formula is to be used, average end area quantities should be taken from a table and the prismoidal correction, from another table, subtracted. The construction of these tables, the construction of diagrams, and the use of both will be found in books on railroad surveying.[1]

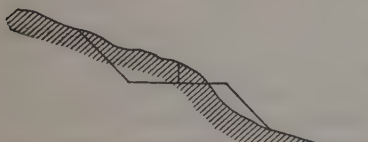


FIG. 194.

[1] See Raymond's "Railroad Field Geometry," Allen's "Railroad Curves and Earthwork," and many others.

CHAPTER XIV

CITY SURVEYING

269. Precision required. Surveying to determine land lines in a town or city does not differ in principle from surveying for similar purposes in the country. The difference lies in the degree of precision required. In the country a square foot of land is worth from a fraction of a cent to, perhaps in extreme cases, five or ten cents. In the heart of a great city it may be worth many dollars, and a single inch frontage of a lot one hundred feet deep may be worth several hundred dollars. The brick wall of a modern office building of ten or more stories, built an inch over the line of the owner's land, would be a costly wall to move, and an error so locating it would virtually place the owner at the mercy of the possessor of the adjoining property. It is readily seen, therefore, that an error in closure of one in three hundred, or one in five hundred, which may be tolerated in farm surveying, would be altogether out of the question in surveys in the heart of a great city.

In such situations a precision of one in fifty thousand should be secured. In villages and small towns a precision of one in five thousand will frequently be sufficiently close, though, if the place has " prospects," it may be well to make closer surveys. The methods of making measurements to the various degrees of precision required are noted in Chapter I.

To lay off an angle so that the position of a line may not depart from its true position by more than $\frac{1}{50000}$ of its length, requires that the angle have no greater error than about four seconds. With a transit reading to thirty seconds this may nearly be accomplished if the angle is measured three times before reading, the reading being divided by three.

An instrument reading to twenty seconds will give ordinarily a little better than the requirement with three measurements, and an instrument reading to ten seconds will give usually the

required result with a single measurement of the angle, though it should always be measured more than once, if for no other reason than to secure a check on the work. Of course the magnifying power of the telescope must correspond to the fineness of the graduations.

It has been stated that angles are read. Bearings are not used, nor are regular traverses run out in city surveys. Bearings are usually worked up from the angles for all the lines of a survey, in order to compute the latitudes and longitudes for determining the error of the survey. Azimuth would do as well.

270. Extent of survey. City land surveys are usually, except where "additions" are being laid out, of small extent, covering perhaps a single lot of 25 feet by 100 feet, more or less. In a well-monumented city the entire survey may be confined to the block in which the lot lies. In more cases, it will be necessary to go several blocks away to obtain monuments for determining the necessary lines. In cities that have been laid out with irregular lines, it may often be necessary to carry the lines and measurements over the roofs of the solid blocks of buildings in order to determine the angle points in the side lines of the lots. Each case involves new difficulties that the surveyor must meet by exercising his ingenuity. Many difficulties are solved by the proper use of the coördinate system, following the general methods outlined in the Appendix, page 433.

If the survey is of a building lot in a large city, some or all of the following items may be required to be shown on the plat submitted by the surveyor in addition to the lot dimensions:

1. Width of streets and alleys adjoining the property.
2. Established width of sidewalks and elevation of sidewalk grade at the building line and curb. This should be referred to city datum.
3. Street gutter grades.
4. Location and elevation of invert or flow line of sewers in adjoining streets, together with dimensions, form, material, etc.
5. Location of gas pipes, electric conduits, waterpipes, etc., both those in the street and those entering the property.
6. Character of surface soil and subsoil, which may require soundings with an auger or drill.
7. Location, dimensions, character of material and elevation of footings, of all walls of adjoining structures.

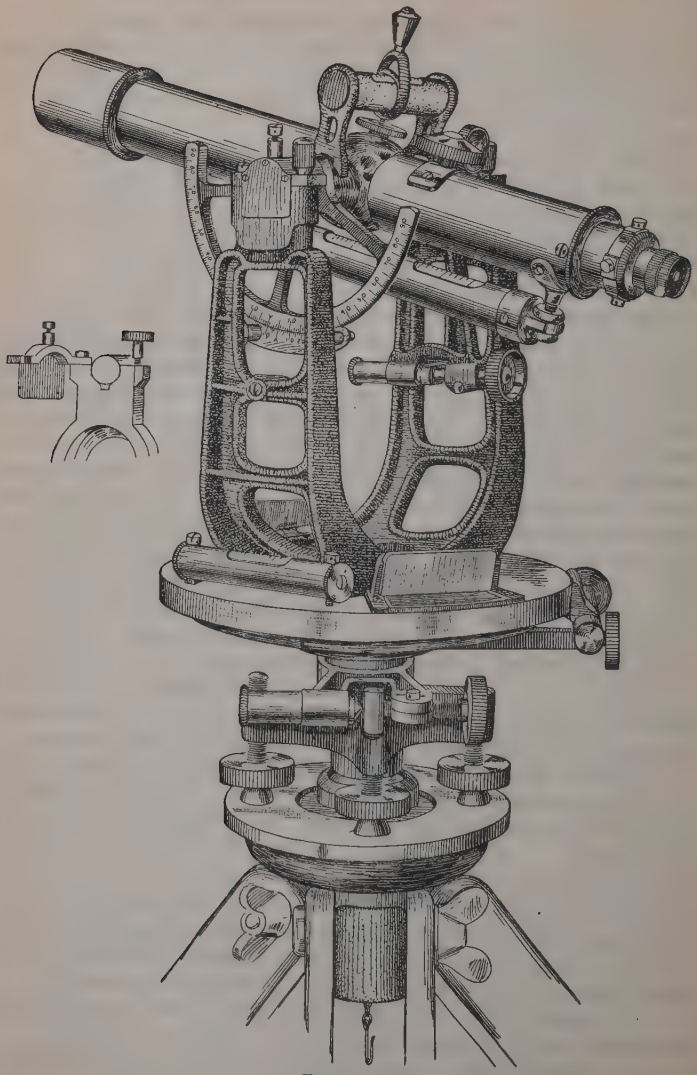


FIG. 195.

8. Character of building, if any, now on the ground, with dimensions, grades, etc., of cellars, subcellars, and street basements.

271. Instruments. A surveyor who undertakes to do city work should be supplied with the best instruments for measuring lines and angles. The tape mentioned in Chapter I, as used in New York City, is a very good tool. It should not be used, however, until it has been tested by the surveyor himself to verify the " pull scale " for various temperatures. The city surveyor should have, in his office, a standard length marked out on the floor, or some other place not subject to change, by which standard he may test his tapes. The National Bureau of Standards in Washington, D.C., will test tapes sent there for that purpose, and will report the temperature, pull, etc., for which they are standard, and their constants. For this service a small fee is required.

The city surveyor's transit should be of high magnifying power, and should read to thirty seconds and preferably to twenty seconds or even ten seconds. It should be well made, and of a pattern to insure stiffness and permanency of adjustment. A good form is shown in Fig. 195.

The compass being of little use in city surveying, the space usually occupied by the compass box is used to form the base of the horseshoe-shaped standards. The transit is made with either three or four leveling screws. The advantage of three screws is wider leveling base, and therefore, with given pitch of thread, a finer adjustment of the horizontal plate for a given turn of the screw. The author prefers four screws, perhaps because he has never become accustomed to the use of three.

272. Description of a city lot. The description of a city lot as found in a deed is either by " lot and block " or by " metes and bounds."

By the first method the shaded lot in Fig. 196 will be described as Lot 7 of Block 61 of the original plat of Iowa City, Iowa, as shown and delineated on a certain map entitled Map of Iowa City, Iowa, and filed in the office of the recorder of the County of Johnson, State of Iowa, June 14, 1832.

By the second method the same lot would be described thus: All that certain tract or parcel of land lying and being in the

City of Iowa City, County of Johnson, State of Iowa, and bounded and described as follows, to wit: Beginning at a point in the northerly line of Washington Street, distant thereon westerly 80 feet from the corner formed by the intersection of the said northerly line of Washington Street with the westerly line of Gilbert Street, and running thence westerly along the said northerly line of Washington Street 80 feet; thence northerly at a right angle with the said northerly line of Washington Street, 150 feet to the southerly line of the alley between the said Washington Street and Iowa Avenue; thence at a right angle easterly along

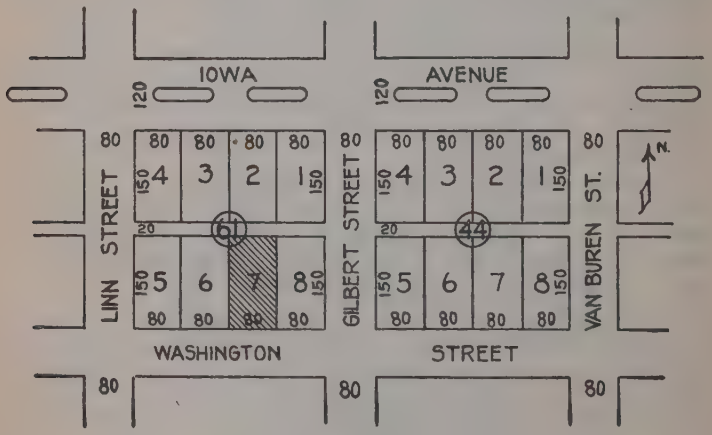


Fig. 196.

the southerly line of said alley 80 feet; thence at a right angle southerly 150 feet to the point of beginning. To this will sometimes be added; "*being lot* 7," etc., as first described.

Careful attention should be paid to the form of this description. There is not too much in it. It is necessary to say 150 feet *to* the — line of the alley, etc., in order that in case of future discrepancy the lot will run to the alley and not stop at 150 feet should that distance fail to reach or overrun the alley line. So with the other duplicate and definite wordings that may seem to the inexperienced surveyor to be unnecessary.

The first method is the better, since surplus and deficiency in original surveys is likely to cause trouble, leaving small un-

sold portions as later surveys are more accurately made, or resulting in overlaps when descriptions for adjoining lots begin at opposite ends of the block, or deficiencies in the last lots sold if the descriptions all begin at one end. In one tract of land with which the author is familiar 600-foot blocks overran quite uniformly 4.84 feet.

273. Finding and marking a city lot. If the description is by lot and block, it will be necessary to refer to the map of the survey mentioned for the data as to the widths of streets, angles in the lines, and positions of lots, before the survey can be begun. The map should show also the character and position of monuments located at the time of the original subdivision of the tract. This it will rarely do if the survey is an old one, and it may not do so even if the survey is a late one.[1]

It will happen often that even though the map shows the positions of the monuments that originally marked the lines, those monuments are not now to be found. In such cases, the best that can be done is to find any monuments having any bearing on the survey in question, and further to note existing lines of permanent improvements that have been long in place, and to endeavor to reconcile these with the figures shown on the map. This work requires the best judgment of the surveyor, for it will be found almost always that the discrepancies are considerable. This is not so true in the centers of very large cities, for here surveys have been gone over and over by most careful workmen, and the lines have become pretty definitely fixed; but in smaller and newer cities there is an endless amount of difficulty. The surveyor should be familiar with local practice and land lines.

Let it be required to locate the four corners of lot 7 of block 61 of Fig. 196, and let it be supposed that monuments exist in Washington Street as shown at *MM*, Fig. 197, but none on the alley line. The surveyor will measure from Linn to Gilbert streets to compare his measurement with that of the recorded plat. He will set points at *a* and *b* at one half and three quarters of the distance from Linn Street. He will set his transit over these points in succession, turn the right angles shown, measure the proper distance to the lot line front and back, and make marks at the corners. The marks will be stakes centered with

[1] Consult Appendix, p. 449.

tacks or marks on buildings or sidewalk. If the lot is occupied by a building or there are obstructions on the side lines, temporary points may be set on the alley line and the back lot line located from these. The surveyor then does such work as may be necessary to locate buildings, sewers, etc. The work is not always so simple as is indicated here, but this book cannot discuss all possibilities. Angles should always be repeated and check measurements made wherever possible.

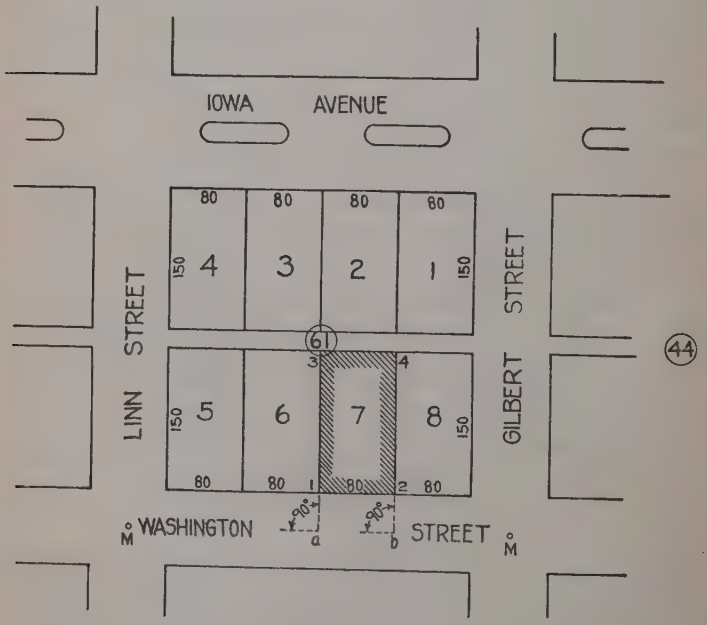


FIG. 197.

274. Discrepancies. If any discrepancies are discovered, they should not be "fudged in" and hidden, but mention should be made of just what is found. If discrepancies are found, the surveyor's work is doubled, for he must be sure the apparent errors are not in his own work. If the discrepancies found are small, no greater than the surveyor would expect to find in repeating his own work — that is, if they are within the

limit of precision required — no note need be made of them. When, for instance, a block that is recorded 500 feet long is found to be 500.04 feet long, and it is required to lay out a lot in any portion of the block, the measurement from one end of the block to locate the lot should be to the recorded measurement as 500.04 is to 500. Such discrepancies may be thus distributed.

If the street lines on both sides of the block are well defined, and the block is found to overrun or fall short of the recorded length, and if the description of the lots sold has been only by lot and block, the error ordinarily should be divided proportionally among the various lots.

If the description is by metes and bounds, it is more difficult to say what to do, and the best the surveyor can do is to report what he finds and ask his employer for further instructions. He must, in no case, locate the lot in what he thinks should be its proper place and attempt to defend such an action as the only proper course. Such procedure has proved a source of much litigation for the owner and loss of reputation to the surveyor.

If the description is by metes and bounds only, the surveyor must determine the starting point by finding the street line on which it lies and the other street line which, with the former, forms the intersection from which the beginning point is located. Frequently this is difficult to do because of lack of monuments or marks. When it is accomplished, the surveyor's work is clear: he follows the description. If he finds discrepancies, he reports what he finds, and then proceeds in accordance with local custom or the orders of the owner. The interpretation of the deed would perhaps usually be the intent of the buyer and seller, if that could be discovered. The surveyor must remember that he has no right to interpret the deed. That is the business of a court. Descriptions in deeds are often inconsistent in themselves and indicate impossible plats.

275. Report. The surveyor should make a report with a scale map of the lot in something the following form:

I hereby certify that I have this day made a survey of Lot 7 Block 61 of the original plat of Iowa City, Iowa, and have found the lines to be as shown on the annexed plat and have set stakes as shown.

Surveyed for _____

At the request of _____

Surveyor.

Elevation grade of curb on east line of lot extended, 805.2
West line, 804.0

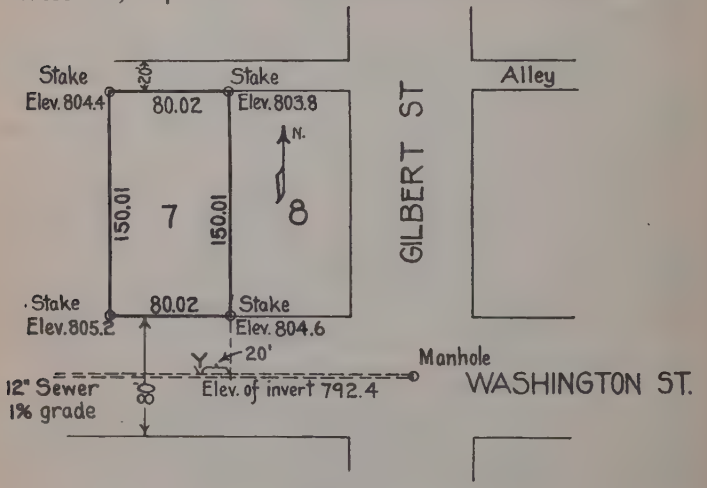


FIG. 198.

Note. If the lot had been described by metes and bounds, the plat should show the distance from the reference point to the point of beginning, as, for instance, from the northwest corner of the intersection of Washington and Gilbert streets to the southeast corner of the lot.

276. The layout of a city. When a surveyor has the opportunity to lay out a new city he should endeavor to impress his clients with the following principles:

1. The general scheme should be rectangular with the lines quartering the cardinal points. This is to bring sunshine on every side of every block and down every street some time of every sunshiny day. There will be no cold and damp north fronts with slowly melting snow and ice in the spring.

2. From the center or business portion of the city wide avenues

should radiate diagonally with the main streets in four directions and at intervals of a half mile to a mile minor avenues should cross these at a right angle. These avenues and the rectangular streets give the shortest practicable distances from one part of the city to another. There should be small parks at the intersections of the avenues.

This layout is suggested by the street arrangement of the City of Washington (Fig. 199), one of the best laid out cities in the world. It was designed by Major L'Enfant of the Engineer

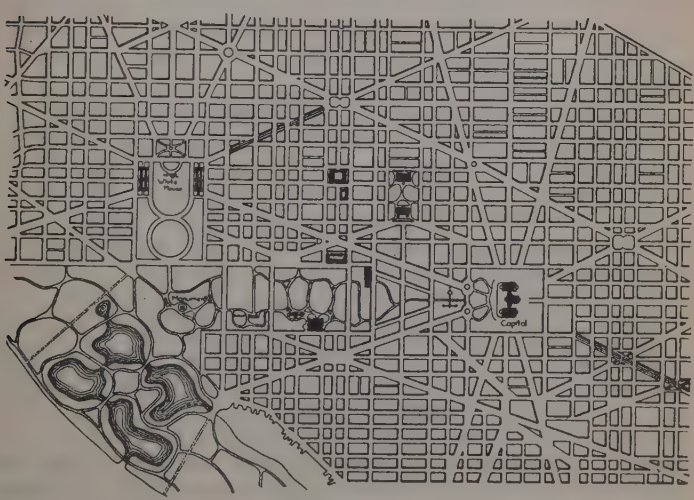


FIG. 199.

Corps of the French Army and was laid out by Andrew Ellicote in 1791. It does not follow rigidly the rule above given but varies from it to conform to the character of the topography. This variation must be introduced in any plan that lies on irregular ground, but need not exist in appreciable degree when the city lies on a flat plain without irregular streams or bodies of water to govern its drainage or street lines.

3. The streets should be wide enough to give plenty of light and air and to accommodate probable travel. In business districts with street cars the streets should be from 120 to 200 feet wide according to the probable size and importance of the city.

No street likely to have car tracks should be less than 100 feet. Streets in residence districts that are to have parkways should be not less than 120 feet wide. Ordinary residence streets may be 80 to 100 feet wide, and minor short courts and ways may be 60 feet. A very common width for residence streets throughout the United States is 60 feet, but it is too narrow.

4. Blocks may be square or twice as long as wide plus a street width. In the business section they should be small, not exceeding 320 feet between property lines, and preferably not over 200 feet by 220 if alleys are used. In the residence districts they may be twice the dimensions of those in the business districts plus a street width to allow for cutting streets through as the business district expands.

In cities laid out in conformity with United States public land lines, as many western cities are, the blocks may be 400 feet from center to center of 80-foot streets. This will make thirteen blocks and two half streets come within the boundaries of a section. In the outer residence districts these blocks may be 400 by 800. Lot frontages should be some aliquot part of the net block length and should also be in multiples of 20 or 25 feet that they may be cut into these smaller sizes as the business district grows.

5. Alleys may be considered necessary in business districts; by some people they are considered desirable in residence districts and in some states subdivisions are required by law to have alleys. Alleys are thought to provide easy means of ingress and egress for the stores of the household, fuel, etc. and to provide suitable location for sewers. If kept clean and wholesome they are harmless, and may be of benefit, but outside of business districts they *never are clean and wholesome*, and often they are not so in the business section. The author believes them to be worse than useless in the residence district.

277. Planning additions. Whenever possible, additions should be so laid out that the streets may be continuations of those in the adjacent subdivided portion of the city. Not enough consideration has been given in the past to this important feature. Unless the ground is very irregular and the addition is a remote suburb, the subdivision should be rectangular. If the ground is irregular and on the extreme outskirts of the city, in a part that will probably never be thickly built up, the additions may be laid out in curved lines to conform

more nearly to the surface of the ground, involving less work in grading streets and lots, and making the tract more beautiful. In such cases the tract will not be cut up into very small lots. No lot should have less than one hundred feet front, and it may be said that such subdivision ordinarily should not be undertaken if the lots are to be less than one acre in area.

In planning a curved subdivision, the streets usually should be located so as to give good drainage to the lots. This will generally mean that the streets will occupy the lower ground. To do this work most satisfactorily, a contour map of the tract is desirable. This is made as described in Chapter XII.

278. Making the survey and map for an addition. The first step in subdividing an addition is to make a complete survey of the entire tract, using the same care as for city work. Many discrepancies of greater or less amounts will be found. The descriptions of the tract are probably from old farm surveys. If no greater discrepancies are found than might be expected from such work, and the corners are well established, no trouble need arise. The new distances and angles are the ones that will be recorded on the map that is to be made. The surveyor will note carefully the intersection of the boundary with the street lines of the adjoining tract, and the directions of those lines. The survey will be very carefully balanced and mapped.

Probably the best method of making the original map is to divide the sheet into squares of, say, one hundred to one thousand units on a side — the number will depend on the size and scale of the map. One of the lines of division will be assumed as the reference meridian, and another, at right angles, as the base parallel. Their intersection may be taken as the origin, or may be given any arbitrary number of hundreds as its coordinates. Each of the corners of the squares will then have for its ordinates quantities expressed in whole hundreds. The coördinates of any point in the survey being determined, it is known at once what square the point falls in, and the point may be measured in from a corner of that square, and be certainly within reach of a single rule length. If the map is small, this method has no advantage over a single meridian on which latitude ordinates are measured and perpendicular to which longitude ordinates are measured.

A system of subdivision will then be planned and drawn on

the map,[1] and then located on the ground. The data for placing the street lines on the ground will not be scaled from the map, but will be computed always. The coördinate system is recommended for this work.

The location on the ground consists in placing monuments at all intersections of street lines. These are placed sometimes at the intersection of center lines and sometimes at the block corners. Neither practice is good. Probably the best place to locate them is in the sidewalk area, at a given offset, say five feet, from the property line. They are less likely to be disturbed here than elsewhere. If the addition is to be laid out on curved lines, the monuments should mark the intersections of street lines and the points on the arcs where the radii change. For methods of locating curves, see Chapter XI.

The monuments should be of stone, durable in quality and about two feet long, with the top dressed to about six inches square and the bottom left rough. The top face should be smooth, with a hole drilled in its center, in which is set with lead a copper bolt with a cross marked in its head. The stone should be set below the disturbing action of frost. A piece of iron pipe may reach to the surface of the ground and terminate with a cast cover. Where stone is costly, concrete, formed of sand and cement, may be used, inside of a piece of stove pipe. The copper bolt should be set as before. Other forms may suggest themselves to the surveyor.

A carefully constructed map finishes the work. This map should show the position of each monument, preferably by giving its coördinates referred to some origin, the relation of the monuments to the property lines, and such further information as is indicated in the Appendix, page 453. If curves are used, the coördinates of the centers and beginning and ending points of the curves should be noted.

279. Subdividing irregular blocks. Diagonal streets will result in some triangular and irregular four-sided blocks. In general, these cannot have alleys, though some of the larger ones may. The subdivision into lots should be such as to give no so-called key lots and to give side lines parallel and at right angles with the street lines so far as possible.

[1] See an excellent paper by J. Stübben, of Cologne, Germany, in Vol. XXIX, "Transactions American Society Civil Engineers."

Thus, the division of the three blocks shown in Figs. 200 and 201 should be the first rather than the second or indeed almost any other division that suggests itself. The interior back lines bisect the angles of the corners. Near the small end of the triangular block the lots may extend from street to street.

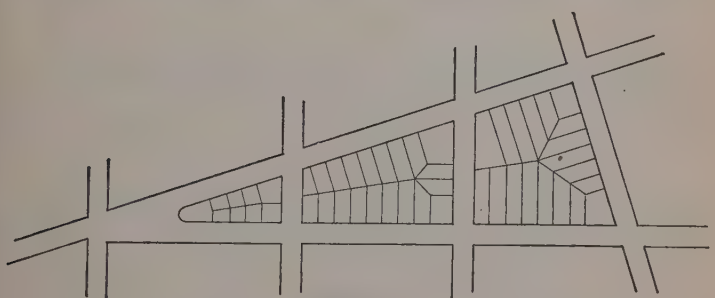


FIG. 200.

When the blocks are of curved subdivisions they should be divided so that the side lines may be as nearly as may be normal to the curves, so that there are no sharp angles and few overlapping portions of lots at the back. Fig. 202, which is approxi-

FIG. 201.

mately that of an actual subdivision, shows an excellent arrangement for a comparatively small plot which must meet rectangular subdivisions on all sides. The contours show how well the streets are planned to fit the drainage and to give sightly building sites.

280. City topographic and cadastral surveys. Cadastral surveys have to do with property lines and technically with property lines in connection with taxation, but in general in this country the term has been used to designate those extensive and precise surveys undertaken by a few large cities for

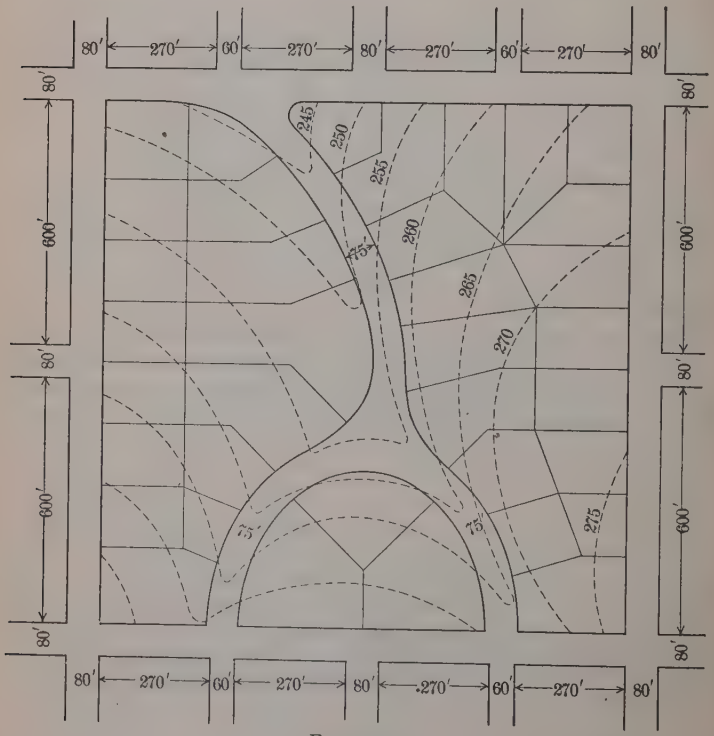


FIG. 202.

the purpose of fixing monuments in various parts of the city and surrounding territory, from which monuments local surveys might be initiated, and mapping streets and property lines with reference to these monuments. In connection with these cadastral surveys careful topographic surveys have been made for

the purpose of securing a map on which the layout of future public improvements might be designed.

The monuments are located by triangulation in which the area is considered a plane, but the work is of the precise character of geodetic surveys, a discussion of which is beyond the province of this book.[1]

The topographic surveys are made by the transit stadia method, which by trial has been found the best of several methods. A light drawing board is carried in the field, or a plane table, and the work sketched at once from the transit stadia notes.

281. Office records. Several forms of records and indexes should be kept in the office; namely, maps, profiles, field books of surveys, monument books, bench mark books and card indexes.

Maps are filed in two ways: by survey number and drawer number, and in block books. The block books are not maps of the surveys made by the individual but are large books on a page of which a city block may be plotted on a scale of from 50 feet to 100 feet to the inch and in which the surveyor will make copies of maps of all the blocks of the city, getting these from the office of the custodian of records, usually the county clerk or recorder. On these block plats should be shown all details of distances, angles, monuments, and relations of lines to those of adjoining blocks, and on them should also be entered from time to time any data of surveys made. If a lot is located on a given date that fact should be noted on the plat, together with any new data obtained by measurement or otherwise during the survey and the names of the owner and person for whom the survey is made.

The blocks should be arranged in the book geographically so that adjoining blocks shall appear in consecutive pages. There should be an index in the front of the book alphabetically by streets or names of tracts or both.

Maps to be filed in drawers will be drawn on tracing cloth and should be of certain uniform sizes. Several sizes may be

[1] For a description of the methods used see Eng. News, 1894–1896, Vol. 32, p. 83, Vol. 33, p. 33, Vol. 34, p. 32, Vol. 36, pp. 307, 312, 364, 400 for the Baltimore Survey; Jour. Assoc. Eng. Soc. 1893, p. 1 for St. Louis Survey; *ibid.* June, 1898, for the Boston Survey, and *ibid.* Oct. 1894, for the Lowell Survey.

used for different purposes and surveys of different extents, but they should all be designed to cut without waste from a standard size of drawing paper used for the largest maps. A sheet 24 × 36 inches will cut into, 18 × 24, 18 × 12, 9 × 12, and 6 × 9 inches. These are good filing sizes, though 24 × 36 does not fit drawers of standard filing cases. Legal cap size is convenient for survey maps made to be filed with deeds. This is $8\frac{1}{2} \times 13$, and cuts from 13 × 17 but not from any of the larger standard sizes. However, roll paper is very generally used in offices having much work and the matter of fitting small sizes is not of great importance. Drawers for filing should be from 1 to 2 inches larger both ways than the maps to be filed and should be not more than $1\frac{1}{2}$ to 2 inches deep, or else too many maps for convenient reference will be put in one drawer. Each drawer should be numbered and each map should be numbered for indexing. Probably each size of map should have a separate series of numbers from 1 on, designated further by a letter or other distinguishing device; thus 6 × 9 maps may be *A*; 9 × 12, *B*, etc. The maps may also be filed by subjects; thus all land surveys will be together, all topographical surveys together, etc. Convenient vertical filing cases are made and preferred by some engineers.

Profiles must be filed in rolls if they are long, but are best made on sheets not exceeding 30 to 36 inches in length and filed flat in drawers. When filed as rolls they should be inclosed in cylindrical tin cases of proper length and filed in racks fitted into cupboards or built up in stalls. The name of the profile in a given case should appear on a paster on the cap of the case and there should be a numerical and letter system for the racks for indexing.

Field Books of surveys should be filed according to subject and number. Lot survey notes should not be put in sewer books, but each kind of survey should have its own separate books. In lot survey books, or private survey books, notes of only one survey should appear on one page, no matter how insignificant the survey, even if it is only a single measurement between two points. Notes should never be taken on pieces of paper, but always in the field book, and everything concerning the survey should appear, including date and hour, names of assistants, condition of the weather, monuments used for reference, points established and method of establishment, and

even pertinent remarks of interested persons having knowledge or claiming knowledge of consequence to the survey. Such notes may be needed at some future time in litigation not expected when the survey is made. The notes should be indexed by owner or employer and by street and blocks.

Monument and bench books should be kept in which nothing but location and description of monuments with reference points, and location, description, and elevation of bench marks is recorded. There should be duplicates of these books for use by different parties, each of which should always carry a monument book and bench book when in the field. Frequently in small places these lists may be printed and pasted into all field books.

Card indexes are the best form of indexes for general office use. Profiles, lot surveys, sewer surveys, topographical surveys, etc., should be indexed separately. It is a good plan to index by subject and by street location, either street or named tract. Under each survey reference should appear separate references to field book and page, map number and drawer, order card and day book or ledger for the record of the charge and payments. To save labor in indexing and without involving a corresponding amount of labor in searching, the subject cards may contain the full information, and the location cards only references to the proper subject card or *vice versa*.

There should be a set of order and cost cards. Each order should be entered on a card when it comes to the office. The assignment of surveyor, date of survey, name of person ordering and for whom ordered, date of report, complete time and cost account, charge, date of bill if made, and of payment.

Complete records such as are indicated in the foregoing paragraphs take time and cost money, which should be charged for in the item of general office expense. The records should never be neglected, as they will become the most valuable single item in the surveyor's office. If possible they should be kept in fireproof vaults.

CHAPTER XV

HYDROGRAPHIC SURVEYING

282. Definition. A hydrographic survey is a survey having to do with any body of water. A topographic survey may be, and frequently is, partly a hydrographic survey.

283. Objects. A hydrographic survey may be undertaken for any one of the following purposes:

1. To determine the topography of a portion of the bed of the sea, a bay, or harbor, or river, in order that it may be mapped for the information of seamen. In this case it is necessary merely to locate the channels, dangerous rocks, and shoals.

2. To determine definitely the configuration of a small portion of the bed of the sea, a bay, harbor, or river, for the purpose of planning works to rest on or in the bed, such as lighthouses, sea walls or docks, bridge piers, etc., or to estimate the quantities of materials to be moved by dredging or blasting, for the improvement of the channels or harbor.

3. To determine the flow of a given river or estuary and the direction of the currents, for the purpose of studying the physics of the stream or for planning waterworks or drainage systems.

4. To secure the necessary information for planning the improvement of marshy shores of a lake or stream by lowering the water level or otherwise.

284. Work of the surveyor. The survey ends with the determination of the required information. The planning of the works and their execution is hydraulic engineering. The laying out of the plan on the ground requires the methods of the surveyor. The work of the surveyor, then, is to make a topographical map of the area to be covered, to compute material moved, to determine cross sections of streams, their velocities, their discharge, the direction of their currents, and the character of their beds, and to lay out projected improvements.

285. General statement of methods. The configuration or topography of the bed of a body of water is determined by sounding; that is, measuring the depth of water. If many points are observed, a contour map of the bottom may be drawn, the water surface being the plane of reference.

The cross section of a stream is determined by taking soundings along a fixed transverse line.

The velocity of flow of a stream is determined by noting how long a float requires to drift a known distance, or by the use of a current meter.

Since the velocity is not the same at all parts of a given cross section, many determinations for velocity must be made before an average for the whole cross section can be obtained.

The discharge of a large stream, like the Mississippi, Missouri, Ohio, or Hudson River, is determined by measuring a cross section and finding the velocity of flow past it. The discharge of a small stream, like any one of the thousands of creeks, is obtained by weir measurements. A weir is a notch in a dam. A dam is constructed across the stream, and a notch is built in it, through which the water flows. From the known size of the notch and the depth of water flowing over it, the flow may be calculated. In common usage the word weir applies to the dam as well as the notch. Insignificant streams may be measured by any simple means that may occur to the surveyor.

The direction of surface currents may be determined by means of floats. Subcurrents are best determined by means of the direction meter.

The character of the bed of a body of water may be ascertained by securing samples in a cup attached to the sounding lead, or by putting tallow in a cavity in the bottom of the lead. Some of the bed will adhere to the tallow. If the character of the bed for some distance below the bottom is required, as when bridge piers are to be founded on firm bottom which is at an unknown depth, various methods are resorted to, according to the depth of water, the soil below, and the difficulties to be overcome. Simple cases may be handled with gaspipe rods driven through the bottom by hammers to a layer of hard material. In other cases drills driven by pile drivers are used, or diamond drills if rock is encountered, and it is necessary to determine its character and depth.

The method of making ordinary weir measurements, and such methods as require the use of surveying instruments and meters, will be described in sufficient detail to enable the student to undertake such work.

SOUNDINGS

286. Making soundings. Deep-sea soundings are made with special elaborate apparatus which will not be described.

Soundings in moderately deep water are made with a weight, known as a lead, attached to a suitable line.

For depths of less than fifteen or twenty feet a pole is best. The lead may be any heavy weight. It is preferably of lead, molded about an iron rod. The form is shown in Fig. 203. The cup at the bottom is for collecting samples of the bottom. There is a leather washer that slides up and down the rod between the bottom of the lead and the top of the cup. This keeps the soil that is collected, in the cup, while the lead is being raised. Very often the cup is omitted, and if samples of the bottom are required, tallow is used, as already described.

FIG. 203.

The line for very deep work may be of wire, but must be used with a reel. For ordinary work, a hemp line or a chain is best. A line must be stretched, but not too much, as it may shrink. It must be tested frequently.

For soundings for navigation charts, the depths are taken in feet to four fathoms, and thereafter in fathoms. The party required to make soundings consists of a leadsman, a recorder, one or more oarsmen, and, when soundings are located from a boat, one or two observers to read the angles. If the boat is located by angles read on shore, there must be one or more observers on shore.

The leadsman casts the lead and announces the depths. For this purpose the line is graduated by suitable tags, so that the depths can be determined readily. In still, deep water the boat may be stopped while the sounding is being taken and recorded, but more often it is not stopped. The leadsman stands in the bow of the boat and throws his lead ahead, so that, as he judges, it will be on the bottom and the line will be vertical when the boat is over it. He will become expert at this after a time.

The recorder notes the number of the sounding and the depth that the leadsman gives him.

The method of using rods will be obvious.

The reference plane for depths is the surface of the water, and as this is continually changing, except in still ponds or lakes, it is necessary to know the stage of the water at the time the soundings are taken. This is done by establishing a gage in the vicinity of the work. The soundings on all navigation charts are referred to mean low tide. The zero of the gage should be set, if possible, at this elevation. The gage may be a graduated board nailed to a pile in a pier in the vicinity of the work and read at hourly intervals during the progress of the sounding. Gages for rivers are frequently inclined, being laid along the shore at right angles to the stream, and the points of equal differences of altitude are determined by leveling. A tide gage that is to remain permanently in place should be self-registering. Such a gage may consist essentially of a float protected by a surrounding house or tube, and attached by suitable mechanism to a pencil that has a motion proportional to the rise and fall of the float. The pencil bears against a piece of graduated paper fastened to a drum that is revolved by clockwork. There will thus be drawn on the paper a profile in which the horizontal units are time, and the vertical units are feet of rise and fall. The stage of the tide for any instant can be read from the profile.

287. Locating the soundings. 1. The soundings may be located by two angles read simultaneously from the opposite ends of a line on shore, the recorder in the boat signaling the observers on shore when a sounding is about to be taken, and again when it is taken.

2. The soundings may be located by establishing two flags on shore in a line nearly normal to the shore line and taking the soundings in line with the range thus formed, the position of the boat being determined by an angle read on shore, or by time intervals. A series of such ranges is laid out, and soundings are taken along each range. The point selected for occupation by the angle-measuring instrument on shore should be so chosen that the angles of intersection with the range lines will be large enough to give good locations. If the shore is narrow and precipitous, the range may be established by placing a flag

on shore and a buoy in the water. The position of the buoy is determined as was described in the first method for locating soundings. The distances between the shore points will be measured for base lines, and the angles will be measured from these base lines. Figure 204 will explain the method. The buoys may be any visible float, anchored in place by any heavy weight attached to the buoy by a rope. The rope must be sufficiently long to permit the buoy to be seen at high tide. At low tide the position of the buoy will not be certain, for obvious reasons. A good kind of buoy is a tapering spar of wood. If it is not readily visible, a flag may be set in the end.

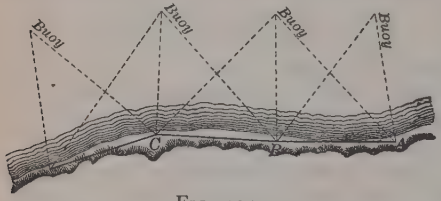


FIG. 204.

3. If many soundings are to be taken at different times on one cross section of a river, they may be located by taking them at the intersection of a series of ranges so laid out as to have their intersections on the required cross section. Figure 205 will explain sufficiently.

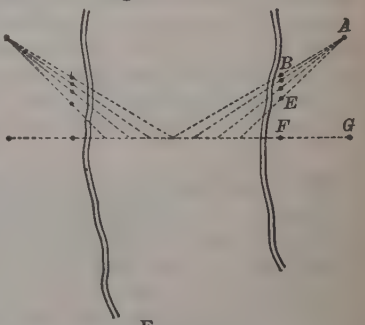


FIG. 205.

4. Again, the soundings may be located by stretching a rope or wire across the channel and taking them at marked intervals along the line. This method is adapted to narrow channels, and is used somewhat in connection with measurement of materials dredged in such channels.

5. On still water, soundings may be located by the transit and stadia when not too far from shore. A thousand feet would be about the limit.

6. Again, they may be located by measuring in the boat, at the time the soundings are taken, two angles to three known points on shore. The angles are measured with a sextant. There are several methods of obtaining from the data thus secured the location of the point where the sounding is taken.

The problem to be solved is known as the "three-point problem."

The most convenient way to plot the point on a map on

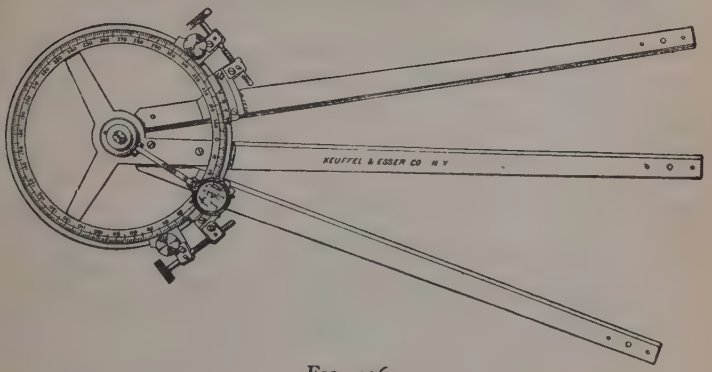


FIG. 206.

which the known shore points have been already plotted, is by the use of a three-arm protractor, shown in Fig. 206. The two measured angles are set on the protractor and the arms are made to coincide with the plotted points. The center then gives the required position of the sounding.

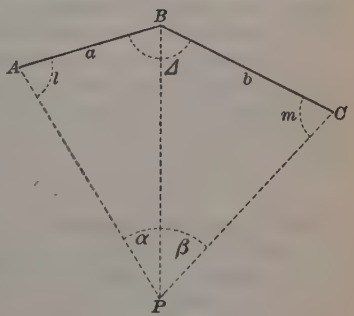


FIG. 207.

If no three-arm protractor is available, draw three lines on a piece of tracing paper so that the two measured angles are included between the lines. Shift the paper over the map till the three lines pass through the three points. The point from which they radiate is now over the position of the sounding, and may be pricked through.[1]

The position of the point may be found algebraically as

[1] These methods fail if the point to be located lies on a circle through the three observed points. Let the student draw a diagram of the conditions and show why the failure. Other methods are described in *Engineering News*, Vol. 68, Sept. 26, 1912, p. 584, and Oct. 31, p. 823.

follows: In Fig. 207, page 391, the known points are A, B, C, determined by the angle $\triangle$, and the sides a and b. The measured angles are α and β, and the required point P. From the triangles APB and CPB there is obtained

$$PB = \frac{a \sin l}{\sin \alpha} = \frac{b \sin m}{\sin \beta}. \quad (1)$$

Also, $$l + m = 360° - (\triangle + \alpha + \beta) = S, \quad (2)$$

whence, $$m = S - l,$$

and $$\sin m = \sin S \cos l - \cos S \sin l. \quad (3)$$

From (1) and (3) determine l, which being found, the triangle APB may be solved and P located. To solve for l, substitute in (1) the value of sin m found in (3), reduce to common denominator, divide by sin l, transpose, and get

$$\cot l = \frac{a \sin \beta + b \sin \alpha \cos S}{b \sin \alpha \sin S} = \cot S \left(\frac{a \sin \beta}{b \sin \alpha \cos S} + 1 \right). \quad (4)$$

288. Occurrence of methods. The sixth method is used almost exclusively in locating soundings in bays, harbors, and off the seacoast. The others are used in connection with river and small lake surveys. Another method by sweeping is used for locating isolated obstructions that would be missed by ordinary methods. A wire sunk at least to the vessel draft depth required is attached to boats and swept through the water. When an obstruction is encountered its position is located.[1]

289. Survey of a harbor, river, etc. In making a complete map of a harbor, coast, or river, a system of triangles of greater or less extent is laid out. The triangulation stations thus locate the prominent points. These points are then used as a basis for locating the soundings. If too far apart, other points are located along shore by running a traverse between triangulation stations. Such a traverse is usually run in any event to determine the detail configuration of the shore line and the topography adjacent thereto. This work is done best with the transit and stadia or with the plane table.

[1] The method is described in the Report of the Chief of Engineers, U.S.A., for 1903, p. 2763, and United States Coast and Geodetic Survey Report for 1905, p. 285.

The map is finished like any topographical map. The contour lines of the bottom are frequently drawn as dotted lines to a depth of four fathoms, and beyond that depth the soundings are given in figures expressing fathoms and quarters. Charts of the various harbors and many portions of the coast of this country are published by the United States Coast and Geodetic Survey, and may be purchased for a small price. Buoys and beacons are located on these charts. They are established to denote shoals and rocks, and are so arranged that in entering a harbor a red buoy with even number is to be passed on the right; a black buoy with odd number is to be passed on the left, and buoys with red and black stripes may be passed on either the right or the left. Buoys in channels are painted with black and white vertical stripes.

Beacons and buoys are different things. A beacon is a permanent fixed signal, usually on a shoal or dangerous rock, while a buoy is a float of some kind, anchored by a chain. It is used to denote either danger or channel.

THE SEXTANT

290. Description. It has been said that when the angles are read in the boat, they are measured with a sextant. The sextant is shown in Fig. 208. It consists of an arc of sixty degrees, which, from the principle of the instrument, gives angles up to one hundred and twenty degrees. *I* is a mirror attached to the movable arm that carries the vernier *V*. The arm is centered under the mirror. This mirror is called the *index glass*. The vernier reads to ten seconds. *H* is another glass, called the *horizon glass*. Its lower portion is a mirror, and its upper portion is unsilvered. *GG* are colored glasses to protect the eye when making observations on the sun. They may be turned back out of the way when not needed.

291. Use. To use the sextant to measure an angle between two terrestrial objects, hold the plane of the arc in the plane of the observer's eye and the two points. The telescope should be directed toward the fainter object. It may be necessary to hold the sextant upside down to do this. Swing the vernier arm till the image of the second point reflected from *I* to *H* to the telescope is seen superimposed on the fainter object seen

directly. Clamp the vernier, bring the images into exact coincidence, using the tangent screws, and read the vernier. The reading is the angle sought. It will be observed that the angle read is not horizontal, unless the distant points are in the same level with the observer. It requires some practice to become expert in the use of the sextant. The eyepieces shown in the figure are of different kinds. There is usually

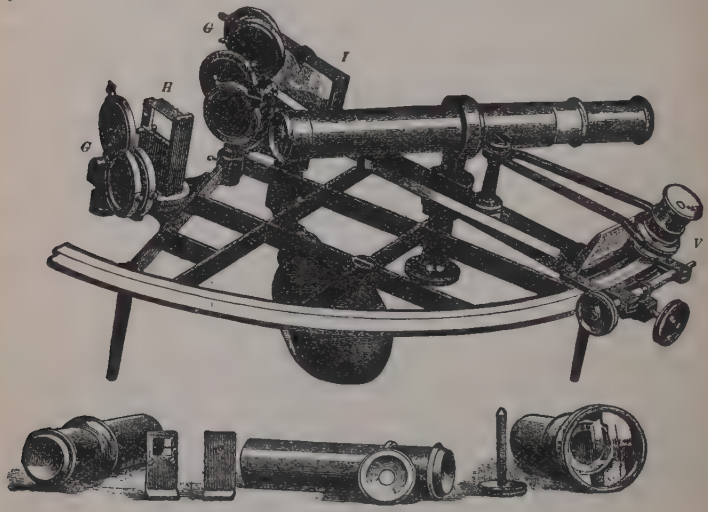


FIG. 208.

one astronomical eyepiece, one for terrestrial work, and one without magnifying power.

The instrument is that used by seamen for observing for latitude and longitude.

292. Theory. The principle on which the sextant is constructed is as follows: If a ray of light is reflected successively from two plane mirrors, the angle between the incident and finally reflected ray is twice the angle of the mirrors.

Referring to Fig. 209, since the angles of incidence and reflection are equal, $i = r$ and $i' = r'$, and from Geometry

$$E = (i + r) - (i' + r') = 2(r - r'),$$

and $$V' = (90° - i') - (90° - r) = (r - r');$$

therefore $E = 2\,V'$, which was to be shown.

To show that the angle read by the vernier on the arc or limb is the angle V', suppose the two mirrors to be parallel and the telescope to be directed to an object infinitely distant, so

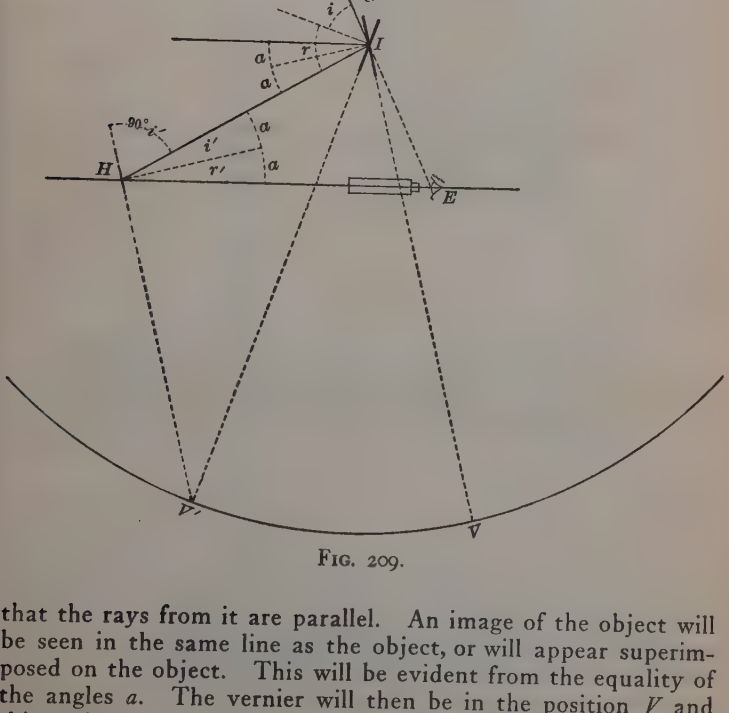


FIG. 209.

that the rays from it are parallel. An image of the object will be seen in the same line as the object, or will appear superimposed on the object. This will be evident from the equality of the angles a. The vernier will then be in the position V and this point of the arc is graduated *zero*. If the vernier is moved now to another point V' for the purpose of getting the reflection of a second point to cover the direct image of the first point, the angle VIV' will equal that at V', the angle

of the mirrors. It is, however, the angle at E that is required, and this is twice the angle through which the vernier has moved. Hence the arc VV', instead of being numbered to give the angle V', is numbered to give at once the angle E. That is, each degree is marked two degrees, etc.

293. Adjustments. There are four adjustments of the sextant:

1. To make the index glass perpendicular to the plane of the limb.
2. To make the horizon glass perpendicular to the limb.
3. To make the line of sight of the telescope parallel to the plane of the limb.
4. To make the vernier read zero when the mirrors are parallel.

1. Bring the vernier to about the middle of the arc. Hold the eye at about the upper G of Fig. 208 and observe the arc near the zero point directly, and the reflected image of the other end of the arc in the index glass. If the glass is perpendicular to the plane of the limb, the reflected and direct portions will seem to form one continuous arc. Adjust the glass, if necessary, by means of the screws at its base. It may be necessary to place slips of thin paper under the base.

2. With the sextant held in a vertical plane, direct the telescope toward a star or other very distant object and note whether the direct and reflected images seem, when the vernier is moved, to separate or overlap laterally. If they do not, the horizon glass is in adjustment. The adjustment is made, when necessary, by the screws of the glass.

3. Place the sextant on a plane surface and direct the telescope to a point not far away. Place the two peep sights shown in Fig. 208 on the extreme ends of the limb and note whether the line of sight through them coincides in elevation with the line of sight of the telescope. If not, adjust the telescope by the screws in its collar. Any other objects of equal altitude will serve as well as the peep sights.

4. Bring the direct and reflected images of a very distant point to coincide, and read the vernier. It should read zero. If it does not, note the error as an index correction to be applied to all angle readings. The error may be corrected by adjusting the horizon glass, but usually this is not done.

MEASURING VELOCITY AND DISCHARGE

294. Position of maximum velocity. The velocity of a stream varies in different portions of a cross section and in

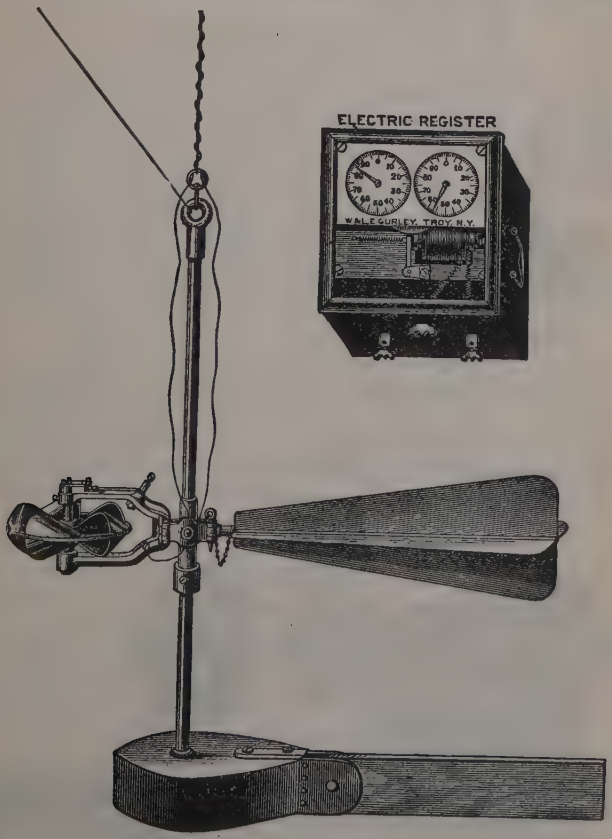


FIG. 210.

different cross sections. The maximum in a cross section is, when there is no wind, at about $\frac{15}{100}$ of the depth in the middle of the channel. The surface velocity may be greater if the

FIG. 211.

wind is favorable. To determine the mean velocity in a given cross section, the velocity in many parts of the section must be found. This is done best with a current meter.

295. Current meters. These are of various patterns. Those shown in Figs. 210–212 are considered good forms. Figure 210 is a meter devised by W. G. Price, United States assistant engineer. There is an electric connection with a register that indicates the number of revolutions of the wheel. The number of revolutions per second is a function of the velocity of the current through the meter.

Figure 211 is a modification of the Price meter, called an audible, or acoustic, meter, and for shallow streams is probably as convenient as any form yet devised. It is so constructed that at each tenth or twentieth revolution a small gong is struck, and the sound is carried through a rubber tube to the ear of the

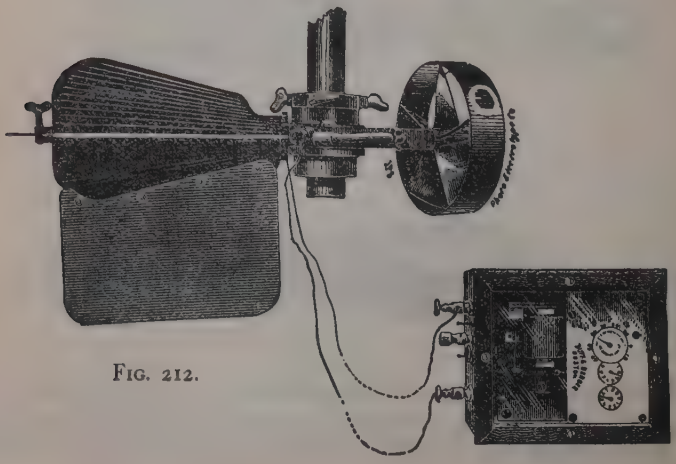


FIG. 212.

observer above in a boat. This meter requires no registering apparatus at all. It is not recommended for deep-sea work, in which a strong, heavy instrument is needed, since it is very light.

Figure 212 is a different form of meter with electric registering attachment.

296. Use of the meter. In use the meter is held in different positions in a cross section whose velocity is required, and in each position the number of revolutions per second is noted. The position of the meter is located by any one of many possible methods that will suggest themselves to the surveyor. The number of revolutions per second bears some relation to the velocity of the current going through the propeller-like wheel of the meter, and the determination of this relation is called *rating the meter*. Every meter must be rated before the velocity of current corresponding to an observed velocity of wheel can be told.

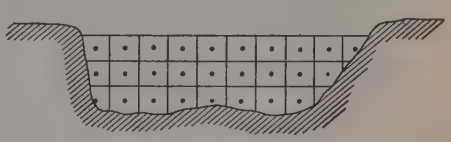


FIG. 213.

The velocity being determined for a given portion of the cross section it is multiplied by the area of the portion observed to get the discharge past the portion. The cross section is divided usually into vertical strips and the mean velocity of each strip determined by getting a meter record at the middle of each unit of depth and averaging. Thus, in the section shown in Fig. 213, the velocities would be taken at each dot, and the noted velocity multiplied by the area of the small figure near whose center or center of gravity the velocity is taken. The resulting products in a vertical strip are added, and the sum is divided by the total area of the vertical strip for the mean velocity of that strip. But the mean velocity of the strip may not be required. If it is the discharge that is wanted, the sum of all the small products in the entire cross section would be the discharge of the stream. Then, if the mean velocity is required, it is found by dividing the total discharge by the area. The small areas in a given strip will all be equal except near the bank or bottom.

Instead of noting the velocities at each unit of depth of a vertical strip, the velocity at six tenths the total depth, or

the average of the velocities at two tenths and eight tenths the depth may be used for mean velocity. The latter method is used generally by the United States Geological Survey. Experiment seems to have indicated that these methods give good results, but the division into small areas is recommended.

A large stream would be divided into wider vertical strips than a small one. The vertical strips near the shore may be

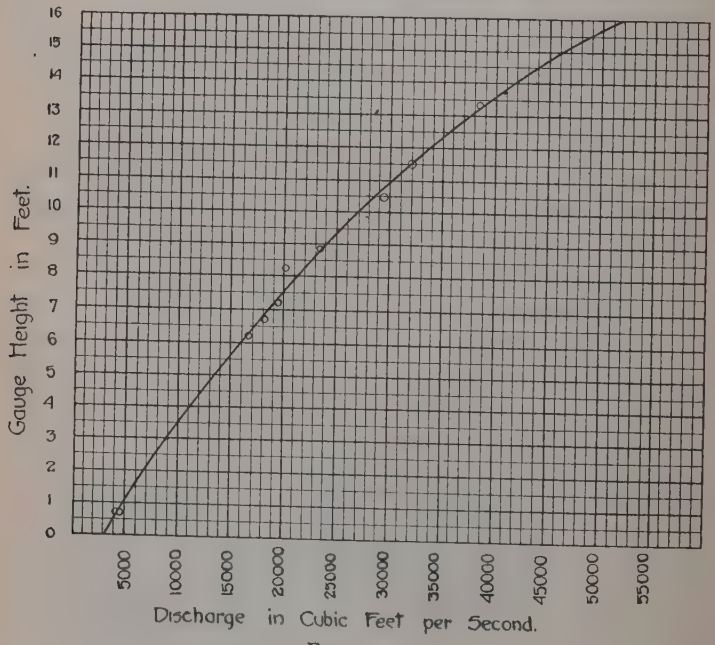


FIG. 214.

narrower than those in the channel if the configuration seems to demand it. The width of each strip is arbitrarily assumed, and they may all be equal or unequal as the configuration of the stream bottom determines. The depth of each strip is determined by sounding. The strip boundaries on the surface may be marked by a tagged line, or on a bridge, if the section is at a bridge crossing, as many are. Such a section is, however, often

undesirable if there are piers, because of irregular eddies near the piers.

The Geological Survey in its work frequently stretches a cable across the stream, and the observer with his meter is suspended from the cable in a carrier and drawn across, stopping at the several stations, marked by tags, to make his observations.

A rating curve or table for a river is made by observing its discharge at a given section for various stages — high and low water — plotting the results on cross-section paper, and drawing a curve through the plotted points. One axis will be gage height of the stream as indicated on a gage board, and the other axis will be second feet (cubic feet per second). When such a diagram has been made it will be necessary only to observe the stage of the river at any time as shown by the gage board and to enter the diagram, or table made from it, at the proper gage point to read the discharge. Figure 214[1] is such a rating curve.

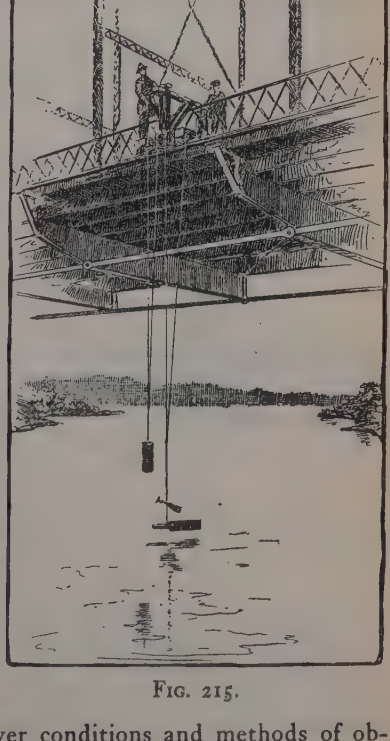


FIG. 215.

A good idea of general river conditions and methods of observing velocity may be had from the following figures.[2] Figure 215 shows a scheme for sounding, sampling, and making meter measurements from a bridge. Figure 216 shows apparatus for meter measurements from a suspended cable. Figure 217 shows a cross section at the New Palz gaging station on the

[1] From Mead's "Water Power Engineering."

[2] Taken from U. S. Geological Survey Water Supply and Irrigation Paper, No. 76.

Walkill River at New Palz, N.Y., with isovels (curves of equal velocity), mean velocity, position, and curves of velocities in horizontal planes of different depths.[1]

Figure 218 shows the effect on the isovels of a smooth ice covering at the same New Palz station.[2]

297. Rating the meter. The meter is rated by moving it through still water at a known rate and noting the revolutions per second. By moving it at different velocities it will become apparent that the velocity is not strictly proportional to the speed of the wheel, but bears a relation that must be expressed by an equation. From a number of observations at different velocities, a diagram may be drawn that will give the velocity corresponding to any observed speed of the wheel. This is done as follows: The meter is moved over a known distance and a stop watch is used to determine the time occupied. The distance divided by the time in seconds gives the velocity in feet per second. The number of revolutions made during the time is recorded, and this divided by the time gives the number of revolutions per second. The following observations were made by Mr. Price, all on a distance of two hundred feet.

FIG. 216.

In the table, R is the whole number of revolutions in the two hundred feet, T the whole time, r the revolutions per second, and v the velocity per second.

Two axes are now drawn at right angles. On one of these axes are laid off the several values of r, and parallel to the

[1] Let the student construct curves of velocities in vertical planes, taking the data from the figure. In particular let the 90-foot plane be used.

[2] Let the student construct a vertical plane velocity curve for the 90-foot plane and compare with that of the same plane without ice covering.

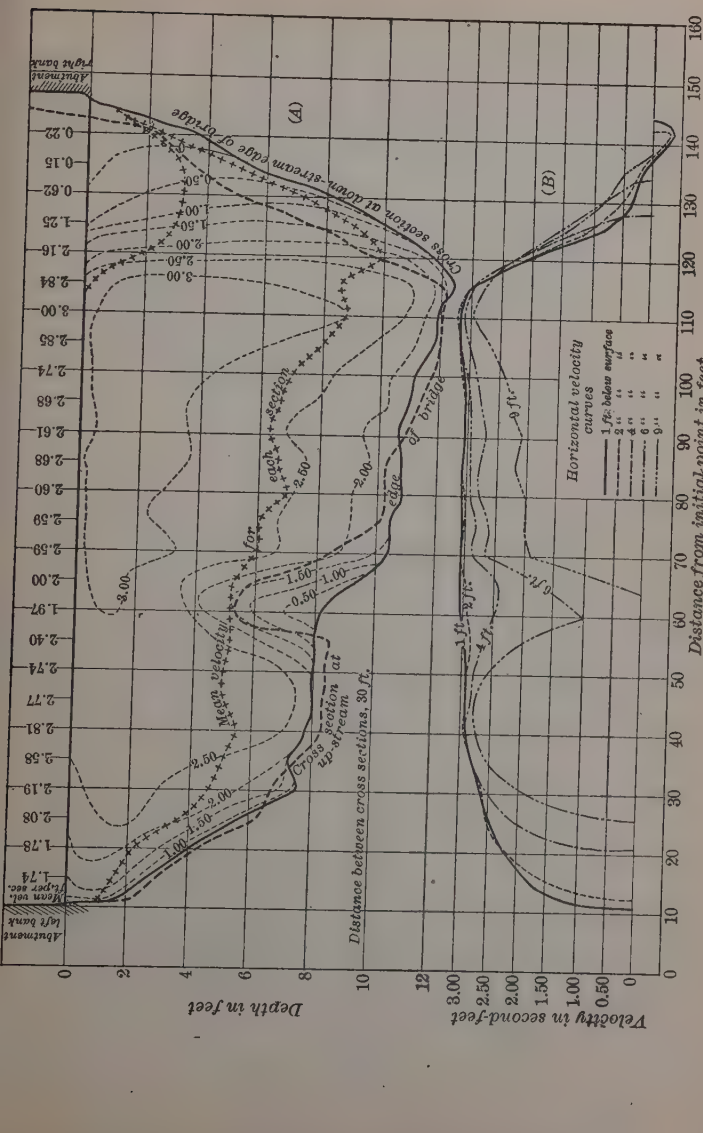

Abutment left bank
Abutment right bank
Cross section at down-stream edge of bridge
Mean velocity for each section
Cross section at up-stream edge of bridge
Distance between cross sections, 30 ft.
Horizontal velocity curves
1 ft. below surface
2 " " "
4 " " "
6 " " "
9 " " "
(A)
(B)
Depth in feet
Velocity in second-feet
Distance from initial point in feet

FIG. 217.

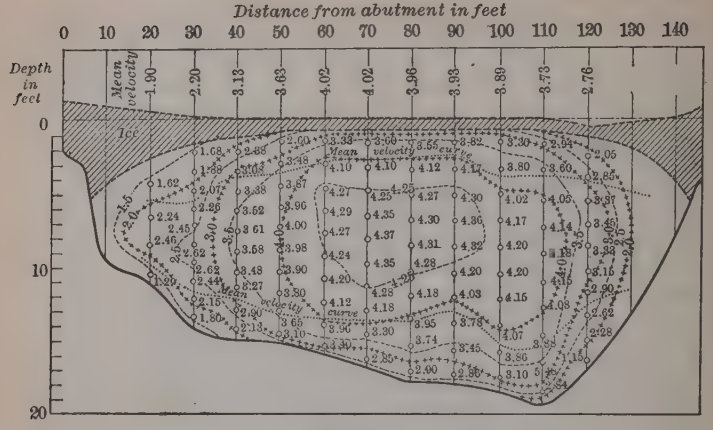


FIG. 218.

other are laid off the corresponding values of v. In Fig. 219, the horizontal axis is that of r, and the vertical axis is the velocity axis. In the first observation, r is 1.886 and v is 3.774. Assuming suitable scales, the observations are plotted on axes of revolutions per second and velocities in feet per second. If cross-section paper is used, the work will be facilitated. Each observation being plotted, it is observed that they all fall nearly on a straight line. It may be assumed that they should all fall

No.	R	T	r	v
1	100	53	1.886	3.774
2	101	44	2.295	4.544
3	101	41	2.464	4.878
4	96	124	0.774	1.613
5	94	152	0.618	1.316
6	90	193	0.466	1.036
7	91	181	0.503	1.105
8	103	28	3.678	7.142
9	100	53	1.886	3.774
10	98	73	1.342	2.740
11	193	29	3.552	6.896

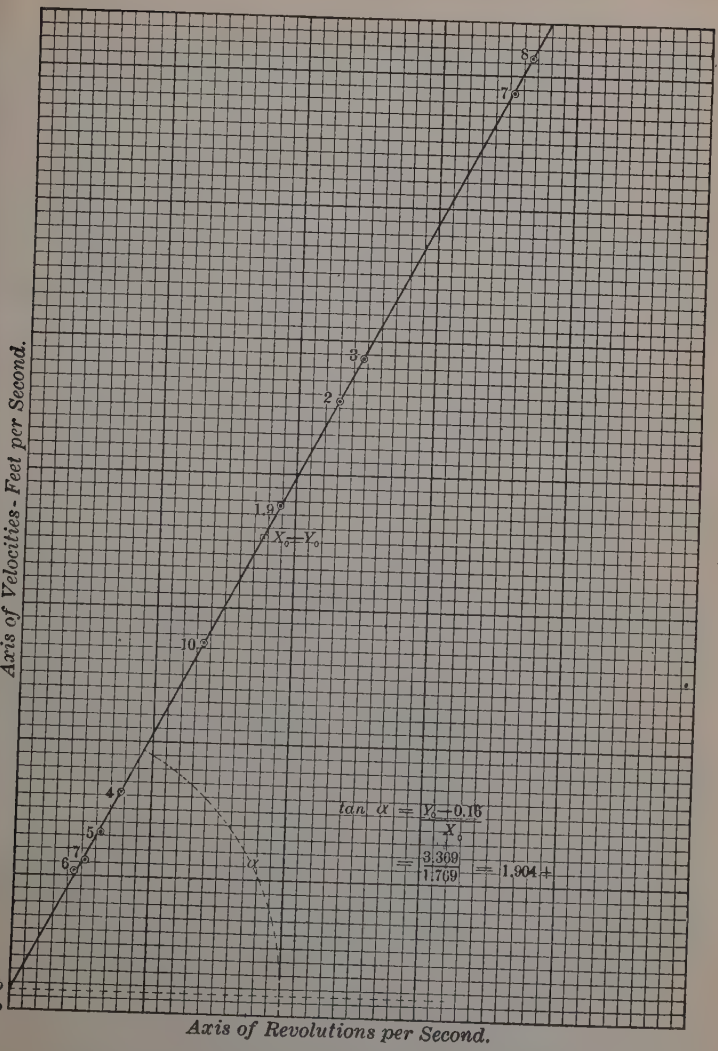

8
7
3
2
1.9
$X_0 - Y_0$
10
4
5
7
6
α
b
o
$\tan \alpha = \frac{Y_0 - 0.16}{X_0} = \frac{3.309}{1.769} = 1.904+$
Axis of Velocities - Feet per Second.
Axis of Revolutions per Second.

FIG. 219.

on a straight line and that they do not because of small errors in the observations. A piece of fine thread is stretched and swung around till it seems to the eye to average the plotted points. This line is that on which it is most probable that the observations should all fall, and, hence, if it is drawn and the meter is held in a running stream and the revolutions per second are noted, the corresponding velocity may be determined by laying off the observed r on its axis and measuring the corresponding v up to the line. If a horizontal line is drawn on the diagram through b, where the line cuts the axis of v, it will be seen that for any value of r the corresponding value of v is $r \tan a + bo$. If b represents bo, we may write

$$v = r \tan a + b.$$

From the values of r and v shown at X_0 and Y_0 in the example given

$$v = 1.904\, r + 0.16.$$

In its general form this equation is

$$v = ar + b,$$

and this is known in Analytical Geometry as an equation of a straight line. This equation is the general equation of the relation between v and r. The rating of the meter then consists in finding values for a and b from observations that give v and r. Since there are two unknown quantities, two independent observations should be sufficient; but since no observation can be perfect, many are taken, and mean values are determined for a and b. This may be done analytically by the method of Least Squares.

If the plotted mean line is not quite straight but has one general slope near the bottom and another higher up, then rather than attempt to derive a complicated equation to fit the whole curve, two straight line equations may be found, one fitting the lower part of the curve and one the upper part.

Example

The following observations[1] for rating a Price acoustic current meter were made by Mr. W. G. Price, June 21, 1895, by the method described by

[1] Kindly furnished to the author by Mr. Price.

him in *Engineering News*, Jan. 10, 1895. The student will use the first, fourth, fifth, and sixth columns. The second and third columns will be understood by a reference to the article in *Engineering News*. Get the equation for rating the meter.

RATING OF PRICE ACOUSTIC CURRENT METER No. 4, JUNE 21, 1895

No.	1ST WIRE Feet	2D WIRE Feet	DISTANCE Feet	REV.	TIME Seconds
1	31.09	79.72	48.63	20	24.4
2	24.84	74.28	48.44	20	22.4
3	37.50	86.32	48.82	20	35.0
4	32.76	82.50	49.74	20	56.0
5	26.21	76.54	50.33	20	56.0
6	13.61	64.21	50.60	20	99.0
7	27.55	77.78	58.23	20	94.6
8	13.45	62.31	48.86	20	25.0
9	23.28	71.27	48.09	20	15.8
10	32.22	80.35	48.13	20	10.2
11	27.84	76.01	48.17	20	7.4
12	32.08	103.95	71.87	30	20.4
13	39.99	87.98	47.99	20	6.0
14	35.08	82.68	47.60	20	5.8
15	24.20	72.84	48.64?	20	6.0
16	38.86	86.39	47.53	20	5.8
17	24.67	72.47	47.80	20	5.2
18	39.13	86.85	47.72	20	5.2
19	35.74	83.51	47.77+	20	11.2
20	20.15	67.97	47.82+	20	11.2
21	30.92	79.17	48.25	20	13.4
22	21.86	70.30	48.44	20	11.6
23	23.67	72.09	48.42+	20	19.4
24	14.47	62.71	48.24+	20	20.0
25	39.27	88.65	49.38	20	32.4
26	46.18	85.82	49.64	20	31.4
27	40.82	90.57	49.75	20	37.4
28	26.14	101.15	75.01	30	64.4

"The center of the meter wheel was 2.1 feet below the water surface. In Nos. 19, 20, 23, and 24 I felt sure the time interval used to start the watch and stop the first wire from passing out, after I had heard the click of the meter, was longer than the time consumed for the same purpose at the end of the run. I therefore put a + mark to indicate that the distance was too

small. No. 15 was doubtful, as the skiff rocked during the trip. The six columns are an exact copy of the field notes."

298. Rod floats. A very good method of obtaining velocities, when a current meter is not available, is to observe the velocities of rod floats in various parts of the stream. For this purpose two ranges are established on shore, one above and one below the section in which it is desired to measure the velocity. The ranges are laid out normal to the stream. A transit is placed on each range, and the rod is started just above the upper range. The transit man on the upper range signals the lower observer when the rod is about to cross the upper range, and just as it crosses, and the lower observer reads an angle to the rod at the instant it crosses the upper range. The operation is repeated when the rod crosses the lower range, the lower observer making the signals, and the upper observer reading the angle to the rod. Each notes the time of crossing the range on which he is stationed. In this way, the path of the rod and the time it takes to travel that path are known. The rod may be of wood, or it may be a long cylindrical tin can. In either case it should be weighted so as to remain vertical and just clear the bottom. The velocity of the float will then be approximately that of the vertical filament of the stream in which it floated. The immersion of the rod should be about nine tenths of the depth of the stream. If this rule is observed, the mean velocity in the vertical filament will be given by $V_m = V_0\ [1 - 0.116\ (\sqrt{D} - 0.1)]$ [1] in which V_0 is the observed velocity of the rod, and D is the ratio of the depth of the water below the bottom of the rod to the total depth of the water.

In small streams, wires or ropes graduated with tags may be stretched across the stream at the upper and lower ends of the stretch of river to be observed, and the time of passing of the float under each wire or rope may be observed.

By observing the velocities in many vertical filaments, the mean velocity of the section may be determined. It would not be a mean of the several observed velocities unless all "filaments" had equal areas. The mean velocity is that which multiplied by the area will give the discharge; hence it is $\frac{\text{Discharge}}{\text{Area}}$. Rods are much better adapted to measurements in

[1] Francis' "Lowell Hydraulics."

smooth-bottomed and sided flumes than in irregular streams. When used in such flumes the results obtained are quite accurate.

299. The discharge. To get the discharge, simply multiply the mean velocity of each filament by its cross-sectional area, and add the products. If the velocity is determined by meter, the work need be done in but one cross section; if by rod floats, it will usually be best to measure two or more sections, preferably at the quarter points of the stretch, from which to get a mean section.[1]

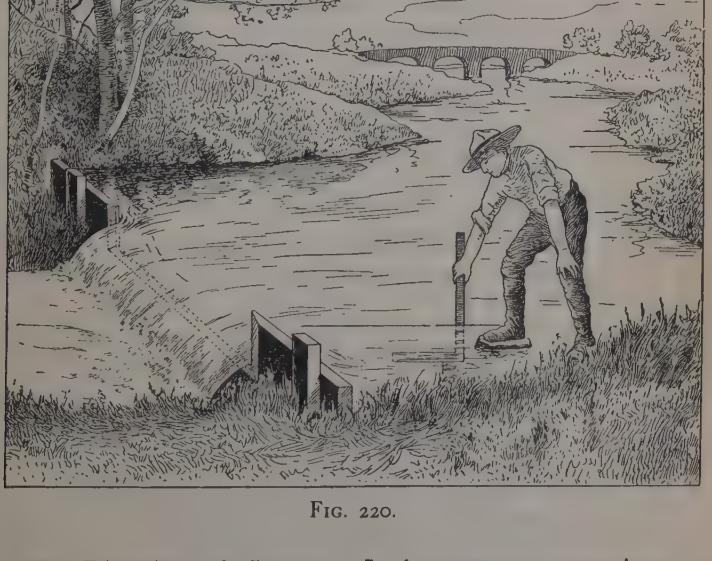


FIG. 220.

300. Direction of Current. It is necessary sometimes to know the direction of both surface and subsurface currents. The necessity may arise in the determination of the proper place to discharge sewage, or in surveys for the improvement of harbors and the approaches thereto. The surface currents may be determined by watching floats. The subsurface currents are

[1] For full details of methods of making stream measurements and gagings on a large scale see Mead's "Water Power Engineering" or Hoyt and Grover's "River Discharge."

best found by a direction meter. This is a form of meter that not only gives velocities, but also shows the direction of the current in which it is placed. It was devised by Messrs. Ritchie and Haskell. It is said to be one of the best forms of meters for measuring currents alone.

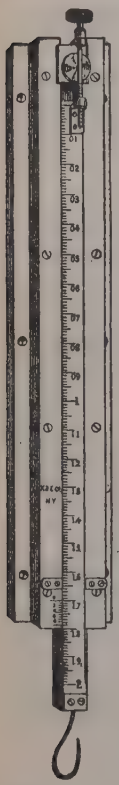


Fig. 221.

301. Weir measurements. Only general field methods likely to be needed by the surveyor will be mentioned. For a small stream a dam is constructed with a rectangular notched board near its middle, the notch being large enough to pass the entire flow of the stream (Fig. 220). If possible, the dam should be built at a narrowed spot below a wide basin, so that there will be little velocity of approach to the weir. It may be necessary to build the weir of several planks and to make it strong. On the shore a little distance upstream from the weir a stake is driven, or a nail is driven into a stump, marking the level of the crest of the weir, which is determined with great care by ordinary leveling operations.

A hook gage, Fig. 221, may be used where many observations and great care are required, but it is more particularly a laboratory instrument. It is placed in a vertical box in the water so that the surface of the water may be still; the height of the hook above the weir crest for a zero scale reading is determined by leveling; the hook is raised or lowered till the point is just in the surface of the water, and the scale is read.

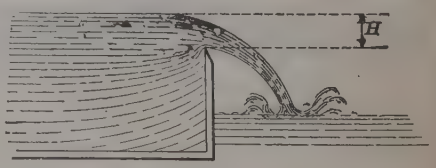


Fig. 222.

The crest of the weir is the bottom of the notch. It should have a sharp edge, preferably a piece of plate or strap iron filed to an edge and set flush with the face of the dam. If for temporary use and there is little or no floating drift, the edge of the plank may be beveled off with a sharp enough bevel so that the water

will fall over clear of the plank as shown in Fig. 222. When the weir is completed, the height of the water over the crest is noted and inserted in the following formula in place of the H:

$$Q = 3.33(l - 0.2\,H)\,H^{\frac{3}{2}},^{1} \qquad (1)$$

in which l is the length of the crest in feet, and Q is the discharge in cubic feet per second. This formula assumes that there are "end contractions"; that is, that the pond extends laterally beyond the edges of the weir and the water curves in at the ends of the weir in passing through, as shown in Fig. 220. If the weir be simply a board across the bottom of a flume so that the water would flow straight through without end contractions the formula is

$$Q = 3.33\,l\,H^{\frac{3}{2}}, \qquad (2)$$

and if there is one end contraction and one smooth side, $0.1\,H$ is used instead of $0.2\,H$ in Equation (1).

To allow for velocity of approach if this is considerable, the discharge is first found by one of the foregoing formulas, and divided by the cross-sectional area of the approaching stream to get an approximate velocity of approach v which is converted into head $h = \frac{v^2}{2g}$ and used in a modified formula

$$Q = 3.33\,l\,[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}] \qquad (3)$$

without end contractions, or

$$Q = 3.33\,(l - 0.2\,H)\,[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}] \qquad (4)$$

with end contractions. Judgment must be used in determining the area for v. If necessary, from the new Q a new v will be found and a more precise Q, etc. Tables of discharge for one foot length of weir and various heads are published, and a limited table will be found at page 502. There are other formulas than those of Mr. Francis, but none simpler nor likely to give better results with ordinary field methods. For minute accuracy such as would be required in laboratory experiments there are certain restrictions to be observed as to relative length of crest to height on the weir, etc., which are fully discussed in any good work on Hydraulics; but these restrictions cannot be wholly observed in ordinary field

[1] "Lowell Hydraulic Experiments," J. B. Francis.

work. For work in which minute accuracy is of great importance they should be studied and observed.

An entire dam in a river is often used as a weir in estimating the discharge. A good deal depends on the form of the dam, the height of the dam, whether the fall is free or obstructed by back water, and whether or not air has free access beneath the falling sheet of water, and the depth of the water on the upstream side.

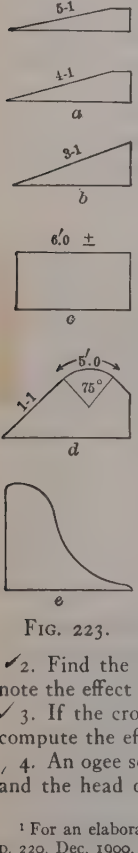


Fig. 223.

For approximate estimates a few standard forms of dams are shown in the figures (224, *a*, *b*, *c*, *d*, *e*), together with estimated variations in discharge from that over a sharp-crested weir.[1] The direction of flow is from left to right.

(*a*) Below 3 feet head about the same as for sharp-crested weirs; above 3 feet from $\frac{6}{7}$ to $\frac{7}{8}$ the discharge for equal heads on sharp crest.

(*b*) About $\frac{1}{3}$ more than over a vertical face sharp crest.

(*c*) About $\frac{1}{4}$ less than over a vertical face sharp crest.

(*d*) About the same as over a sharp crest up to heads of $1\frac{1}{2}$ feet. Thereafter a gradual increase to $\frac{1}{7}$ more than over a sharp crest at 5 feet head.

(*e*) From slightly more at a few inches head to $\frac{1}{3}$ more at heads above one foot than over a sharp-crested weir.

Examples

1. Find the discharge through an 8-foot weir under a head of 1.4 feet without end contractions.

2. Find the flow for the same conditions but with end contractions, and note the effect of the contractions.

3. If the cross section of the stream above the weir is 100 square feet, compute the effect of velocity of approach on the result of Example 1.

4. An ogee sectioned dam like the last section in Fig. 223 is 300 feet long, and the head on it is 2 feet. What is the discharge?

[1] For an elaborate discussion of this subject see Transactions of Am. Soc. C. E., Vol. 44, p. 220, Dec. 1900. The results here given are derived from that discussion.

CHAPTER XVI

MINE SURVEYING

SURFACE SURVEYS

302. Coal mines. The surface surveys in connection with coal mining operations consist in making land surveys of the property that may be owned by the mining company, and locating all shafts, tunnel openings, buildings, railways, roads, and other structures. In addition to this it is well, if the area owned is extensive, to make a complete topographical survey of the property. Frequently, a mining company mines coal under the lands of many small holders and pays a royalty for the coal so mined. It is necessary therefore to know the positions of the land lines of all owners under whose land it is probable that operations may extend. The location of these lines is best performed by careful stadia surveys — reading the stadia to the smallest possible unit. (There may be cases of sufficient importance to warrant most careful transit and tape work.) The topographical survey may be carried on at the same time. Indeed, the location of the land lines becomes merely an incident in the topographical survey. The positions of the corners are determined by computing their latitudes and longitudes.

Among other points that will be located in the topographical survey, or land survey, will be the positions of exploration drill holes that have been put down in advance of mining operations for the purpose of locating veins, or beds, as they are frequently called. The entire survey may or may not be hung on a system of triangles. If it is of any considerable extent, it is best to reference it thus for the sake of the checks it gives on the field work, even though not more than two or three triangles are used. The whole work, except the side shots, is best plotted by latitudes and longitudes, these being computed with sufficient accuracy for this purpose by the aid of a trigonometer. The side shots, except those to land corners, may be plotted with a

protractor. The surface map should show all outcroppings of mineral.

303. Metal mines. Surface surveys for metal mines may be of the same character as those for coal mines, but in the United States there are certain laws governing the size and form of mining claims that are located on public land, and the method of surveying them.

304. Form of mining claim. By the provisions of these laws, a mining claim may not exceed fifteen hundred feet in length along the vein nor six hundred feet in width (three hundred feet each side of the vein). It may be of any length or width less than these limits, according to local custom; but no rule may be made that will compel the claim to be less than twenty-five feet on each side of the vein. The side lines need not be straight or parallel, but the end lines must be both. The miner has the right to follow the vein he has discovered into the ground to any depth, even should it depart so much from the vertical as to pass beyond the vertical planes through the side lines, but he may not mine beyond the vertical planes through the end lines.

When claims intersect, the subsequently located claim is said to conflict with the previous one and carries with it only the area that is not in conflict. The foregoing rules apply to what are called lode claims.

A *lode* is the name sometimes given to a vein of ore.

A *placer* claim is a claim taken for the purpose of obtaining gold or silver or other metal from the surface materials, as in washing gold from a river bed, or from gravel, in the mines worked by hydraulic process in California. Such a claim may not exceed twenty acres, and, if located on surveyed lands, its lines must conform to the legal subdivisions. A placer claim carries no right to mine beyond the vertical planes through its bounding lines. It carries the right to mine a vein discovered within its boundaries after the claim is located, but not if the vein is known to exist when the claim is located, unless specially mentioned.

305. Surveying the claim. When a miner has found a vein of ore, he has a claim staked out on the ground, conforming to the legal requirements, using a width and length that he thinks

best. He also posts the notices required by law. This survey may be made by any surveyor. After the miner has expended five hundred dollars in working his claim he is entitled to have it deeded to him by the government. This deed is called a *patent* and the claim is said to be *patented.* For the purpose of this patent, a survey is made from which to write a description. This final survey, on which the deed to the miner is based, must be made by a United States officer, known as a United States deputy mineral surveyor. Such officers are under bonds of ten thousand dollars, and are required to pass an examination before receiving their licenses. They make the final survey and tie it to a corner of the public surveys or to a special mineral survey monument, and return to the surveyor general of the state the notes, with an affidavit that the required five hundred dollars have been expended on the mine.

The form of field notes and the instructions issued to United States deputy mineral surveyors by the surveyor generals of the various mining states may be had on application to the surveyor general's office. They should be in the possession of any one having to do with United States mineral land surveys.

306. Surface monuments. The surface monuments of a mine that is to be worked for any length of time should be of a permanent character. At least two of them should be located near the entrance to the mine, and these two should be particularly permanent, for use in connecting the underground and surface surveys. These may be auxiliary monuments located for the express purpose of connecting the two surveys. They may be built of stone or brick, with a properly centered copper bolt to mark the exact spot. The bolt may be removable for the purpose of substituting a signal. These monuments must be constructed with a view to being proof against disturbance by frost, or travel, or by the operations about the mine. A true north and south line permanently monumented is perhaps the best base for all surveys.

UNDERGROUND SURVEYS

307. General statement. Underground surveys consist of traversing, leveling, and sometimes measuring volumes. The necessary traverses are run to determine the location of the

various portions of the mine, so that a map or a plan of the mine can be made. Leveling is done to determine the relative elevations of the different parts of the mine, the grade of the tunnels or drifts, the direction and amount of dip of the vein, etc. The quantity of ore mined is sometimes determined by measuring the volume excavated in the mine, and the measurement of the quantity of ore " in sight " is usually by the volumetric method. The ore mined is usually measured by weight after it is brought out of the mine.

The only differences between underground and surface surveys are due to the difficulties encountered in working in dark, cramped passages. These necessitate certain modifications in surface methods to make them applicable to underground work.

308. Definitions. The following are a few definitions of technical terms used in mining.[1] There are many others that the mining surveyor must know.

Shaft. A pit sunk from the surface.

Adit. A nearly horizontal passage from the surface, by which a mine is entered and through which the water that collects in it is removed. In the United States an adit is usually called a tunnel, though the latter, strictly speaking, passes entirely through a hill, and is open at both ends.

Drift. A horizontal passage underground. A drift follows the vein, as distinguished from a cross-cut, which intersects it, or a level or gallery, which may do either.

Level. A horizontal passage or drift into or in a mine. It is customary to work mines by levels at regular intervals in depth, numbered in their order below the adit or drainage level.

Cross-cut. A level driven across the course of a vein.

Winze. An interior shaft, usually connecting two levels.

Strike. The direction[2] of a horizontal line, drawn in the middle plane of a vein or stratum, not horizontal.

Dip. The inclination of a vein or stratum below the horizontal. The dip is necessarily at right angles[3] with the strike or course, and its inclination is steeper than that of any other line drawn in the plane of the vein or stratum.

[1] Taken from "A Glossary of Mining and Metallurgical Terms" by R. W. Raymond, Mining Engineer.

[2] Azimuth.

[3] That is, the direction of the vertical plane in which the amount of the dip is measured is at right angles.

Pitch. The inclination of a vein, or of the longer axis of an ore body.

Incline or *Slope.* A shaft not vertical; usually on the dip of a vein.

Stope. To excavate ore in a vein by driving horizontally upon it a series of workings, one immediately over the other, or *vice versa.* Each horizontal working is called a stope (probably a corruption of step), because when a number of them are in progress, each working face being a little in advance of the next above or below, the whole face under attack assumes the shape of a flight of steps. The process is called *overhand* stoping, when the first stope is begun at a lower corner of the body of ore to be moved, and, after it has advanced a convenient distance, the next is commenced above it, and so on. When the first stope begins at an upper corner, and the succeeding ones are below it, it is *underhand* stoping. The term "stoping" is applied loosely to any subterranean extraction of ore except that which is incidentally performed in sinking shafts, driving levels, etc., for the opening of the mine.

309. Location and form of station marks. In underground work, stations cannot be stakes driven in the floor of the mine. The reasons are obvious. Where the roof is solid rock, holes may be drilled in it and wooden plugs inserted, in which may be driven a nail or tack. A good form is a horseshoe nail with flattened head, in which a triangular hole is cut with an apex of the triangle toward the head end of the nail. A nail of this description, called a *spad*, is on the market, and is quite generally used in many portions of the country in preference to all other devices. A plumb line suspended from such a nail will always hang from the same point. A small round opening about one eighth inch in diameter is preferred by some. When the roof is soft and the drift timbered, temporary stations may consist of nails driven in the timbers. Probably it is better in such cases to reference the station by two points on the side of the drift. If the two points are properly located, and the distance from the vertical through the station to each of them is recorded, the station may be recovered at any time.

When about to occupy a station, transfer it to the floor with a plumb line and set a temporary mark consisting of a piece of slate with a cross scratched on it, or a small metallic cone carried

for the purpose. Some engineers prefer to center the transit under the roof point, using the center mark on the telescope. Stations underground should be plainly marked. White paint is good if kept bright. The stations in a mine should be numbered consecutively, and the same number should not be used twice. If there are different levels, the stations may be numbered 100, 101, etc., on the first level, 200, 201, etc., on the second level, and so on. They should be numbered at the station points as well as in the notebook. Probably as good a way as any to do this is to drive round-headed nails or tacks in the timbering near the station, locating them all in the same position relative to the station. The tacks may be driven in a small board or plug fastened to the side of the drift near the station. The nails are arranged to give the number of the station, thus for 324 and for 302, a washer being used for the 0. Practically this method was followed in the New Almaden mine in California. The nail heads can be felt and read, even though not clearly visible.

As the mine is extended and new stations are added, they should receive their proper numbers and be permanently located. There is no branch of surveying where method and system count for so much as in mine surveys.

310. Instruments used. For all angle work in mine surveys, except the location of short tortuous passages, the transit is the only suitable instrument. The work must be done with great precision. Results need not be so precise, perhaps, as in high-class city work, but a precision of one in five thousand should certainly be secured. It is best to use azimuths and perform the traverses as already described under traversing with the transit.

The transit should be a complete transit or tachymeter, so that leveling may be done either directly or by vertical angles. The vertical angle method is the better ordinarily, as it saves much time in the mine. The tripod should be either short or adjustable in length. A diagonal eyepiece is desirable for pointings at high inclinations. The wires are made visible by means of a reflector described in Art. 176, or by a hollow axis and reflector within the telescope. Often enough light is secured by having an assistant flash a candle near the objective. It is

necessary sometimes to point upward or downward at a considerable inclination, as when an incline is to be traversed. Pointing up is accomplished by means of an auxiliary diagonal eyepiece, but pointing down cannot be done when the angle is so great that the line of sight cuts the plates of the instrument.

Various devices have been invented to overcome this difficulty. Sometimes an auxiliary telescope is mounted above, and parallel with, the main telescope of the transit, as in Fig. 224. In some transits, the auxiliary telescope is mounted on the end of the horizontal axis, as in Fig. 225. Some transits are made with the standards leaning forward far enough to permit the line of sight to clear the plates when turned vertical. Others are made with double standards, one as usual, and one bracketed to the ordinary standards. The telescope is made so as to be readily changed from one bearing to the other. If the side telescope is used, the transit should be set over a point to one side of the station occupied, or the line of sight of the auxiliary telescope should be directed to a point as much to one side of the true station as the auxiliary telescope is to one side of the main telescope. This is done conveniently by having the object sighted at, as a candle, fastened to a flag at the proper distance from it. The flag is held on the true station.

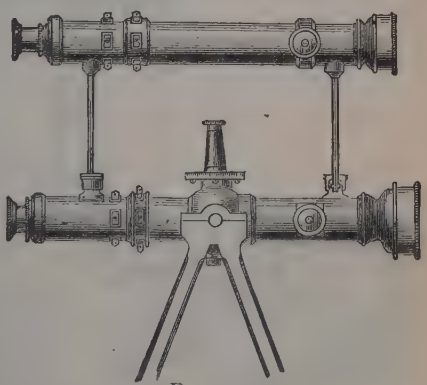


FIG. 224.

For tortuous passages in which a transit cannot be set up, so crooked that one end cannot be seen from the other, resort is had to what is sometimes called a German dial, and a hanging clinometer. One of the best forms of the dial is shown in Fig. 226, page 421.

The compass is hung on a wire or line stretched from one station to the next, and as it levels itself like a mariner's compass, it may be read properly. The bearings are not depended on, but angles are read. Passages so tortuous as to require

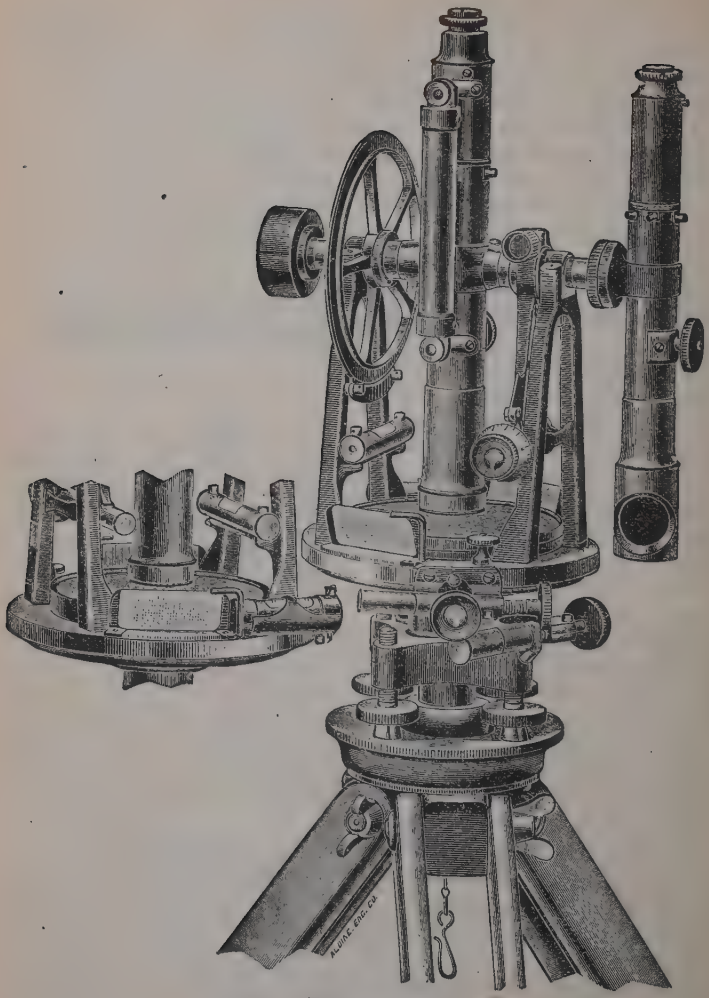


FIG. 225.

the use of such methods are not usually of great length, and the resulting errors are small. They are not likely to occur in coal mines, but occur frequently in quicksilver mines, and sometimes in gold and silver mines. Coal mines usually involve fewer irregularities of alignment than any other class of mines.

The linear measurements are made with a steel tape. In some cases of long, straight drifts, a long tape, two hundred feet, or three hundred feet, may be used to advantage; but usually a tape either fifty feet or one hundred feet long is best. The tape should be as long as possible, and no more stations should be used than seem necessary. It is very necessary to expedite work in a mine, because mining work in a drift must be largely suspended while the survey is in progress. The flat wire tapes are best for mine work, since they are not so easily broken as the ribbon tapes. A pocket tape, graduated to hundredths of feet, may be carried for the necessary plus measurements at stations, but the better way is to have an extra end foot graduated to 100ths. Etched graduations are generally preferred to raised sleeves. "Metallic" tapes are most practicable for side measurements.

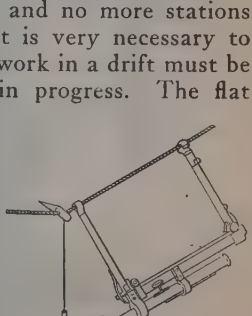


FIG. 226.

If a level rod is used, it must be a short rod. Bench marks are best placed on the side of the drifts where they will not be disturbed, and should occur in pairs, one in the roof and one in the floor to check errors arising from vertical movements of floor or roof. Notes must show clearly whether rod readings are with the rod erect (on the floor) or inverted (on the roof). A general system should be adopted.

311. Devices for making stations visible. The object observed at a station is usually a plumb line made visible by holding a piece of oiled paper or milk glass behind it and a candle behind the transparency. A piece of tracing cloth serves well. It may be carried in the pocket, is not easily broken or worn out, and may be renewed when soiled. The plumb line may be illuminated by holding a light in front of it, shielded from the observer.

Another method is to sight the flame of a candle or lamp,

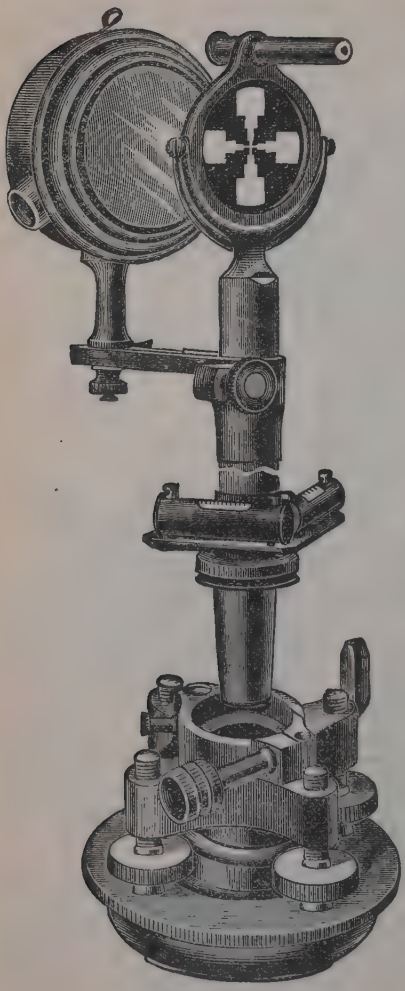


Fig. 227.

properly centered over the station. Another way is to use a plummet lamp, which is a lamp made in the form of a plummet. Perhaps the neatest method, at least in coal mines, is to use a second and third tripod on which a target lamp, such as is shown in Fig. 227, can be mounted. The lamp is behind the target from the observer, and the target may be accurately centered over the station point. The target and transit are interchangeable on the tripods, the particular form shown being used with a special transit that is interchangeable above the leveling screws. The method of use is somewhat as follows: The back station is occupied by a target, as is also the forward station, and that at which the observation is to be made is occupied by the transit. When the observation is complete, the tripod at the back station is carried to the next forward station, the others remaining in place. The back lamp is placed on the tripod that was occupied by the transit while the latter is being carried to the tripod occupied by the previous forward lamp, which in turn is carried to the next forward station. The lamps should

neither of them be moved till the observations at the station occupied by the transit are complete. This is a very expeditious method of work.[1]

The entire angular work may be done first, and the linear measurements made afterwards, or *vice versa*, or the two may be carried on at the same time. The latter method will probably consume more of the miner's time, since the linear work would retard the angle work. The linear work may be carried on without seriously interfering with the work of the mine. The extent or character of the work will guide the surveyor in choosing his methods. When lateral drifts depart at frequent intervals from one long tunnel or drift, the stations at the angle points of the main drift should be first located and the substations to be used in running the lateral drifts should be set in the line of the main drift. This may be done at the time the angles of the main drift are measured. In this case, the angle work should be done first.

When an extensive survey of a mine is to be made, the surveyor should go through the mine and lay out his program of work as closely as may be. It will almost always be possible to mark out the main stations before taking the transit into the mine, or at least before setting it up.

312. Notes. In an underground traverse in which notes are to be taken at intervals along each course to objects on the sides, a good form is as follows: Prepare the page of the notebook by ruling a column down the center, in which to write distances and azimuths, and leave the sides to note the objects measured to, and such other information as need be recorded. The form would be similar to that shown on page 293, except that the column for distances would also contain the azimuths and would be in the middle of the page instead of at one side. Notes taken on the right of a line that is run out should be noted on the right of the column, and those taken on the left should be recorded on the left.

A line should be drawn across the page between each two courses.

[1] An important point in favor of the three-tripod system, when long tapes are used for measuring, is that all ordinary distances may be measured from center of the telescope to center of backsight or foresight flame, the vertical angle being read and noted. Having then the length of the line of sight and its angle, the true horizontal distance between stations is easily calculated. Oftentimes this is a much safer course than to rely on presumably horizontal measurements along the floor of the gangways. — J. J. ORMSBEE, *Mining Engineer*.

For the purpose of the computation of latitudes and longitudes, the azimuths and distances may be copied in more compact form in the office notebook.

If elevations are carried by vertical angles, a good form is that given in Art. 231 for traversing with the transit and stadia, with sufficient vertical space given each course to write the details of the line along the side or right-hand page under remarks. The center line up the middle of the right-hand page may be used to represent the transit line, and the objects may be sketched to scale on this page. There should always be two sets of notes in existence, kept in different places, and when not in use they should be stored in fireproof vaults.

CONNECTING SURFACE AND UNDERGROUND WORK

313. When the mine is entered by a tunnel. In order that the underground workings may be plotted with reference to the surface lines, or even so that they may be properly oriented, it is necessary to connect the underground surveys with those on the surface. The simplest case occurs when the mine is entered by an adit or drift. From a point in the surface survey, located conveniently for the purpose, run a traverse into the drift.

314. When there are two shafts. If the mine is entered by shafts, and there are two or more of these some distance apart, the surface and underground surveys are connected by running a traverse on the surface and another underground between the shafts. The azimuth of the line joining the shafts is computed from the surface survey; and that of the same line underground, from the underground survey, a zero azimuth being assumed for this purpose. The two computed azimuths will differ by the angle between the true zero and the assumed zero, and the assumed underground azimuths may be corrected accordingly. The precise points at the shafts are made the same by using plumb lines. In deep shafts the lines are best made of piano wire, and the plummet must weigh from ten to twenty-five pounds. The line is first lowered with a light bob, and the heavy one is attached at the bottom. The line may be lowered by a reel. Signals may be given for lowering or raising by tapping on the guides or pipes in the shaft. Wherever possible,

this is an excellent method, since small errors of position of the bobs are not multiplied.

Another method, that may be applied in shallow mines, is to use the transit and auxiliary telescope, either setting the points in the bottom in line with a given line on top, or setting the transit in the bottom by trial, so that the line of sight may be in line with a surface line. Perhaps a better way than the latter is to set the transit on the bottom and, with the horizontal motion clamped, set two points in a line on opposite sides of the shaft at the surface, which line is produced into the mine as the first course of the traverse. The points located on the surface are connected afterwards with the surface survey. This method of using the transit and auxiliary telescope is adapted only to shallow shafts that are tolerably free from smoke. The better method is that of using the plummets. Work may be carried from one level to another through winzes and air shafts by the same method. As high vertical angles occur in this work, double centering is an absolute essential to correct work.

Levels are carried down the shafts by measuring with a steel tape; or, what amounts to the same thing, by lowering a weighted wire and then measuring the wire. If the tape is used, and is shorter than the shaft, it is first hung on a nail near the surface, the elevation of the nail being determined by leveling. The reel and observers are then slowly lowered in the cage until the tape is all paid out, when a second nail is driven at a measured distance from the first. A piece of white cloth is fastened to this second nail, to make it visible; and the cage is raised, the tape unhooked, the cage again lowered to the second nail, and the operation repeated from there down. A nail is left for a bench mark opposite each level.

315. When the mine is entered by one shaft. To connect a surface survey with an underground survey through a single deep shaft is the most difficult task the mining surveyor will encounter. If the mine is of small extent and the shaft shallow, it may be done with the transit and auxiliary telescope, as already described in the preceding article. But, if the shaft is deep, it cannot be done this way. The method is to use two plumb lines. The utmost care is necessary in their use, for from a base line from six to twelve feet long must be produced a survey one or two or more miles long. It may be necessary

to drive a second shaft a mile or two distant, and it must be driven to meet an underground drift. An error of one tenth of an inch in a base line ten feet long will cause a shaft two miles distant to be driven nearly nine feet out of place. Two methods of using two plumb lines have given satisfactory results.

1. In Fig. 228, M and M' are surface monuments, and AB the two plumb lines. The distance AB is measured carefully, and the distance MA and MB and the angles at M. The azimuth of AB is then determined. T is the transit set in the mine below. The angles at T, and the distances AT and BT are measured. The triangle TAB is then solved, from which the azimuth of TD becomes known.

2. This method probably is not as good as the second, which consists in suspending the plumb lines so that the transit may be set up in line with them below. The difficulty in all this work is that the plumb lines are never still, but oscillate continually. The mean position of the line is best determined by placing a fine scale behind it and noting the amplitude of the vibrations. The transit is pointed to the mean position. The oscillations are not in a straight line, but are more or less elliptical; hence, the scale must be placed a little behind the wire, but should be as close as possible to it, in order to avoid parallax. The transit is set up in the mine as close to the nearest line as possible (this will be from ten to fifteen feet distant), and each line is separately observed and the transit gradually brought by trial into line with the mean positions of both wires. The two plummets should hang in water, or, better, in some more viscous liquid, such as oil, or even molasses. The oscillations will be slow, a line three hundred and twenty-five feet long requiring about ten seconds for one complete vibration in air. Of course the cages cannot be in use during the operation.

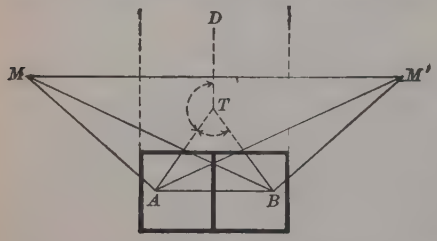


FIG. 228. — Two compartment shaft.

The points from which the bobs are suspended must be as firm as possible, and as well defined, so that the line joining them may be connected with the surface surveys. Two transits are

really required to insure good work. One is left on the surface with an observer to see that there is no movement of the points of suspension, during the operation, or to note what change, if any, takes place. It must be known that the lines do not hang against any projections on the sides of the shaft. This may be told, in some instances, by looking up the shaft; in others, by passing a candle or other light slowly around the wire at the bottom, and observing from the top that it is visible throughout the motion. In case neither method is practicable, a system of movements of the wire may be arranged. When the observer below is ready, he will signal by rapping on the pipes or guides, and the observer above will, at agreed intervals of time, move the wire out one inch, two inches, etc. The observer below notes whether corresponding movements take place there.

On account of the stoppage of machinery this connection of surface and underground surveys is an expensive operation. It should nevertheless, in important cases, be performed several times, and a mean of the results taken for the first course of the underground traverse. When the bottom of the shaft can be seen from the top, a method used at the Severn tunnel will give good results. A wire is stretched from one side of the shaft, about one hundred or more feet into the drift, and accurately aligned for the few feet visible by a transit on the surface. In the case of the Severn tunnel the wire was stretched over horizontal screws, the wire lying in the groove between the threads. The wire was then moved laterally for aligning by turning the screws.

MAPPING THE SURVEY

316. Metal mines. Three maps are necessary for a complete representation of a mine with many levels running in different directions: 1. The plan; 2. a longitudinal section; 3. a transverse section. Such a set of maps is shown in Plate IV at the back of the book. The notes taken by the surveyor should be such as to enable him to make these three maps. These maps may be in addition to one of the surface survey, and, if so, there would be four maps necessary for a complete representation of a mining property. It is usual to tint the portions shown on the plans as worked, and the different levels are sometimes tinted with different colors to distinguish them.

If several lodes are shown on one map, it will be better to color each lode working with a single color to distinguish it from the other lodes.

317. Coal mines. The representation of a coal mine is usually a simpler matter than the mapping of a metal mine. The coal ordinarily lies in beds nearly horizontal, and the workings may often be represented by a plan alone, the different levels being distinguished by colors, and the elevations being written in. It is customary to show on the map the direction and amount of dip of the seams. There are a number of symbols used to represent various objects; but the practice is not uniform. Arrows may be used to show the direction of air currents, blue for inlets, and red for outlets, etc. The plan should show all the details of the mine, all stop walls, all doors, all sumps or reservoirs, all pumps or machinery, the location of faults, etc.

It is necessary that complete, correct maps should be kept up, so that if any connections are to be made, they may be correctly directed. Very serious accidents have occurred from incorrect maps of mines.

318. Scale. The state of Pennsylvania provides that there shall be kept at each mine a plan on a scale of one to twelve hundred, or one hundred feet per inch, for the use of the state inspector. The English law requires the plan to be on a scale of twenty-five inches per mile, or about one half that required in this country. Other countries require different scales. The maps of the mining claims made by the deputy mineral surveyors are to a scale of two hundred feet to the inch. This would seem to be large enough for working drawings, except for details of structures.

As in the case of notes, two maps should be in existence and kept in different places. A good example of a coal mine map is shown in Plate V at the back of the book.

319. Problems. The problems that arise in mining surveying call for some ingenuity in handling Trigonometry and Descriptive Geometry, although they may all be solved without a knowledge of the latter subject by name.

Some problems, such as seeking a lost vein, require the principles of Geology rather than those of Mathematics.

The commonest problems are the following:

1. The determination of the course and distance from a point in one compartment of a mine to a point in another compartment. By course is meant azimuth and vertical angle. This includes such problems as the determination of the distance and direction to run a tunnel to tap a shaft.

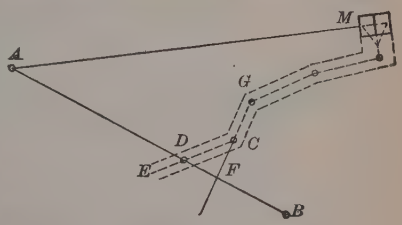


FIG. 229.

2. Knowing the strike and dip of a vein, to determine the length of a shaft or drift started at a given point and run in a given direction to meet the vein, or to determine the direction to run a tunnel from a given point so as to cut the vein in the shortest possible distance, and to find the distance required.

It is believed that the student can solve such problems as follow, if he remembers that the pitch or dip is always measured in a vertical plane normal to a horizontal element of the vein, and that the strike is the direction of such horizontal element. However, two examples will be explained.

1. *To find a point in a mine where it is crossed by the vertical plane through a property boundary.* In Fig. 229 *AB* is the boundary on the surface. One end is connected with the mine through the shaft at *M* and the mine traverse is run to a point, as *E*, which it is thought is beyond the intersection. The whole can then be plotted and the approximate position of the intersection determined.

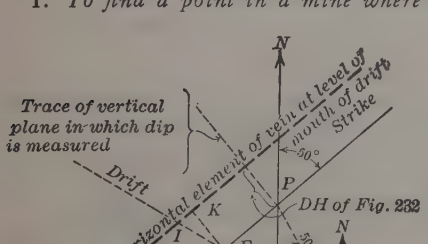


FIG. 230.

To find it mathematically, treat the survey as one with two sides wanting and find them (see Art. 123). Thus the bearings of *AD* and *CD* will be known, but their lengths will be unknown. If the survey is not plotted, a course *GC* that

is supposed to cross the boundary may be assumed to cross and its point of intersection found as at F which is discovered to be beyond C. Then the triangle CFD may be solved for CD, and if it is found that D falls beyond the end of the course CE, a second trial finding two wanting sides is necessary.

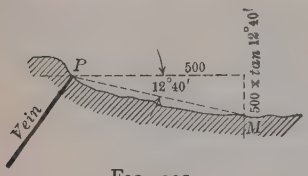


FIG. 231.

2. From a point of outcrop of a vein whose strike is N. 50° E. and dip 75° northerly, the mouth of a cross cut drift bears S. 30° E. 500 feet, vertical angle − 12° 40′. (*a*) The drift being run on a rising grade of 0.5 per cent at what distance from the mouth will it reach the vein, if run N. 60° W.? (*b*) In what direction should it run to meet the vein in the shortest distance?

(*a*) Showing as much as can be shown in plan or horizontal projection in Fig. 230, P is the point of outcrop, M the mouth of the drift, the other parts being named in the figure. M is below P and below all points of the horizontal element through P by 500 tan 12° 40′ if the 500 is horizontal measurement (or 500 sin 12° 40′ if it is inclined measurement), a vertical plane through PM showing as in Fig. 231. If the drift were run level, it would intersect that horizontal element of the vein that lies 500 tan 12° 40′ below P. That element will be drawn parallel to the strike and distant from it by 500 tan 12° 40′ tan 15° as shown in Fig. 232, which shows the trace of the vein on a vertical plane through P normal to the strike. This distance, shown also on Fig. 230, equals KE of that figure. The drift would therefore intersect the vein at I, Fig. 230, if run level. ME is first obtained by solving the triangle MEP, in which one side and all the angles are known. Then EI is EK sec. KEI or $\frac{EK}{\cos 20°}$. $ME + EI$ is the distance if run level. But the drift is run on an upgrade of 0.5 per cent = 0° 17′ +. This will make very little difference in the length, but to show the principle the change will be found. In Fig. 233, EI is the element of the vein cut by a vertical plane through MI of Fig. 230. The angle EIM is not just 75°, since the cutting plane is not normal to the strike. But EJ of Fig. 233 is known and $JI = EI$

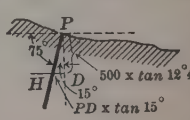


FIG. 232.

of Fig. 230, whence EIJ (Fig. 233) is found by $\tan EIJ = \frac{EJ}{JI}$. Then in the triangle MIL all the angles and MI are known to find ML, which is the result finally required.

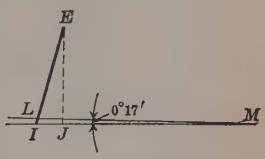


FIG. 233.

(*b*) To meet the vein in the shortest distance on the 0.5 per cent grade the drift should run normal to the strike, or N. 40° W. To meet it at the shortest possible distance it should run N. 40° W. on an upgrade that would make it normal to the plane of the vein or 15°, which is of course impossible, since this would build it in the air.

EXAMPLES

1. From the following data of the survey of a tunnel and shaft for a connection, determine the azimuth, length, and grade of the connecting drift. First draw a map of the survey and indicate the connecting drift.

From a monument at the mouth of the tunnel run in the tunnel, azimuth 36° 30′, 436 feet, vert. ∠ + 1° 00′; thence azimuth 52° 10′, 200 feet, vert. ∠ + 1° 10′, to point near breast of tunnel.

From the monument at the mouth of the tunnel run on the surface, azimuth 86° 30′, 232 feet, vert. ∠ − 3° 30′; thence azimuth 40° 20′, 636 feet, vert. ∠+ 13° 50′, to center of shaft 110 feet deep.

2. The strike of a vein of ore observed on the point of an outcrop is N. 36° W. or S. 36° E. The dip of the vein is found to be 13° from the vertical and to the northeast (right angle to the strike). From a point which bears from the before-mentioned point of outcrop N. 36° E. 300 feet, vert. ∠ − 15° a cross-cut tunnel is to be driven level and in a direction S. 55° W. How long must it be to reach the vein? What will be the difference if the tunnel is run on a one per cent grade?

3. If, in the above example, the tunnel is to run S. 60° W., how long must it be?

4. How long must it be, if the dip is to the southwest, the tunnel being run S. 54° W.?

5. How long must it be, if the dip is to the southwest and the tunnel is run S. 50° W.?

6. In Example 4 what would be the azimuth, grade, and length of the shortest tunnel that could be run to reach the vein?

500 N.
400 N.
300 N.
200 N.
100 N.
00 N.
100 E.
200 E.
300 E.
400 E.
500 E.
600 E.
700 E.
800 E.
900 E.
1000 E.
A
B
C
D
E
F
a
b
c
d
N. 403.66
E. 881.69
N. 207.46
E. 914.26
N. 289.08
E. 100.00
183.60
198.88
S. 9° 31' 00" E.
S. 9° 24' 30" E.
S. 2° 32' 30" W.
N. 19° 41' 00" E.
N. 80° 30' 00" E.
N. 19° 01' 30" E.
N. 80° 00' 18" E.
N. 69° 04' 14" E.
N. 80° 38' 00" E.
N. 80° 52' 30" E.
S. 34° 32' 30" E.
S. 8° 45' 52" E.
S. 7° 54' 30" E.
S. 88° 30' 00" E.
S. 0° 35' 30" E.
N. 89° 37' 45" E.
N. 69° 51' 30" E.
N. 75° 18' 44" E.
N. 0° 25' 30" W.
N. 0° 25' 30" E.
N. 0° 25' 30" W.
N. 0° 25' 30" E.
N. 0° 25' 30" W.
N. 0° 25' 30

APPENDIX

I. EXAMPLES AND PROBLEMS IN SURVEYING AND COMPUTING BY COÖRDINATES[1]

Model Example

1. To determine the coördinates of the corners *Abcdef* (Fig. 234) and plot the survey, the corner *e* and bearing *fe* being unknown, but point *e'* in prolongation of *fe* and bearing of *de*, as well as other corners, known:

Run the random traverse *ABCDEF*. Make a rough sketch of the traverse and place on it lengths of lines and angles measured. (Note that angles and not azimuths or bearings are read. This is careful work where angles are repeated several times, and hence it is impracticable to read azimuths.) Balance the angles so that their sum will be "twice as many right angles less four as the figure has sides," distributing any error equally among the various angles. In the example the angles "close."

Following a previous survey in this district, assume bearing of *DE* to be S. 9° 31′ E., and compute bearings of remaining lines and place them on the sketch.

Assume coördinates of point *A* 200 N. and 100 E. to bring all coördinates positive.

Compute the coördinates of the other corners of the traverse as shown on page 434. The student should note the systematic layout of the work. The error of closure, 0.05 E. and 0.14 N., is distributed as in balancing a survey, and the balanced coördinates are used in all subsequent work.

Plot the traverse as shown in Fig. 234. Check the plotting by scaling the lengths of the lines.

To locate the property corners:

Corner *A* is a corner of the random.

Course *Af* was actually run on the ground.

Calculate from the recorded angles the bearings of the lines joining random corners and true corners, and compute the latitude and longitude

[1] The problems given here were furnished to the author by Mr. John H. Myers, Jr., A.B., C.E., Assistant Engineer Department of Public Works, Brooklyn, N.Y. The examples are from practice in New York City and were treated as here indicated.

Coördinates of *A*			+ 100.00 E.			+ 200.00 N.
N. 80° 52′ 30″ E.	log sin 80° 52′ 30″	9.994469		log cos 80° 52′ 30″	9.200273	
515.63	log 515.63	2.712338	08	log 515.63	2.712338	75
		2.706807	+ 509.16 E.		1.912611	+ 81.77 N.
Coördinates of *B*			609.08 E.			+ 281.75 N.
N. 8° 27′ 00″ W.	log sin 8° 27′ 00″	9.167159	+ 606.16 E.	log cos 8° 27′ 00″	9.995260	281.77 N.
48.19	log 48.19	1.682957		log 48.19	1.682957	65
		0.850116	− 7.08 W.		1.678217	+ 47.66 N.
Coördinates of *C*			602.00 E.			+ 329.40 N.
N. 75° 41′ 00″ E.		9.986299	662.62 E.		9.393191	336.43
300.53		2.477888	18		2.477888	29
		2.464187	291.79 E.		1.871079	74.31 N.
Coördinates of *D*			893.18 E.			403.69
S. 9° 31′ 00″ E.		9.218363	893.21		9.218363	403.74
183.60		2.263873			2.263873	10
		1.482236	30.35 E.		2.257855	181.67 S.
Coördinates of *E*			923.53 E.			222.59 N.
S. 79° 01′ 30″ W.		9.991983	911.56 E.		9.279623	111.67 N.
819.96		2.913793	98		2.913793	13
		2.905776	804.96 W.		2.193416	156.16 S.
Coördinates of *F*			118.55 E.			66.46 N.
N. 7° 54′ 30″ W.		91.38582	118.66 E.		9.995850	66.37 N.
134.85		2.129851			2.129851	
		1.268433	18.55 W.		2.125701	133.57 N.
Coördinates of *A*			100.00			200.00
			100.03			200.14
Error			0.05 E.			0.14 N.

differences of these lines, and from these the coördinates of the corners. They will be found to be:

b, 597.42 E., 282.05 N.
c, 585.10 E., 353.95 N.
d, 881.69 E., 403.66 N.
f, 118.59 E., 70.33 N.

The computations are systematically arranged as follows, the computations for *c*, *d*, and *f*, being shown:

Cc ...	N. 34° 32′ 30″ W.	9.753587		9.915776		
	29.81	1.474362	602.00 E.	1.474362	329.40 N.	
		1.227949	16.90 W.	1.390138	24.55 N.	
	c		585.10 E.	. . . .	353.95 N.	
Dd ...	S. 89° 51′ 30″ W.	9.999990		7.417968		
	11.49	1.060320	893.18 E.	1.060320	403.69 N.	
		1.060319	11.49 W.		0.03 S.	
	d		881.69 E.	. . . .	403.66 N.	
Af ...	S. 8° 09′ 30″ E.	9.152010		9.995582		
	131.00	2.117271	100.00 E.	2.117271	200.00 N.	
			18.59 E.		129.67 S.	
	f		118.59 E.	. . . .	70.33 N.	

By Problem 1, Art. 127, find the length and bearing of the courses the coördinates of whose ends are known. They will be found to be as indicated on the figure. The arrangement of the work for a single course is shown below.

cd		
353.95		585.10
403.66		881.69
49.71		296.59
1.696444		
9.993984	N.	80° 29′ 07″ E.
9.224288	tan	9° 30′ 53″
2.472156	cos	do.
2.478172		300.73

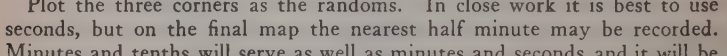
To find the corner *e* find the coördinates of *e′*, then the bearing of *e′f* by Problem 1, Art. 127. The bearing of *de* is known to be S. 9° 24′ 30″ E. By Problem 2, Art. 127, find the coördinates of *e*, the intersection of *de* and *fe′*. The quantities found appear on the sketch. Prob. 2 is arranged thus:

	403.66 N.	881.69 E.	
	70.33 N.	118.59 E.	
	333.33	763.10	
	2.522874		
	2.882581	N. 66° 24′ 14″ E.	
	9.640293	23° 35′ 46″	
	9.962081		
	2.920500		2.920500
75° 48′ 44″	9.986547	13° 49′ 04″	9.378097
89° 37′ 48″	0.000009		0.000009
	2.907056		2.398606
	807.34 *ef*		198.88 *de*

From the length and bearing of *de* and coördinates of *d* find coördinates of *e*, and from coördinates of *e* and *f* find bearing of *fe*.

Plot the three corners as the randoms. In close work it is best to use seconds, but on the final map the nearest half minute may be recorded. Minutes and tenths will serve as well as minutes and seconds and it will be even better if one has instrument and tables for decimal divisions of the degree.

If the area is required, it is obtained as follows, making the computations by logarithms. The work is here carried to a needless number of decimals.



200.00 × 597.42 = 119484.0000	100.00 × 282.05 = 28205.0000
282.05 × 585.10 = 165027.4550	597.42 × 353.95 = 211456.8090
353.95 × 881.69 = 312074.1755	585.10 × 403.66 = 236181.4660
403.66 × 914.20 = 369025.9720	881.69 × 207.46 = 182915.4074
207.46 × 118.59 = 24602.6814	914.20 × 70.33 = 64295.6860
70.33 × 100.00 = 7033.0000	118.59 × 200.00 = 23718.0000
997247.2839	746772.3684
746772.3684	

250474.9155 ÷ 2 = 125237.4577 square feet = 2.875 acres

Examples

1. In Fig. 234 determine whether the line *fd* passes to the north or south of the corner *b*; and how far *b* is from *fd*, measured on a line at right angles to *fd*.

2. In Fig. 235 the line *ABCDEF* represents a portion of a survey which has been made to locate a street or railroad. It has been decided to connect the lines *AB* and *EF* by a circular curve of radius 716.779 feet, and it is required to find how far from *B* the curve will start and how far from *E*

it will end. The method in outline is as follows: Assume coördinates for the point B, and by means of the connecting courses work up coördinates for the point E. By Problem 1, Art. 127, find the bearing of BE and the logarithm of its length. Then by Problem 2 find the distances from B and from E to the intersection I of AB and EF produced. Then the radius multiplied by the tangent of $\frac{1}{2}$ the angle GOH, which angle is equal to the

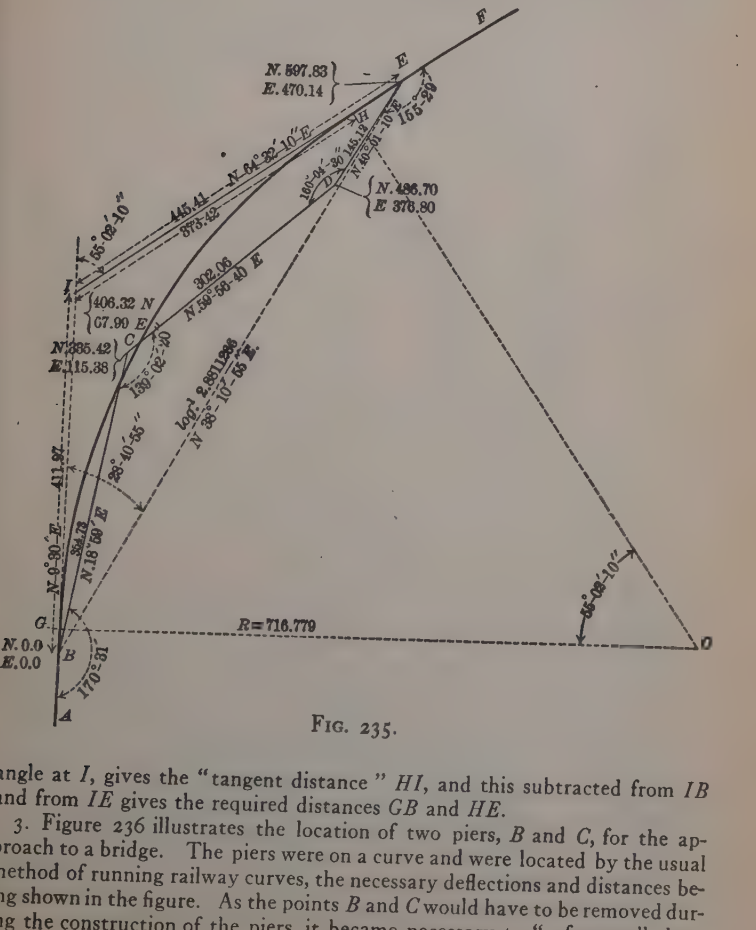


FIG. 235.

angle at I, gives the "tangent distance" HI, and this subtracted from IB and from IE gives the required distances GB and HE.

3. Figure 236 illustrates the location of two piers, B and C, for the approach to a bridge. The piers were on a curve and were located by the usual method of running railway curves, the necessary deflections and distances being shown in the figure. As the points B and C would have to be removed during the construction of the piers, it became necessary to "reference" them

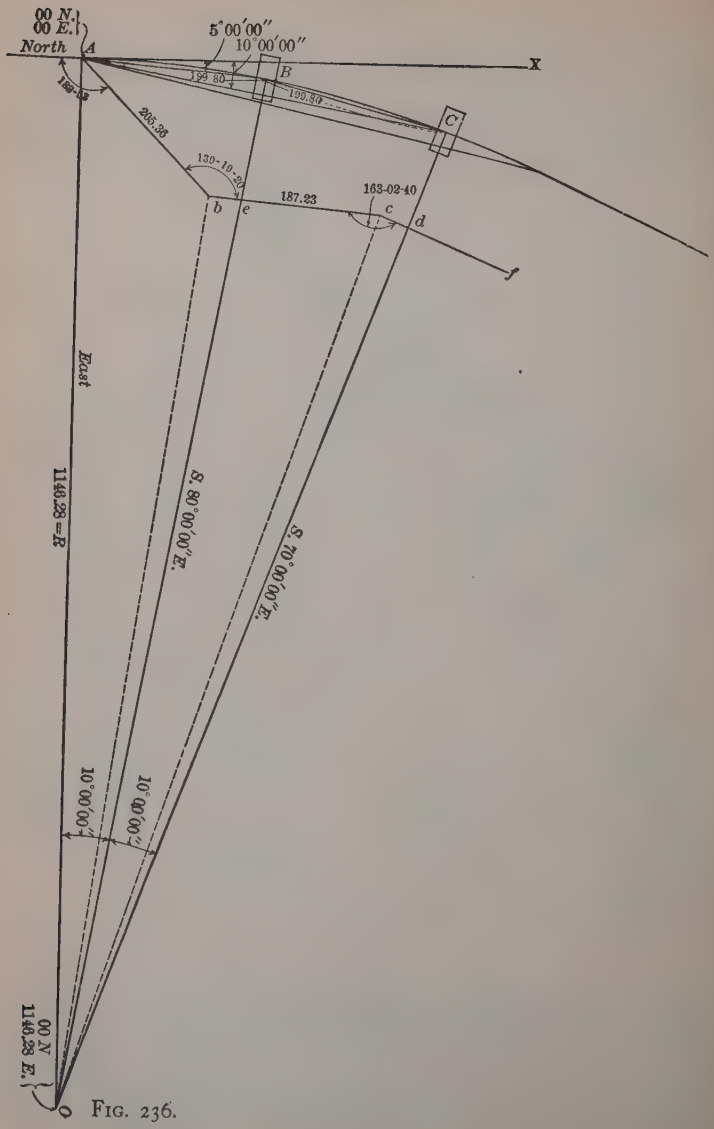

00 N.
00 E.
North
A
5°00′00″
10°00′00″
B
X
199.80
199.80
C
205.36
139-19-20
187.23
163-02-40
b
e
c
d
f
East
1146.28 = R
S. 80°00′00″ E.
S. 70°00′00″ E.
10°00′00″
10°00′00″
00 N
1146.28 E.
O

FIG. 236.

so that they could be recovered easily and quickly at any time. This was done by placing two or more points on the radial lines through the centers of the piers, since the intersections of these lines with the lines AB and AC would determine the points B and C. If the ground had been solid, the instrument could have been set at B and at C, and, by turning off the proper angles from A, the radial lines BO and CO could have been established easily. This, however, it was impossible to do, as the ground was so swampy as to render the instrument too unsteady for good work. Therefore the following method was adopted:

Beginning at A, a random traverse $Abcf$ was run on firm ground, and the angles and distances were measured as shown. The distances from b and

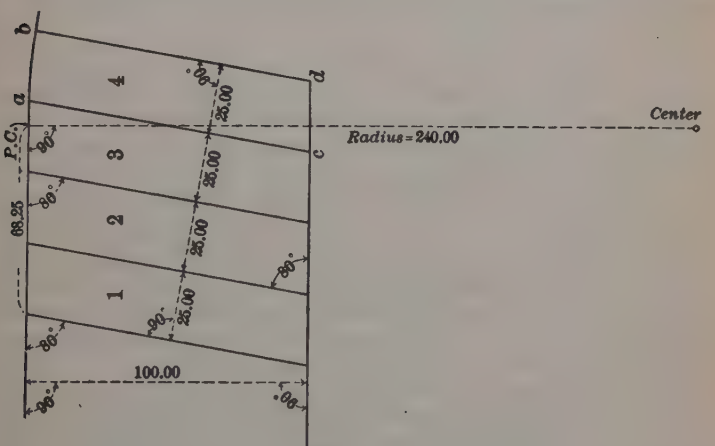


FIG. 237.

from c at which the radial lines BO and CO would intersect this traverse were then calculated, points were set at e and d, and the angles beB and cdC were measured to see whether their actual and calculated values agreed. This having been done, the instrument was set at e and at d, and by backsighting on B and C and reversing, any number of points could be established on the radial lines BO and CO. The points B and C are thus referenced. The problem for the student therefore is to calculate the distances be and cd. It will be well to use the tangent AX as a meridian and to make the coördinates of A 00 N and 00 E. (Problem 2, Art. 127.)

4. From the data given on Fig. 237 calculate the depths ac and bd of lot number 4.

II. THE JUDICIAL FUNCTIONS OF SURVEYORS[1]

BY JUSTICE COOLEY OF THE MICHIGAN SUPREME COURT

When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his action by facts which lead him away from the results which theoretically he ought to reach. Observation warrants us in saying that this remark may frequently be made of surveyors.

In the state of Michigan all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed it is impossible to determine by the record, with the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plots were more accurately surveyed, as indeed they should have been, for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear. The fact probably is that town surveys are quite as inaccurate as those made under authority of the general government.

It is now upwards of fifty years since a major part of the public surveys in what is now the state of Michigan were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold and the work of improvement begun. A generation has passed away since they were converted into cultivated farms, and few, if any, of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were

[1] A paper prepared for the Michigan Society of Surveyors and Engineers.

handy, but often they were not, and the witness trees must be relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing were substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary.

If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determined the boundary line between the proprietors. However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though the effect be to make one half-quarter section ninety acres and the one adjoining but seventy; for parties buy or are supposed to buy in reference to those monuments, and are entitled to what is within their lines, and no more, be it more or less. *McIver* v. *Walker*, 4 Wheaton's Reports, 444; *Land Co.* v. *Saunders*, 103 U. S. Reports, 316; *Cottingham* v. *Parr*, 93 Ill. Reports, 233; *Bunton* v. *Cardwell*, 53 Texas Reports, 408; *Watson* v. *Jones*, 85 Penn. Reports, 117.

While the witness trees remain there can generally be no difficulty in determining the locality of the stakes. When the witness trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men whose theoretical education is supposed to make them experts, who think that when the monuments are gone the only thing to be done is to place new monuments where the old ones should have been, and where they would have been if placed correctly. This is a serious mistake. The problem is now the same that it was before, — to ascertain, by the best lights of which the case admits, where the original lines were. The mistake above alluded to is supposed to have found expression in our legislation; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed.

An act passed in 1869, Compiled Laws, § 593, amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows:

"*Second.* Extinct interior section corners must be reëstablished at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

Third. Any extinct quarter-section corner, except on fractional lines, must be reëstablished equidistant and in a right line between the section corners; in all other cases at its proportionate distance between the nearest original corners on the same line."

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near.

To estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct. If the monument was set for an interior-section corner, but did not happen to be "at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that, notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word "extinct" may be used in this connection: one the sense of physical disappearance; the other the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man lose his deed he shall lose his land altogether.

But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between state and federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether state or federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hear-

ing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged.

The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost until after he has thoroughly sifted the evidence and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary when no other is attainable; and the surveyor should inquire when it originated, how, and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the state statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or may not have been trustworthy; but to bring these lines into discredit when the people concerned do not question them not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance and perhaps discredit, since in a legal controversy the law as well as common sense must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. *Stewart* v. *Carleton*, 31 Mich. Reports, 270; *Diehl* v. *Zanger*, 39 Mich. Reports, 601; *Dupont* v. *Starring*, 42 Mich. Reports, 492. And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession must often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees and no monuments to govern except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these, without looking farther. Now it may perhaps be known in a particular case that a certain monument still remaining was the starting point in the original survey of the town plot; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plot to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plot, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and use for many years, and the proprietors in a certain block

have laid off their lots in reference to this practical location. Two lot owners quarrel, and one of them calls in a surveyor that he may be sure that his neighbor shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was, or not, and the first result is, he notifies the lot owners that there is error in the street line, and that all fences should be moved, say, one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course, if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot owners generally will allow the new survey to unsettle their possessions, but there is always a probability of finding some one disposed to do so. We shall then have a lawsuit; and with what result?

It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute, road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. *McNamara* v. *Seaton*, 82 Ill. Reports, 498; *Bunce* v. *Bidwell*, 43 Mich. Reports, 542. Public policy requires that such lines be not lightly disturbed, or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment, or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were at another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go, as an officer of the law, is to express his opinion where the monument should be, at the same time that he imparts the information to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer and intrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument, for any considerable period, as strong, if not conclusive, evidence of such settlement. The peace of the community absolutely requires this rule. *Joyce* v. *Williams*, 26 Mich. Reports, 332. It is not long since that, in one of the leading cities of the state, an attempt was made to move houses two or three rods into a street, on the ground that a survey under which the street had been located for many years had been found on more recent survey to be erroneous.

From the foregoing it will appear that the duty of the surveyor where boundaries are in dispute must be varied by the circumstances. 1. He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has

reason to believe that agreements of the parties, express or implied, have rendered them unimportant. By monuments in the case of government surveys, we mean, of course, the corner and quarter stakes: blazed lines or marked trees on the lines are not monuments; they are merely guides or finger posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. 2. If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any state statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts; giving due prominence to the acts of parties concerned, and always keeping in mind, *first*, that neither his opinion nor his survey can be conclusive upon parties concerned; *second*, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and rules that will govern theirs. On town plots if a surplus or deficiency appears in a block, when the actual boundaries are compared with the original figures, and there is no evidence to fix the exact location of the stakes which marked the division into lots, the rule of common sense and of law is that the surplus or deficiency is to be apportioned between the lots, on an assumption that the error extended alike to all parts of the block. *O'Brien* v. *McGrane*, 29 Wis. Reports, 446; *Quinnin* v. *Reixers*, 46 Mich. Reports, 605.

It is always possible when corners are extinct that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course it is desirable that all such agreements be reduced to writing; but this is not absolutely indispensable if they are carried into effect without.

Meander Lines. — The subject to which allusion will now be made is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that the meander lines are boundary lines, and all in front of them remains unsold. Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river; and

while some courts have held that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the state, while conceding to the owners of the banks all riparian rights. The practical difference is not very important. In this state the rule that the center line is the boundary line is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation in some cases with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the center line of the river. Of course the original survey supposes that each purchaser of land on the stream has a water front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and in some cases, if it were continued in the same direction to the center line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation, or other valuable use of the water, while the other might have a water front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule: Extend the line of division between the two parcels from the meander line to the center line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and it is not limited to government surveys, but applies as well to water lots which appear as such on town plots. *Bay City Gas Light Co.* v. *The Industrial Works*, 28 Mich. Reports, 182. It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out.

When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes

very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive; but if he adopts one that follows, as nearly as the circumstances will admit, the general rule above indicated, so as to divide, as near as may be, the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties and may constitute a sufficient guide when peculiar cases arise. Each riparian lot owner ought to have a line on the legal boundary, namely, the center line of the stream, proportioned to the length of his line on the shore; and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream, when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the original admeasurement. Wherever that was the case, the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of governmental sale afterwards, for the reason that the rights of the bank owners are fixed by their purchase: when making that, they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which their purchase would include under it cannot afterward be taken from them. It is believed, however, that the federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the Great Lakes.

On all the little lakes of the state which are mere expansions near their mouths of the rivers passing through them — such as the Muskegon, Père Marquette, and Manistee — the same rule of bed ownership has been judicially applied that is applied to the rivers themselves; and the division lines are extended under the water in the same way. *Rice* v. *Ruddiman*, 10 Mich. Reports, 125. If such a lake were circular, the lines would converge to the center; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for some considerable distance, in order to determine where one may lawfully moor vessels or rafts, for the

winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. *Lorman* v. *Benson*, 8 Mich. Reports, 18; *People's Ice Co.* v. *Steamer Excelsior*, recently decided.

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them.

As regards the lakes and ponds of the state, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not:

1. To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them?

2. What public rights exist in them?

3. If there are islands in them which were not surveyed out and sold by the United States, can this be done now?

Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the Great Lakes. Perhaps it should be held that the boundary is at low-water mark, but improvements beyond this would only become unlawful when they became nuisances. Islands in the Great Lakes would belong to the United States until sold, and might be surveyed and measured for sale at any time. The right to take fish in the lakes, or to cut ice, is public like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore owners. But so far as these public rights can be the subject of ownership, they belong to the state, not to the United States; and, so it is believed, does the bed of a lake also. *Pollard* v. *Hagan*, 3 Howard's U. S. Reports. But such rights are not generally considered proper subjects of sale, but, like the right to make use of the public highways, they are held by the state in trust for all the people.

What is said of the large lakes may perhaps be said also of many of the interior lakes of the state; such, for example, as Houghton, Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size — say, even a mile across — there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon before. But the general rules for the extension of

shore lines, which have already been laid down, should govern such cases, or at least should serve as guides in their settlement.[1]

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the states may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation.

If the state should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as a trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the state could not drain, even for this purpose, against the will of the shore owners unless their rights were appropriated and paid for.

Upon many questions that might arise between the state as owner of the bed of a little lake and the shore owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not and cannot be judicial officers, but in a great many cases they act in a *quasi* judicial capacity with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value.

III. THE OWNERSHIP OF SURVEYS, AND WHAT CONSTITUTES A SURVEY AND MAP[2]

THERE seems to be a difference of opinion among surveyors as to how much of the information obtained, and how much of the work done in making a survey, shall be furnished to the individual for whom the survey is made. Many surveyors keep what are called "private notes." All men doing business as surveyors keep notes of all surveys in a convenient form

[1] Since this address was delivered, some of these questions have received the attention of the Supreme Court of Michigan in the cases of *Richardson* v. *Prentiss*, 48 Mich. Reports, 88, and *Backus* v. *Detroit*, Albany Law Journal, vol. 26, p. 428.

[2] A paper prepared by the author for "The Polytechnic," the student journal of the Rensselaer Polytechnic Institute, from which it is taken.

for ready reference. The extent to which these notes are private has not been comprehended rightly by all surveyors, and hence has resulted the difference of opinion mentioned.

This article is an attempt to present a side of this question that has not been fully considered heretofore. An endeavor has been made also to point out to the young surveyor a line of action expedient for him to follow, which will at the same time be found advantageous to the community in which he works.

In this discussion the question arises at once, "What constitutes a survey?" and the answer obviously depends on the object of the survey. This discussion will be confined to land surveys; that is, to surveys made for the purpose of subdividing a large tract of land into smaller parcels to be sold; or of determining the boundary of a tract the description of which is known; or of determining the description when the boundaries are known.

The principle to be enunciated applies to any other survey as well, be it railroad, canal, bridge, or topographical survey. Indeed, it is well understood in all such surveys, but seems to be ignored by many engineers having to do with land surveys.

A survey is the operation of finding the contour, dimensions, position, or *other particulars* of any part of the earth's surface, . . . tract of land, etc., and *representing the same on paper*.

In making a survey it is necessary to set certain points, called monuments or corners, and to determine a description of these points. These items, therefore, become a part of the survey. Then a map must be drawn. This map, to be a faithful representation of the ground and the work done, should, together with the notes, show all of the items mentioned.

The object of establishing monuments or corners and describing them is twofold: to mark on the ground the boundaries of the tract, and to secure definite information as to its location with reference to other points or tracts, so that from this information the land may at a future time be found. For a complete survey, therefore, the corners must be fixed, information that will preserve their location must be obtained, and the facts must be delineated on a map with accompanying notes.

To whom belongs this survey? It would appear to be evident that it belongs to the individual who pays to have it made. It is not readily seen in what way the survey, or any part of it, becomes the sole property of the surveyor. He may keep a copy of his notes to facilitate his future work; but he has not the shadow of a claim to a single note, the time for taking which has been paid for by his employer. If his charge for his work is on a time basis, there can be no question as to the correctness of the above statements. If he contracts to do the work for a definite sum for the entire job, he may take as much time as he likes, and may keep as many private notes as he desires; but he is bound in honor to return to his employer the complete survey; and, if he does so, the private notes would thereafter be of no great assistance to him in securing further employment, particularly when it is

remembered that men of repute do not bid against each other for professional work. His reputation for accuracy and honesty will be a far more potent factor in securing employment than any set of private notes fairly obtained.

It is true that a great many surveyors hold a different opinion, and purposely return their maps and notes in such condition, that, while they may serve the purpose for which they are primarily made, they do not tell the whole story, nor enough to make it easy for another surveyor to relocate the tract surveyed. When this is done, the person ordering the survey does not receive what he pays for. Something is withheld. It seems to need no argument to show that this is radically wrong.

But there is another reason for condemning this practice. The correct and permanent location of all public land lines, as streets, alleys, etc., as well as the permanent location of party lines between private owners, is a matter of the gravest importance, and no information that will at all serve to fix such lines in their correct positions for all time, should be withheld from the owner who pays for the survey, be it private citizen, municipality, county, or state.

The records of monuments and street lines made by a city engineer are no more his private property than are the records in the offices of the clerk, auditor, or treasurer, the property of the individuals who held office at the time the records were made. The correctness of the position assumed has been indicated by court decisions.

A great deal of laxity is shown in the conduct of offices of city engineers and county surveyors. The methods of regulating the pay of these officers has doubtless had much to do with this. It is frequently the case that the surveyor receives no salary, but is allowed to collect certain specified fees for work performed, and this gives color to his claim that his work is private and belongs to him. That this is not true concerning the public work he does is evident from what has preceded. That the records of work done for private citizens are not the property of the public needs no demonstration; but such work belongs to those citizens for whom it was done.

A different policy should be pursued with regard to these offices. In every case such an office should be a salaried one, with such salaried assistants as may be necessary. Certain fees should be prescribed for performing the various kinds of work that the surveyor may be called upon to do within the limits of the territory of the political division whose servant he is. These fees should cover all work connected with public construction and public or private land lines, and should be returned to the public treasury.

Their amount may be regulated, from time to time, so that they shall aggregate a sum sufficient to pay the expenses of the office. They should, of course, not cover work of a private character not having to do with land lines. But the entire public is interested in the permanency of land lines, and all records concerning them made by a public official should become public property. The permanency of land lines is too important a matter to be subject to avaricious and jealous rivalry; and all the surveyors in a

given district should coöperate to preserve, in their correct places, all lines within the district.

To this end, the returns of every surveyor made to the owner should be thoroughly complete Maps made for filing as public records should be so finished as to enable any surveyor to relocate the land without the least uncertainty as to the correctness of his work. That this is done in very few instances is well known to every surveyor who has had occasion to examine public records for data for surveys which he has been called upon to make.

Because of the fact that in most cases neither owners nor attorneys have been fully posted as to what constitutes a complete description, sufficient for relocation, and because surveyors have been willing to let matters stand as they were, great carelessness has arisen in the practice of making and filing maps for record.

While in some states good laws exist prescribing what shall appear on a map before it will be received as a public record, in more states there is nothing whatever to guide either owner, surveyor, attorney, or recorder in the matter. In the county records in such states, anything that is made up of lines and figures and labeled "this is a map," is considered a sufficient basis for a correct description and location of the property it purports to represent — no matter whether it is drawn by hand, photo-lithographed, or simply printed with "rule" and type. The records are full of auctioneers' circulars, manufactured in a printing office from information coming from nobody knows where, filed at the request of the auctioneer's clerk, with no name of owner or other interested party attached, except as the name of the auctioneer appears in the accompanying advertisement. Further than this, these maps are frequently purposely distorted to create a favorable impression of the property to be sold. Wide streets are shown where only narrow ones exist, streets appear opened for the full width where they have been opened for but half their width, subdivisions are indicated as rectangles that really may not be even parallelograms, etc. Such maps as these frequently form the only basis for the description and location of the property they are supposed to represent. Such misrepresentations are bad, *very* bad for those who buy; but is the information given by these circulars much worse than that furnished by many of the maps made by surveyors and filed at the request of the owners?

On these plots, if of "additions," we find lines indicating the boundaries of blocks and lots, all of which blocks and lots are numbered; the names of streets appear in neat letters; a few dimensions, possibly all linear dimensions, will be given; the streets or blocks may be delicately tinted, and the whole set off with a fine border and title. As an exhibition of the draughtsman's skill, these maps are perhaps valuable. As a source of information as to the location of the lines they purport to show, they are worth little more than the auctioneer's circular. Perhaps they have a few more figures, and the presumption may be a little stronger that the figures are correct.

Examine one of these maps closely. There will be found no evidence that a monument has been set in the field; not an angle is recorded, though the lines may cross at all sorts of angles; and dimensions are given that do not agree among themselves, so that the angles cannot be calculated. There will be found no name signed except, possibly, that of the surveyor, who thus advertises what we shall charitably call his stupidity.

Frequently no monuments are set except small stakes at the corners of the blocks; but even the fact that such stakes have been set is not recorded on the plot. One who is acquainted with the practice of surveyors in a given district knows at what points to look for such stakes, and if they have been set and not pulled out to make room for a fence post or building, he may succeed in finding them. Some surveyors are accustomed to set stakes a certain distance away from the point the stake is supposed to mark, but no mention of this fact appears on the map. In fact, the map is so drawn that no one but the surveyor who made it can write a description of any one of the parcels of land shown, or correctly locate it on the ground. Furthermore, the surveyor himself finds it impossible, after the lapse of a few years and the destruction of his "private marks," to rerun any one of the lines exactly as originally laid out.

It is easy to see to what this leads — impossible descriptions of property, giving opportunity for differences in judgment as to interpretation of what was intended; disputes as to position of party lines; costly litigation and expensive movement of structures begun or completed; and the actual shifting of lines back and forth by different surveyors, or even by the same surveyor, honestly trying to locate the lines properly.

The writer has seen enough trouble of this sort to indicate to him that a radical change is needed in the field work and mapping of cities, towns, and additions, not to mention farms and other tracts of land that it may be necessary to lay out and describe. So long as fallible man is responsible for the accuracy of surveys, maps, and descriptions of properties, so long will there be errors; but that it is possible to reduce their number greatly by proper regulation the writer is fully persuaded. What we have been describing are not maps at all, or at most they are very imperfect maps, and "what constitutes a map?" thus seems to be a very pertinent question.

A map of a city, town, or addition, or other tract of land, serving as a basis for the description of property, should furnish all the information necessary for the proper description and location of the various parcels shown, and also of the whole piece. It should show further the exact location of the whole tract relatively to the lands immediately adjoining; particularly should this be done when an offset or angle in a street line occurs. To accomplish these things, there should appear on the map the following items:

1. The lengths of all lines shown.
2. The exact angle made by all intersecting lines.
3. The exact position and character of all monuments set, with notes of reference points.

4. The number of each block and lot.

5. The names of all streets, streams or bodies of water, and recognized landmarks.

6. The scale.

7. The direction of the meridian and a note as to whether the true or magnetic meridian is shown. (It should be the true meridian.)

8. The angles of intersection made by the lines of adjoining property with the boundaries of the tract mapped.

9. The exact amount of offset in lines that may extend from the outside through the tract mapped.

10. A simple, complete, and explicit title, including the date and the name of the surveyor.

All this is necessary to make the map valuable for *description and location* of the property it represents.

Of course monuments will not be shown if none have been set, and very frequently none are set, either from carelessness on the part of the surveyor, or an unwillingness on the part of the owner to pay their cost. Monuments of a permanent character should be set at each corner of a tract surveyed, and at least two, visible the one from the other, should be on the line of each street. If these monuments are not placed on the center lines of the streets, they should be at *uniform* distances from the center or property lines. If placed with reference to the center line they should all be on the same side of the center. In streets extending east and west the monuments should all be on the north of the center, or they should all be on the south, and at uniform distances. In streets extending north and south the monuments should all be on the east of the center, or all on the west. Uniformity in such practice saves a vast amount of time.

Monuments may be set at uniform distances from the block lines, in the sidewalk area, and this is an excellent practice. The stakes or monuments set at the corners of the blocks in additions, or town sites, should never be the *only* stakes or monuments set in the tract.

That the map may be reliable there should appear on it the following:

1. The certificate of the surveyor that he has carefully surveyed the land, that the map is a correct representation of the tract, and that he has set monuments (to be described) at the points indicated on the map.

2. The acknowledged signature of all persons possessing title to any of the land shown in the tract, and, if possible, signatures of adjoining owners.

3. If of an addition, the acknowledged dedication to public use forever of all areas shown as streets or roads.

4. If a street of full width, whose center line is a boundary of the tract, is shown, the acknowledged signature of the owner of the adjoining property, unless his half of the street has been previously dedicated.

It has been stated already that, in some states, a map may be filed at the request of any person, and without signature. This practice frequently leads to trouble. The writer knows of cases in which owners of large tracts of land have had those tracts subdivided and have taken land of adjoining

non-resident owners for street purposes without the consent or knowledge of those owners. When, at a later day, the owners of the land so taken have objected and attempted to close half of the street, trouble of a serious character has arisen. The same trouble has occurred where streets have been run through narrow gores of land and have subsequently been completely closed, leaving houses built on the mapped property without outlet. Time and again have cases of this sort come to the knowledge of the writer.

Having pointed out certain evils, it remains to suggest a remedy. It lies in the enactment of laws governing these matters. There should be included in the statutes of every state a law explicitly defining what shall appear on every map filed for reference, and making it a misdemeanor to file a map that does not strictly conform to the requirements. In the absence of such laws it is believed that the young surveyor can assist greatly in a much-needed reform, by following the principles suggested in this paper as the correct ones, and avoiding the errors here indicated.

It is hoped that those graduates of our engineering schools who drift into this class of work will be guided by a higher principle than that which actuates the surveyor who covers up his tracks, at the expense of his employer, in order to secure a monopoly of the business of the locality. The young surveyor can spend his energies to greater advantage in devising new and better methods of work, than in inventing ways for hiding information that belongs to his employer. Certainly a thorough education should so broaden the young man's views as to make it impossible for him to be controlled by those meaner instincts which, if indulged, lead ever to narrow the vision and prevent one from perceiving the greater problems that continually present themselves for solution.

IV. GEOGRAPHICAL POSITIONS OF BASE LINES AND PRINCIPAL MERIDIANS GOVERNING THE PUBLIC SURVEYS

The system of rectangular surveying, authorized by law, May 20, 1785, was first employed in the survey of United States public lands in the state of Ohio.

The boundary line between the states of Pennsylvania and Ohio, known as "Ellicott's line," in longitude 80° 32′ 20″ west from Greenwich, is the meridian to which the first surveys are referred. The *townships* east of the Scioto River, in the state of Ohio, are numbered from south to north, commencing with No. 1 on the Ohio River, while the *ranges* are numbered from east to west, beginning with No. 1 on the east boundary of the state, *except* in the tract designated "U. S. military land," in which the townships and ranges are numbered, respectively, from the south and east boundaries of said tract.

During the period of one hundred and nine years since the organization of the system of rectangular surveying, numbered and locally named principal meridians and base lines have been established, as follows:

The first principal meridian begins at the junction of the Ohio and Big Miami rivers, extends north on the boundary line between the states of Ohio and Indiana, and roughly approximates to the meridian of longitude 84° 48′ 50″ west from Greenwich. The ranges of the public surveys in the state of Ohio, west of the Scioto River, are, in part, numbered from this meridian. For further information in regard to numbering of townships and ranges of the early surveys in Ohio, the reader is referred to the state map prepared in the General Land Office.

The second principal meridian coincides with 86° 28′ of longitude west from Greenwich, starts from a point two and one half miles west of the confluence of the Little Blue and Ohio rivers, runs north to the northern boundary of Indiana, and, with the base line in latitude 38° 28′ 20″, governs the surveys in Indiana and part of those in Illinois.

The third principal meridian begins at the mouth of the Ohio River and extends north to the northern boundary of the state of Illinois, and with the base line in latitude 38° 28′ 20″, governs the surveys in the state *east* of the third principal meridian, with the exception of those projected from the *second principal meridian*, and the surveys on the *west*, to the Illinois River. This meridian is nearly coincident with 89° 10′ 15″ of west longitude from Greenwich.

The fourth principal meridian begins at a point on the right bank of the Illinois River, in latitude 40° 00′ 30″ north, and longitude 90° 28′ 45″ west from Greenwich, and with the base line running west from the initial point, governs the surveys in Illinois west of the Illinois River and west of that part of the *third* principal meridian which lies north of the river.

The fourth principal meridian also extends north through Wisconsin and northeastern Minnesota, and, with the south boundary of Wisconsin as its base line, governs all the surveys in the former and those in the latter state lying east of the Mississippi River, and the third guide meridian west (of the fifth principal meridian system), north of the river.

The fifth principal meridian starts from the old mouth of the Arkansas River, and, with the base line running west from the old mouth of the St. Francis River, governs the surveys in Arkansas, Missouri, Iowa, North Dakota; those in Minnesota, west of the Mississippi River and west of the third guide meridian north of the river; and in South Dakota all east of the Missouri River, and the surveys on the west side of the river to a limiting line following the third guide meridian (of the sixth principal meridian system), White River, and the west and north boundaries of the Lower Brulé Indian Reservation. This meridian is nearly coincident with 91° 03′ 42″ longitude west from Greenwich.

The sixth principal meridian, which is approximately the meridian of 97° 23′ west longitude from Greenwich, extends from the base line coincident with the north boundary of Kansas in latitude 40° north, *south* through

the state to its south boundary, in latitude 37° north, and *north* through Nebraska to the Missouri River; and governs the surveys in Kansas and Nebraska; the surveys in Wyoming, except those referred to the *Wind River meridian* and base line, which intersect in latitude 43° 01′ 20″ north, and longitude 108° 48′ 40″ west from Greenwich; the surveys in Colorado, except those projected from the *New Mexico* and *Ute meridians*, the latter intersecting its base line in latitude 39° 06′ 40″ north and longitude 108° 33′ 20″ west from Greenwich; and the surveys in South Dakota extended, or to be extended, over the tract embracing the Pine Ridge and Rosebud Indian reservations.

In addition to the above-mentioned *numbered* principal meridians, other principal meridians with *local names* have been established as follows:

The Michigan meridian, in longitude 84° 22′ 24″ west from Greenwich, with a base line in latitude 42° 26′ 30″ north (eight miles north of Detroit), governs the surveys in Michigan.

The Tallahassee meridian, in longitude 84° 16′ 42″ west from Greenwich, runs north and south from the initial point on the base line at Tallahassee, in latitude 30° 28′ north, and governs the surveys in Florida.

The Saint Stephen's meridian, in longitude 88° 02′ west from Greenwich, begins at the initial point (Ellicott's corner), on the base line, in latitude 31° north, extends south to Mobile Bay and north to latitude 33° 06′ 20″, and governs the surveys in the southern district of Alabama, and in Pearl River district lying east of the river and south of the Choctaw base line, in latitude 31° 52′ 40″ north, in the state of Mississippi.

The Huntsville meridian begins on the northern boundary of Alabama, in latitude 34° 59′ north, longitude 86° 34′ 45″ west from Greenwich, extends south to latitude 33° 06′ 20″ north, and governs the surveys in the northern district of Alabama.

The Choctaw meridian begins on the Choctaw base line, latitude 31° 54′ 40″ north, longitude 90° 14′ 45″ west from Greenwich, runs north to the south boundary of the Chickasaw cession, in latitude 34° 19′ 40″ north, and governs the surveys east and west of the meridian, and north of the base line.

The Chickasaw meridian begins on the north boundary of Mississippi in latitude 34° 59′ north, longitude 89° 15′ west from Greenwich, extends south to latitude 33° 48′ 45″ north, and governs the surveys in north Mississippi.

The Washington meridian begins on the base line in latitude 31° north, longitude 91° 9′ 15″ west from Greenwich, extends *north* to the Mississippi River, and governs the surveys in the southwestern angle of the state of Mississippi.

The Saint Helena meridian begins at the initial point of the *Washington meridian*, in latitude 31° north, and longitude 91° 09′ 15″ west of Greenwich, extends *south* to the Mississippi River, and governs the surveys in the Greensburg and southeastern districts of Louisiana, east of the Mississippi River.

The Louisiana meridian, in longitude 92° 24′ 15″ west of Greenwich, extends from the Gulf of Mexico to the north boundary of Louisiana, and, with the base line through the initial point conforming to the parallel of 31° north latitude, governs all the surveys in the state west of the Mississippi River.

The New Mexico meridian, in longitude 106° 53′ 40″ west from Greenwich, extends through the state, and with the base line, in latitude 34° 15′ 25″ north, governs the surveys in New Mexico, *except* those in the northwest corner of the state, referred to *Navajo meridian* and base line, which have their initial point in latitude 35° 45′ north, longitude 108° 32′ 45″ west from Greenwich.

The Salt Lake meridian, in longitude 111° 54′ 00″ west from Greenwich, has its initial point at the corner of Temple Block, in Salt Lake City, Utah, extends north and south through the state, and, with the base line, through the initial, and coincident with the parallel of 40° 46′ 04″ north latitude, governs the surveys in the territory, except those referred to the *Uintah meridian* and base line projected from an initial point in latitude 40° 26′ 20″ north, longitude 109° 57′ 30″ west from Greenwich.

The Boisé meridian, longitude 116° 24′ 15″ west from Greenwich, passes through the initial point established south 29° 30′ west, nineteen miles distant from Boisé City, extends north and south through the state, and, with the base line in latitude 43° 46′ north, governs the surveys in the state of Idaho.

The Mount Diablo meridian, California, coincides with the meridian of 121° 54′ 48″ west from Greenwich, intersects the base line on the summit of the mountain from which it takes its name, in latitude 37° 51′ 30″ north and governs the surveys in the state of Nevada, and the surveys of all central and northern California, *except* those belonging to the Humboldt meridian system.

The Humboldt meridian, longitude 124° 08′ west from Greenwich, intersects the base line on the summit of Mount Pierce, in latitude 40° 25′ 12″ north, and governs the surveys in the northwestern corner of California, lying west of the Coast range of mountains, and north of township 5 south, of the Humboldt meridian system.

The San Bernardino meridian, California, longitude 116° 56′ 15″ west from Greenwich, intersects the base line on Mount San Bernardino, latitude 34° 07′ 10″ north, and governs the surveys in southern California, lying *east* of the meridian, and that part of the surveys situated *west* of it which is *south* of the eighth standard parallel south, of the Mount Diablo meridian system.

The Willamette meridian, which is coincident with the meridian of 122° 44′ 20″ west from Greenwich, extends *south* from the base line, in latitude 45° 31′ north, to the north boundary of California, and *north* to the international boundary, and governs all the public surveys in the states of Oregon and Washington.

The Black Hills meridian, longitude 104° 03′ west from Greenwich, with

the base line in latitude 44° north, governs the surveys in the state of South Dakota, north and west of White River, and west of the Missouri River (between latitudes 45° 55′ 20″ and 44° 17′ 30″), the north and west boundaries of the Lower Brulé Indian Reservation, and the west boundary of range 79 west, of the fifth principal meridian system.

The Montana meridian extends north and south from the initial monument on the summit of a limestone hill, eight hundred feet high, longitude 111° 38′ 50″ west from Greenwich, and, with the base line on the parallel of 45° 46′ 48″ north latitude, governs the surveys in the state of Montana.

The Gila and Salt River meridian intersects the base line on the south side of Gila River, opposite the mouth of Salt River, in latitude 33° 22′ 40″ north, longitude 112° 17′ 25″ west from Greenwich, and governs the surveys in the territory of Arizona.

The Indian meridian, in longitude 97° 14′ 30″ west from Greenwich, extends from Red River to the south boundary of Kansas, and, with the base line in latitude 34° 30′ north, governs the surveys in Oklahoma *east* of 100° west longitude from Greenwich.

The Cimarron meridian, in longitude 103° west from Greenwich, extends from latitude 36° 30′ to 37″ north, and, with the base line in latitude 36° 30′ north, governs the surveys in Oklahoma *west* of 100° west longitude from Greenwich.

V. UNITED STATES PUBLIC LAND SURVEYS

RESTORATION OF LOST OR OBLITERATED CORNERS AND SUBDIVISIONS OF SECTIONS

The following statements embrace everything of importance contained in a circular issued by the General Land Office in Washington for the guidance of surveyors in relocating lost or obliterated corners of the public land surveys and in subdividing sections.

The rules given are in the nature of the case not mandatory, but are suggestions based on United States court decisions. Many decisions have been made in accordance with the suggestions herein contained. When questions arise not covered by these rules, and the surveyor is in doubt as to the proper procedure, he should apply for advice to the General Land Office in Washington, always describing fully the corner, giving section, range, and township.

A United States Deputy Surveyor, in retracing old lines or subdividing sections for individuals, does so in his private capacity only, and proceeds under the same rules that would govern any surveyor in the same work.

A distinction must be made between lost and obliterated corners.

Lost corners. A lost corner is one whose position cannot be determined beyond reasonable doubt either from original marks or reliable external evidence.

Obliterated corners. An obliterated corner is one in which no visible evidence remains of the work of the original surveyor in establishing it, but which may be reëstablished beyond reasonable doubt by evidence of competent persons.

Various methods of subdivision. Present practice requires section lines to be initiated from the corners on the south boundary of a township to be subdivided, and to close on existing corners on the east, north, and west boundaries, unless the north boundary be a base line or standard parallel.[1] This results in a single set of corners on all exterior lines except base lines and standard parallels, on which there are two sets of corners.

A previous practice resulted in two sets of corners on all exteriors, — the corners set when the exteriors were run, and those set when the township was subdivided, — no attempt having been made to close on existing corners on the north or west boundaries of the township, but closure having been made on the east boundary.

Still another practice resulted in three sets of corners on the range lines, no attempt having been made to close on any exteriors.

The more recent general instructions provide that no new township exteriors or section lines shall depart from a true meridian, or east and west line, by more than 21 minutes of arc, and that where a random line is found liable to correction beyond this limit, it must not be so corrected, but must be run as a true cardinal course, and a closing corner set on the line to which it closes. This produces, when new work closes on old irregular work, a double set of corners on exteriors.

It is necessary that surveyors attempting to rerun old lines shall be familiar with the original notes and field methods, some of which have been exceptional. The following synopsis of congressional action will be of assistance.

Acts of Congress. *Act of May* 20, 1785, *U. S. Land Laws, Edition of* 1828, *p.* 349. Provided that the "Western Territory" should be divided into townships of six miles square, by lines running due north and south, and others crossing them at right angles as near as may be; that the first north and south line should begin on the Ohio River at a point due north from the western terminus of a line run as the south boundary of the State of Pennsylvania, and the first line running east and west should begin at the same point and extend through the whole territory. In the initial surveys under this law only exterior lines were run, mile corners being established on these exteriors, but the plats were marked by subdivisions into sections one mile square, numbered as shown on page 277.

Act of May 18, 1796, *U. S. Statutes at Large, vol.* 1, *p.* 465, *U. S. Revised Statutes, Sec.* 2395. Applied to "the territory Northwest of the River Ohio and above the mouth of the Kentucky River"; provided for dividing such lands as had not been already surveyed or disposed of "by north and south lines run according to the true meridian, and by others crossing them at

[1] See page 290.

right angles, so as to form townships of 6 miles square"; provided that "one half of said townships, taking them alternately, should be subdivided into sections containing, as nearly as may be, 640 acres each, by running through the same each way parallel lines at the end of every two miles, and by marking a corner on each of said lines at the end of every mile." This act provided also for the method of numbering sections that is still in use, for which see page 278.

Act of May 10, 1800, *amending the foregoing act, U. S. Statutes at Large, vol.* 2, *p.* 73, *U. S. Revised Statutes, Sec.* 2395. Provided, that townships west of the Muskingum, which are directed to be sold in quarter townships, should be subdivided into half sections of 320 acres each, as nearly as may be, by running parallel lines through them from east to west, and from south to north, one mile apart, marking corners each half mile on the east and west lines, and each mile on the north and south lines; that interior lines of townships intersected by the Muskingum, and of all townships lying east of that river, which had not been previously actually subdivided into sections, should also be run and marked; that, when the exterior lines of the townships thus to be subdivided into sections or half sections, should exceed or fall short of six miles, the excess or deficiency should be noted and added to or subtracted from the western or northern ranges of sections, according as the error might occur in running from east to west or from south to north; that the northern and western tiers of sections should be sold as containing only the quantity expressed on the plats, and all others as containing the full legal quantity.

Act of June 1, 1796, *U. S. Statutes at Large, vol.* 1, *p.* 490. Regulated the grants of land appropriated for military services, etc., and provided for dividing the "U. S. Military Tract" in the state of Ohio into townships five miles square, each to be subdivided into quarter townships containing 4000 acres.

Act of March 1, 1800, *U. S. Statutes at Large, vol.* 2, *p.* 14. Amended the foregoing act, and authorized the Secretary of the Treasury to subdivide the quarter townships into lots of 100 acres bounded as nearly as practicable by parallel lines, 160 perches in length by 100 perches in width. These subdivisions were at first only made on the plats in the Secretary's office. Subsequently, when a sufficient number of lots had been taken to warrant surveys, the surveys did not in all cases agree with the plats, and some fractional lots shown on the plats were entirely crowded out.

Act of February 11, 1805, *U. S. Statutes at Large, vol.* 2, *p.* 313, *U. S. Revised Statutes, Sec.* 2396. Directed the subdivision of the public lands into quarter sections; that all corners marked in the field should be held to be the true corners of the sections or quarter sections they were intended to mark; that corners of half sections and quarter sections not marked should be placed as nearly as possible equidistant from those two corners which stand on the same line; that the boundary lines actually run and marked should be held to be the true boundaries of the subdivisions for which they were intended, and that the lengths of such lines as returned by the survey-

ors should be held to be the true lengths; that the boundary lines not actually run and marked should be ascertained by running straight lines from the established corners to the opposite corresponding corners, except in fractional townships, where no such opposite corners exist, in which cases the boundaries were to be run from the established corners due north, south, east, or west, as the case may be, to the water course, Indian boundary line, or other external boundary of the fractional township.

Act of April 24, 1820, *U. S. Statutes at Large, vol.* 3, *p.* 566, *U. S. Revised Statutes, Sec.* 2397. Provided for the sale of public lands in half-quarter sections, and that the lines dividing the quarter sections should be run north and south; that fractional sections containing 160 acres or more should be divided as nearly as practicable into half quarters under special rules to be prescribed by the Secretary of the Treasury, but that fractional sections of less than 160 acres should not be divided.

The important requirements of the act of May 24, 1824, are given on page 278. The act refers to surveys of lands bordering lakes, bayous, etc.

Act of April 5, 1832, *U. S. Statutes at Large, vol.* 4, *p.* 503, *U. S. Revised Statutes, Sec.* 2397. Directed the subdivision of the public lands into quarter-quarter sections; that half sections should be divided by lines running east and west, and that fractional sections should be subdivided under rules to be prescribed by the Secretary of the Treasury.

Under the last provision it was directed that fractional sections containing less than 160 acres, or the residuary portion after the subdivision into as many quarter quarters as possible, should be divided into lots, each containing as nearly as practicable the quantity of a quarter-quarter section, by so laying out the subdivision that the lots should be 20 chains wide, which distances were to be marked on the plat of subdivision, together with the areas of the quarter quarters and the residuary fractions.

The last two acts provided that the corners and contents of half-quarter and quarter-quarter sections should be ascertained as nearly as possible by the methods prescribed in the act of 1805.

General principles. From the foregoing synopsis of the land laws it will be evident:

1. That the boundaries of the public lands established and returned by the duly appointed government surveyors, when approved by the surveyors general and accepted by the government, are unchangeable.

2. That the original township, section, and quarter-section corners established by the government surveyors must stand as the true corners which they were intended to be, whether they be in place or not.

3. That quarter-quarter corners not established by the government surveyors shall be placed on the straight lines joining the section and quarter-section corners, and midway between them, except on the last half mile of section lines closing on the north and west boundaries of the township, or on other lines between fractional sections.

4. That all subdivisional lines of a section running between corners established in the original survey of a township must be straight lines, run-

ning from the proper corner in one section line to its opposite corresponding corner in the opposite section line.

5. That in a fractional section where no opposite corresponding corner has been, or can be, established, any required subdivision line of such section must be run from the proper original corner in the boundary line due east and west, or north and south, as the case may be, to the water course, Indian reservation, or other boundary of such section, with due parallelism to section lines.

RULES OF PROCEDURE

Examination of ground. To restore an extinct corner, first look for marks of the original surveys, and, if possible, identify the original locus by the aid of the mound, pits, line trees, bearing trees, etc., described in the field notes of the original survey. If the corner cannot be thus determined, clear and convincing testimony of citizens should be taken as to the original position. By whichever method the corner is located, its position should be checked by measurements to known corners. What will constitute sufficient evidence in a given case can not be stated. Much is here left to the fidelity, skill, and judgment of the surveyor.

Corner on most important line. It is a general principle that a corner shall be established on the more important (hence, presumably, the first surveyed) line on which it is supposed to be: thus, a corner returned as being 78 chains from a given other interior corner and on a township or range line, should be located on the township or range line regardless of the distance from the other interior corner, unless the condition of the range line or township line is such as to indicate that it was not a true line. The surveyor must be guided by his judgment.

Similarly, a closing corner established on a standard parallel, township, or range line, or state boundary, should be held to be on such important line even though the corner itself actually may exist erroneously located off the line. Such erroneous corner, together with the one from which it was run, may be held to indicate the *location* of the minor line, but its length must be governed by its intersection with the previously run more important line. But it is only where it is evidently impossible to obey the law literally that the surveyor is justified in such methods.

Principle of proportionate measurements. Lost or obliterated corners are to be restored *on the lines on which they were originally located* by proportionate measurements from the nearest existing corners of the same class on those lines. By proportionate measurement is meant, "*As* the original field-note distance between the selected known corners *is to* the new measure of said distance, *so is* the original field-note length of any part of the line *to* the required new measurement thereof." Thus, if on a township line the corner common to sections 2, 3, 34, and 35, is missing, and the first corners found on either side are those common to 3, 4, 33, and 34, and to 1 and 36, then the distance between these two corners should be measured, and the

required corner should be placed on line between them at a distance from 3, 4, 33, 34, found by the proportion:

The required distance *is to* the recorded distance between the same points *as* the newly measured distance between the existing corners *is to* the recorded distance between those corners.

Discrepancies between new and original field-note measurements between selected original corners will not in any manner affect measurements beyond those corners.

Corners of the same class. Nearest existing corners used to relocate lost or obliterated corners should be of the same class as the corner to be restored; that is, if the lost corner is a standard corner on a base line, standard parallel, or correction line, the corners used in its restoration should be also standard corners, and not ordinary closing corners. They may be either township, section, quarter-section, or meander corners, or closing corners *established during the survey of the standard line as corners on that line.* No other closing corners may be thus used, but closing corners thus established will be treated as standard corners, when they are lost or obliterated, and relocated accordingly.

The general principle governing is, that corners must be reëstablished by proportionate measurements from other corners on the same line and of the same original survey.

Corners common to four townships. 1. When the corner has been located by measurements on one line only — as on a guide meridian — proceed as already indicated for any standard corner. 2. When the original location depended on measurements on two lines intersecting at right angles, proceed as follows: First run a straight line between the nearest identified corners north and south of the required corner on the meridional line, and set a temporary corner at the proper proportionate distance. Next proceed similarly with the latitudinal line. Thus two temporary corners will be determined. From that on the meridional line run east or west, and from that on the latitudinal line run north or south, the directions depending on the particular case, until the lines thus run intersect, and establish the corner at the point of intersection.

Interior-section corners. Proceed as for corners common to four townships. When several corners next to the required corner are gone, simply follow the rule, measuring on both lines to the nearest corners that can be found, and establish the corners by proportionate measurement. In all cases the lines will be retraced where they were originally placed if it is possible to determine by existing original marks where the original location was. When this is impossible, the lines will be run straight between the selected original corners.

Closing corners on standard lines. These corners should be reëstablished by measuring from the adjacent quarter-section, section, or township corner to the next preceding or succeeding corner in the order of original establishment, and locating the missing corner by proportionate measurement.

Quarter-section corners. But one set of quarter corners are actually marked on township lines, and this is the set established when the township line was run. These corners will be established as already described by proportionate measurements from the adjoining original section corners. When two sets of section corners exist, the second set being closing corners for the township south of the township line, care must be exercised that the wrong section corners are not taken. Quarter-section corners for the township south of the line will be established, ordinarily, midway between the corresponding adjacent section corners, but such corners must be established so that the areas of the quarter sections south shall be proportionate to the areas originally returned. The same procedure holds for quarter corners on range lines when they are the corners of quarters east of the range line.

On closing lines betweeen fractional sections, reëstablish the quarter corner by proportionate measurements 40 chains, more or less, from the last interior-section corner. The entire closing line must be measured and any discrepancy distributed between the two portions of the line.

Interior quarter corners are established by proportionate measurements between adjacent section corners, care being taken to follow the old line, if its location is in evidence, and otherwise running straight between the section corners.

Double corners on township and range lines. The corners established when the exterior lines were run belong to the sections north or west of those lines, while those established as closing corners during subdivision belong to sections south and east of the exterior lines.

If one is missing and the other found, determine to which set the existing corner belongs, remeasure distances to known corners of the same survey, and reëstablish the missing corner by proportionate measurement from the existing corner, checking by measurements to the opposite corresponding section corner.

If both are missing, and the one belonging to the exterior survey is wanted, first find the nearest corners of the exterior survey on the exterior line, and establish the missing corner on a straight line between them — unless the original line is in evidence and is not straight — by proportionate measurements, then check by retracing the section, north or west, to which the corner belongs.

If both are missing, and the one belonging to the subdivision survey is wanted, first mark the exterior line in the vicinity, then retrace the section line closing on the missing corner, and establish a temporary corner at the intersection with the exterior line. Verify the position by measurements to known objects and corners on the exterior line noted in the original field notes, and make such correction in the position as may seem proper.

Triple corners on range lines. But two of the set of three corners are actually section corners, no attention having been paid, in the subdivision either east or west, to the corner set in the original exterior survey, except that in some instances the distances from the closing corners to such original exterior corner have been noted. If one or two of a set are missing, and

tones of their distance from one another were made, identify the existing corner and proceed as for double corners. If no distances were noted originally, carefully retrace the range line, marking it plainly, and then retrace the section lines closing on the missing corners, to intersection with the range line, and reëstablish the corners at the intersections. Great care should be exercised to retrace the lines in the original location, when possible. If all three corners of the set are missing, proceed as already directed, establishing the exterior corner first.

Meander corners. To reëstablish meander corners, first remeasure not less than three section lines between known corners of the township in which the meander corner lies, to determine the relation between the old and the new measures, then carefully retrace the section line on which the meander corner was placed, and reëstablish the corner at the proper proportionate distance from the nearest known corner on that line.

Meander lines, not being boundaries usually, the actual water line usually being the boundary, are not of great importance; but meander corners are of importance as fixing the position of the section lines on which they occur. In states whose laws prescribe that dried lake beds are state property, the meander line assumes more importance.

Rules for running division lines between private owners of riparian accretions are a matter of state legislation, and not subject to general rule of the U. S. Land Office.

Corners of land sold. When corners of land already sold become misplaced or lost, either by accident or design, and no evidence exists as to the original location that will enable the surveyor to establish the corner to the satisfaction of the owners affected, the relocation can only be made on the order of a court of competent jurisdiction. The authority of the United States to make changes, or prescribe rules, ceases with the disposal of the lands.

Permanent corners. When, in accordance with the preceding rules, a corner has been properly relocated, a permanent corner should be established, and notes to witness objects should be taken and recorded.

Subdivision of sections. To establish the center corner of a section, connect the opposite quarter corners by straight lines and place the center corner at their intersection. It should be noted that in the northern and western tiers of sections the quarter corners are placed 40 chains from the interior-section line south or east of the township or range line. It should be remembered also that when double corners exist on township or range lines, the quarter corner of the section south or east, as the case may be, was not set in the original survey, and that it should be set to suit the areas returned on the official plats, adopting proportionate measurements.

In subdividing fractional sections, where there is no opposite corner in one or more directions, the proper procedure, if the original work had been correctly done, would be to run from the established corners due north, south, east, or west lines, as the case might be, to the boundary of the fractional section. Since the section lines, supposed by law to be due east and

west or north and south lines, rarely are so, it is necessary to retrace the section boundaries supposed to be parallel with the required division line, and then run that line on a mean bearing. If but one of the two section lines exists, run parallel to that one.

To subdivide quarter sections into quarter quarters, establish the quarter-quarter corners on the four sides of the quarter section midway between the adjacent corners. Connect the opposite quarter-quarter corners by straight lines and place the center corner at the intersection of those lines. On north and west tiers of quarter sections, the quarter-quarter corners will be placed 20 chains from the quarter corners next south of the township line or next east of the range line, as the case may be. For the subdivision of fractional quarter sections proceed as in the subdivision of fractional sections.

Where to find records. Copies of the original plats and field notes of the public land surveys may be had from the United States surveyors general of the states in which the surveys occur, except in the following states, where application should be made to the officer named:

ALABAMA: Secretary of State, Montgomery. ARKANSAS: Commissioner of State Lands, Little Rock. ILLINOIS: Auditor of State, Springfield. INDIANA: Auditor of State, Indianapolis. IOWA: Secretary of State, Des Moines. KANSAS: Auditor of State and Register of State Lands, Topeka. MICHIGAN: Commissioner of State Land Office, Lansing. MISSISSIPPI: Commissioner of State Lands, Jackson. MISSOURI: Secretary of State, Jefferson City. NEBRASKA: Commissioner of Public Lands and Buildings, Lincoln. OHIO: Auditor of State, Columbus. WISCONSIN: Commissioner of Public Lands, Madison.

VI. THREE POINT PROBLEMS WITH THE PLANE TABLE

U. S. COAST AND GEODETIC SURVEY METHOD

THREE visible points whose positions have been plotted on the plane table sheet are used for the purpose of locating on the sheet the point occupied by the instrument. The occupied point is chosen because it is a convenient point from which to map surrounding topography. If the instrument can be properly oriented and the occupied position correctly plotted with respect to the other plotted points, time will be saved that would otherwise be used in running a traverse from some known point to the desired point occupied. The method here described for locating the point and orienting the instrument is a method by trial.

The instrument is oriented as nearly as may be by estimation, or by the use of the compass or declinator. The *declinator* is a box with an arc of about 30° in which box there is a magnetic needle. A magnetic north and south line may be drawn on the paper; the box placed on the line, and the tables swung until the needle reads zero. The line on the paper may be

drawn when the table is over a known point, by making the declinator read zero when the table is properly oriented for the lines radiating from the occupied point, and drawing a line along the edge of the declinator or along the edge of the alidade when the compass on, or against, the alidade reads zero.

When the orientation has been made as nearly as possible, lines are drawn through the three plotted points with the alidade pointing to the distant points respectively. If the orientation is right, the three lines will meet in a point which is the point sought. If this point happens to lie on the great circle through the three known points, the three lines will meet in a point anyway and the point sought cannot be determined. It is necessary, therefore, to know when selecting the position to be occupied that it is not on or near a great circle through the three known points. It is desirable to know, also, whether the chosen position is within the great triangle made by the three known points, whether it is inside or outside the great circle through the three points, and in which segment of that circle of those made by the three sides of the great triangle. The reasons for this will appear later.

If the three lines drawn through the three points do not meet in a common point they will form a small triangle, called the *triangle of error*, or two will be parallel, or approximately so, and crossed by the third. To find the point sought from the lines drawn constitutes the solution of the three point problem.

Lehmann's method of solution is the simplest and most direct, and applies under all circumstances. The directions are stated in the form of rules.

The term "point sought" will be understood to mean the true position on the sheet of the projected point representing the station occupied. The surveyor is assumed to be facing the signals, and the directions right and left are given accordingly.

Rule 1. — The point sought is always distant from each of the three lines drawn from the three fixed points, in proportion to the distances of the corresponding actual points from the station occupied, and it will always be found on the corresponding side of each of the lines drawn from the fixed points. That is, if it is on the right side of one line, it is on the right side of each one of the other two, and if on the left side of one, it is on the left side of each one of the other two.

The simplest case for the application of this rule occurs when the station to be determined is within the triangle formed by the three fixed points; the point sought must then be within the triangle of error to satisfy the conditions. (See Fig. 239 *e*.)

Demonstration of the first part of Rule 1. Referring to Figure 238, *A*, *B*, *C* are the plotted representations of the three known points; *D* is the point occupied, but the table being improperly oriented the three lines drawn from *A*, *B*, and *C*, form the triangle of error *ab*, *ac*, *bc*. The true point lies on a circumference through *A*, *B*, and *ab*; also on a circumference through *A*, *C*, and *ac*; and also on a circumference through *B*, *C*, and *bc*; and, hence, the point *D* is at the intersection of these three circumferences. If perpendiculars be drawn from *D* to the three lines $A-ab$, $B-bc$, and $C-ac$,

then since in the circumference A, B, ab, the angles at A and B are equal subtending the same chord $D-ab$,

$$Da : Db :: DA : DB.$$

Similarly $$Db : Dc :: DB : DC;$$ etc.

If the surveyor can determine on which side of any one of the lines the required point lies, and can get the distances of the several points from the point sought, he can then solve the problem by locating the point at the proper proportional distances from the three lines and on the proper side of those lines. He is assisted in doing this work by the second and third rules, which are not necessary but are convenient to use.

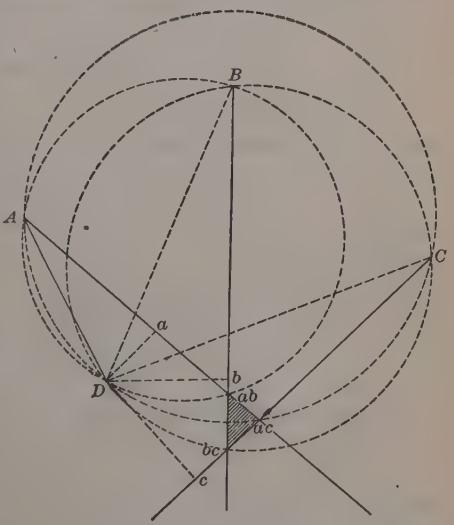


FIG. 238.

RULE 2. — When the point sought is without the great circle it is always on the same side of the line from the most distant point as the intersection of the other two lines. (See Fig. 239 *a*.)

RULE 3. — When the point sought falls within either of the three segments of the great circle, formed by the sides of the great triangle, the line drawn from the middle point lies between the point sought and the intersection of the other two lines. (See Fig. 239 *c*, *d*, *f*.)

Application of rules. — In practice the topographer first decides the relation of the new station with reference to the fixed points, whether it is within the great triangle or in one of the segments or outside the great circle. Then he determines the position of the point sought with reference to one line (if within one of the segments or without the great circle by Rule 2 or 3); it follows then, from Rule 1, that it must be on the corresponding side of the other two lines. Finally, he estimates the relative distances of the three actual points from him and marks the position of the point sought a proportionate distance from the three lines.

Repetition. — When the true point has been estimated and marked on

the sheet in accordance with the foregoing rules, a new orientation is made. If the lines from the three stations now intersect at that point, it proves the

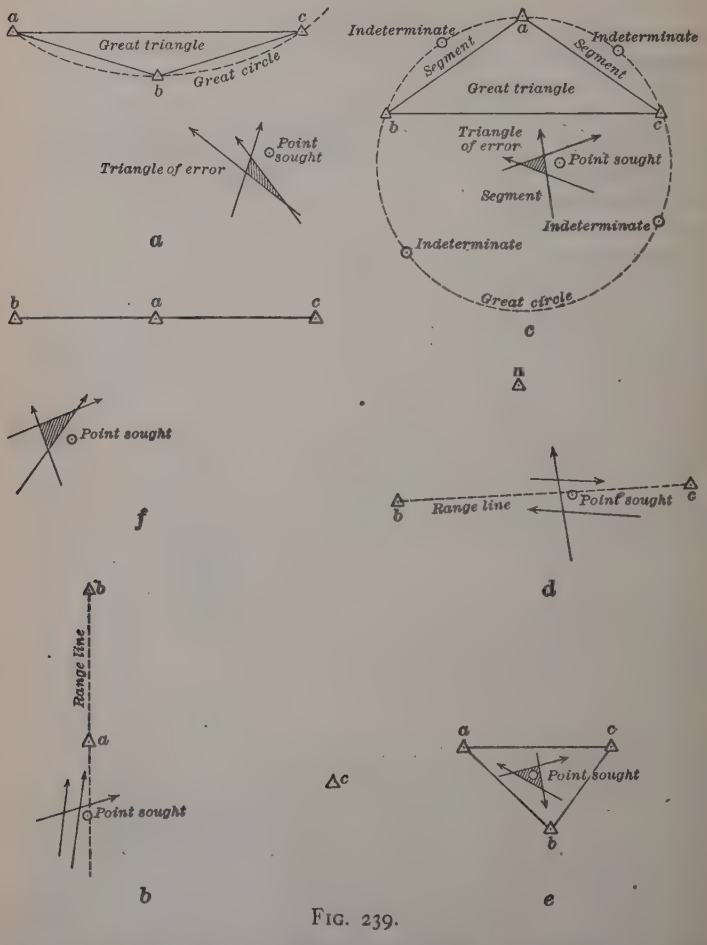


FIG. 239.

estimate to have been correct and the position is determined. If a new triangle of error is formed, it indicates an erroneous estimate, and the operation must be repeated.

Illustrative Examples

Figure 239 *a*: When the point sought is without the great circle, the intersection of the lines from *B* and *C* fall to the right of the line from *A*, the most distant point; therefore (Rule 2), the point sought must be on its right and also (Rule 1) on the right of the line from *B* and *C*. Its exact position is then estimated according to Rule 1.

Figure 239 *b*: When the point sought is on or near the prolongation of a range line, it must be outside the parallel lines on the side of the line to the nearest fixed point of the range. In the figure it will be seen that the point sought must be outside the lines from *A* and *B*, and to their right to satisfy Rule 1, and also to the right of the line from *C*.

Figure 239 *c*: When the point sought is on the circle passing through the three fixed points, the position is indeterminate, as the three lines will intersect at one point, although the table is imperfectly oriented. Another selection of points must be made.

Figure 239 *c*: When the point sought falls within one of the segments of the great circle, the line drawn from *A*, the middle point, is to the right of the intersection of the lines from *B* and *C*; therefore (Rule 3) the point sought must be on its right side, and also (Rule 1) to the right of the line from *B* and from *C*. Locate it exactly according to Rule 1.

Figure 239 *d*: When the point sought is on or near the range line between the fixed points, the point sought must be between the parallel lines to satisfy the conditions of Rule 1. Its position with reference to the intersecting line follows from the same rule. In the figure the point sought being between the lines from *B* and *C*, is to the right of each, therefore it is to the right of the line from *A*.

Figure 239 *e*: When the point sought falls within the great triangle, it must fall within the triangle of error. No other position would satisfy the conditions of Rule 1.

Figure 239 *f*: When the three fixed points are in a straight line. In this case the points are considered as being in the circumference of a circle of infinite diameter and the point sought always lying in one of the segments of the great circle. The treatment of this case is then identical with that of Figure 239 *c*.

The preceding cases are all examples of the conditions which may occur when the table is deflected to the right. By turning the printed side of the illustration to the light and looking at the figures through the paper, they will appear reversed, and they will then be examples of conditions which may occur when the table is deflected to the left.

VII. FIELD METHOD FOR DETERMINING REFRACTION CORRECTION TO THE SUN'S DECLINATION [1]

At any given hour note the time in seconds required for the sun to change its azimuth by 10 minutes of arc, and note the sun's altitude at the same time to the nearest half degree. Call the time required n, and the altitude, h.

From the following table, which may be copied in a notebook for field use, find the value of N corresponding to the determined value of h, and substitute it in the equation

$$\text{Correction, } c, \text{ in seconds of arc} = 100\ (N \div n). \qquad (1)$$

h	N	h	N
10°	131″ 45	30	36″ 14
15	86 24	40	22 9
20	62 15	50	13 6
25	47 11	60	7 4
30	36	70	3

The values of N are tabulated for a temperature of 50° F. and a barometric pressure of 30 inches. To adapt the results to other temperatures and pressures: diminish c one per cent for each 5° above 50° F. and increase c one per cent for each 5° below 50° F.; and diminish c one per cent for each 300 feet elevation above the sea. Except for considerable variations of temperature and pressure, these corrections may be neglected.

To determine n and h, set up the transit, bring the sun into the field of view, using a colored eyepiece, or, receiving the image on a card held back of the eyepiece, bring one edge of the sun tangent to the vertical wire and note the time to the nearest second. By the vernier tangent screw shift the alidade to the right (to the left in the southern hemisphere) through 10′ and note the time when the sun's edge, previously observed, again comes tangent to the vertical wire. The difference in the observed times is n, and h is read from the vertical circle.

EXAMPLE

Vernier		Watch			Vertical circle: 32° 08′
°		h.	m.	s.	$n = 57$
170	00	4	05	30	$N = 33$
170	10	4	06	27	$c = 58''$

[1] This method is due to Prof. George C. Comstock, Director of Washburn Observatory. See "Bulletin of the University of Wisconsin," Science Series, vol. I, No. 3.

It is true generally that a precision of 20″ is ample. When this is so and the sun's altitude is not less than 10°, we may substitute for N, in equation (1), $(1200 \div h)$, and divide by 60 to reduce c to minutes. The formula then becomes

$$c, \text{ in minutes of arc, } = \frac{2000}{hn}, \qquad (2)$$

and the table for N may be dispensed with. This final formula, so very simple in form, is almost always sufficiently precise for solar transit work.

VIII. TABLES

TABLE I

CORRECTION TO ONE HUNDRED UNITS MEASURED ALONG THE SLOPES GIVEN

UNITS' RISE IN 100	CORRESPONDING VERTICAL ANGLE	CORRECTION
1.02	0° 35′	0.005
2.01	1° 09′	0.020
3.03	1° 44′	0.046
4.02	2° 18′	0.081
5.01	2° 52′	0.125
6.00	3° 26′	0.179
7.00	4° 00′	0.244
8.02	4° 35′	0.320
9.01	5° 09′	0.404
10.01	5° 43′	0.497
20.01	11° 19′	1.617
30.00	16° 42′	4.218
40.00	21° 48′	7.151
50.00	26° 34′	10.559

TABLE II[1]

CORRECTION COEFFICIENT FOR TEMPERATURE AND HYGROMETRIC CONDITIONS

This correction is used when no hygrometric observations have been made. To the difference in altitude found in Table III for the given barometer

[1] Computed from Tables I and IV, Appendix 10, "U. S. Coast Survey Report" for 1881.

readings is added algebraically the product of that difference and the correction below given, according to the formula, Diff. Alt. = (Diff. by Table III) $(1 + c)$.

Sum O. T.[1]	Corr. Coeff.[2]	Sum O. T.	Corr. Coeff.	Sum O. T.	Corr. Coeff.
0°	− 0.1024	70°	− 0.0273	140°	+ 0.0471
10	− 0.0915	80	− 0.0166	150	+ 0.0575
20	− 0.0806	90	− 0.0058	160	+ 0.0677
30	− 0.0698	100	+ 0.0049	170	+ 0.0779
40	− 0.0592	110	+ 0.0156	180	+ 0.0879
50	− 0.0486	120	+ 0.0262		
60	− 0.0380	130	+ 0.0368		

[1] Sum of Observed Temperatures. [2] Correction Coefficient.

TABLE III[1]

Barometric Elevations

Giving altitudes above arbitrary sea level (barometer reading 30 inches) for various barometer readings B.

To determine difference of elevation of two points having barometer readings B and B_1, take from the table the altitudes corresponding to B and B_1, and correct *their difference* by Table II. The corrected difference is the quantity required.

B	A	Diff. for .01	B	A	Diff. for .01	B	A	Diff. for .01
Inches	*Feet*	*Feet*	*Inches*	*Feet*	*Feet*	*Inches*	*Feet*	*Feet*
11.0	27,336	− 24.6	12.1	24,740	− 22.4	13.2	22,368	− 20.6
11.1	27,090	24.4	12.2	24,516	22.2	13.3	22,162	20.4
11.2	26,846	24.2	12.3	24,294	22.1	13.4	21,958	20.1
11.3	26,604	24.0	12.4	24,073	21.9	13.5	21,757	20.0
11.4	26,364	23.8	12.5	23,854	21.7	13.6	21,557	19.9
11.5	26,126	23.6	12.6	23,637	21.6	13.7	21,358	19.8
11.6	25,890	23.4	12.7	23,421	21.4	13.8	21,160	19.8
11.7	25,656	23.2	12.8	23,207	21.2	13.9	20,962	19.7
11.8	25,424	23.0	12.9	22,995	21.0	14.0	20,765	19.5
11.9	25,194	22.8	13.0	22,785	20.9	14.1	20,570	19.3
12.0	24,966	22.6	13.1	22,576	20.8	14.2	20,377	19.1
12.1	24,740		13.2	22,368		14.3	20,186	

Taken from Appendix 10, "U. S. Coast and Geodetic Survey Report" for 1881.

TABLE III — *Continued*

B	*A*	DIFF. FOR .01	*B*	*A*	DIFF. FOR .01	*B*	*A*	DIFF. FOR .01
Inches	*Feet*	*Feet*	*Inches*	*Feet*	*Feet*	*Inches*	*Feet*	*Feet*
14.3	20,186	— 18.9	17.9	14,070	— 15.2	21.5	9,077	— 12.6
14.4	19,997	18.8	18.0	13,918	15.1	21.6	8,951	12.6
14.5	19,809	18.6	18.1	13,767	15.0	21.7	8,825	12.5
14.6	19,623	18.6	18.2	13,617	14.9	21.8	8,700	12.5
14.7	19,437	18.5	18.3	13,468	14.9	21.9	8,575	12.4
14.8	19,252	18.4	18.4	13,319	14.7	22.0	8,451	12.4
14.9	19,068	18.2	18.5	13,172	14.7	22.1	8,327	12.3
15.0	18,886	18.1	18.6	13,025	14.6	22.2	8,204	12.2
15.1	18,705	18.0	18.7	12,879	14.6	22.3	8,082	12.2
15.2	18,525	17.9	18.8	12,733	14.4	22.4	7,960	12.2
15.3	18,346	17.8	18.9	12,589	14.4	22.5	7,838	12.1
15.4	18,168	17.6	19.0	12,445	14.3	22.6	7,717	12.0
15.5	17,992	17.5	19.1	12,302	14.2	22.7	7,597	12.0
15.6	17,817	17.4	19.2	12,160	14.2	22.8	7,477	11.9
15.7	17,643	17.3	19.3	12,018	14.1	22.9	7,358	11.9
15.8	17,470	17.2	19.4	11,877	14.0	23.0	7,239	11.8
15.9	17,298	17.1	19.5	11,737	13.9	23.1	7,121	11.7
16.0	17,127	16.9	19.6	11,598	13.9	23.2	7,004	11.7
16.1	16,958	16.9	19.7	11,459	13.8	23.3	6,887	11.7
16.2	16,789	16.8	19.8	11,321	13.7	23.4	6,770	11.6
16.3	16,621	16.7	19.9	11,184	13.7	23.5	6,654	11.6
16.4	16,454	16.6	20.0	11,047	13.6	23.6	6,538	11.5
16.5	16,288	16.4	20.1	10,911	13.5	23.7	6,423	11.5
16.6	16,124	16.3	20.2	10,776	13.4	23.8	6,308	11.4
16.7	15,961	16.3	20.3	10,642	13.4	23.9	6,194	11.4
16.8	15,798	16.2	20.4	10,508	13.3	24.0	6,080	11.3
16.9	15,636	16.0	20.5	10,375	13.3	24.1	5,967	11.3
17.0	15,476	16.0	20.6	10,242	13.2	24.2	5,854	11.3
17.1	15,316	15.9	20.7	10,110	13.1	24.3	5,741	11.2
17.2	15,157	15.8	20.8	9,979	13.1	24.4	5,629	11.1
17.3	14,999	15.7	20.9	9,848	13.0	24.5	5,518	11.1
17.4	14,842	15.6	21.0	9,718	12.9	24.6	5,407	11.1
17.5	14,686	15.5	21.1	9,589	12.9	24.7	5,296	11.0
17.6	14,531	15.4	21.2	9,460	12.8	24.8	5,186	10.9
17.7	14,377	15.4	21.3	9,332	12.8	24.9	5,077	10.9
17.8	14,223	— 15.3	21.4	9,204	— 12.7	25.0	4,968	— 10.9
17.9	14,070		21.5	9,077		25.1	4,859	

TABLE III — *Continued*

B	*A*	DIFF. FOR .01	*B*	*A*	DIFF. FOR .01	*B*	*A*	DIFF. FOR .01
Inches	*Feet*	*Feet*	*Inches*	*Feet*	*Feet*	*Inches*	*Feet*	*Feet*
25.1	4,859		27.1	2,770		29.1	830	
		— 10.8			— 10.0			— 9.4
25.2	4,751	10.8	27.2	2,670	10.0	29.2	736	9.3
25.3	4,643	10.8	27.3	2,570	10.0	29.3	643	9.3
25.4	4,535	10.7	27.4	2,470	9.9	29.4	550	9.2
25.5	4,428	10.7	27.5	2,371	9.9	29.5	458	9.2
25.6	4,321	10.6	27.6	2,272	9.9	29.6	366	9.2
25.7	4,215	10.6	27.7	2,173	9.8	29.7	274	9.2
25.8	4,109	10.5	27.8	2,075	9.8	29.8	182	9.1
25.9	4,004	10.5	27.9	1,977	9.7	29.9	91	9.1
26.0	3,899	10.5	28.0	1,880	9.7	30.0	00	9.1
26.1	3,794	10.4	28.1	1,783	9.7	30.1	— 91	9.0
26.2	3,690	10.4	28.2	1,686	9.7	30.2	181	9.0
26.3	3,586	10.3	28.3	1,589	9.6	30.3	271	9.0
26.4	3,483	10.3	28.4	1,493	9.6	30.4	361	9.0
26.5	3,380	10.3	28.5	1,397	9.5	30.5	451	8.9
26.6	3,277	10.2	28.6	1,302	9.5	30.6	540	8.9
26.7	3,175	10.2	28.7	1,207	9.5	30.7	629	8.8
26.8	3,073	10.1	28.8	1,112	9.4	30.8	717	8.8
26.9	2,972	10.1	28.9	1,018	9.4	30.9	805	— 8.8
27.0	2,871	— 10.1	29.0	924	— 9.4	31.0	— 893	
27.1	2,770		29.1	830				

TABLE IV

POLAR DISTANCE OF POLARIS

For January 1 of years named

1894	1897	1900	1903	1906	1909	1912	1915	1918	1921
1° 15.43′	1° 14.50′	1° 13.55′	1° 12.62′	1° 11.68′	1° 10.75′	1° 09.82′	1° 08.88′	1° 07.97′	1° 07.03′

$$\text{Sin of azimuth at elongation} = \frac{\text{sin polar distance}}{\text{cosine latitude}}.$$

Latitude = altitude of Polaris at culmination ± polar distance − refraction correction given below.

Latitude or Altitude	Correction, Minutes	Latitude or Altitude	Correction, Minutes
20°	2.60	50°	0.80
30	1.65	60	0.55
40	1.13		

TABLE V[1]

Amount and Variation of the Magnetic Needle from its Mean Daily Position

The letters E and W indicate which side of the mean position the needle points.

Season and Position in Latitude	Local Mean Time; Morning Hours						
	6^h	7^h	8^h	9^h	10^h	11^h	12^h
	′	′	′	′	′	′	′
December, January, February:							
Latitude 37° to 49°	0.7 E	1.1 E	1.9 E	2.2 E	1.5 E	0.1 W	1.8 W
Latitude 25° to 37°	0.1 W	0.1 E	1.0 E	2.0 E	2.2 E	1.1 E	0.5 W
March, April, May:							
Latitude 37° to 49°	2.6 E	3.8 E	4.4 E	3.5 E	1.2 E	1.6 E	3.8 W
Latitude 25° to 37°	1.6 E	2.8 E	3.3 E	2.6 E	1.1 E	0.6 W	1.9 W
June, July, August:							
Latitude 37° to 49°	4.0 E	5.6 E	5.7 E	4.5 E	1.7 E	1.6 E	4.1 W
Latitude 25° to 37°	2.4 E	4.0 E	4.2 E	2.9 E	0.5 E	1.6 W	2.8 W
September, October, November:							
Latitude 37° to 49°	1.8 E	2.6 E	3.1 E	2.5 E	1.0 E	1.5 E	3.3 W
Latitude 25° to 37°	0.9 E	2.1 E	2.6 E	2.1 E	0.6 E	0.9 W	2.1 W

Season and Position in Latitude	Local Mean Time; Afternoon Hours						
	0^h	1^h	2^h	3^h	4^h	5^h	6^h
	′	′	′	′	′	′	′
December, January, February:							
Latitude 37° to 49°	1.8 W	2.9 W	2.8 W	2.1 W	1.3 W	0.7 W	0.2 W
Latitude 25° to 37°	0.5 W	1.5 W	1.8 W	1.6 W	1.0 W	0.4 W	0.1 W
March, April, May:							
Latitude 37° to 49°	3.8 W	4.8 W	4.6 W	3.8 W	2.5 W	1.4 W	0.7 W
Latitude 25° to 37°	1.9 W	2.6 W	2.8 W	2.4 W	1.6 W	0.9 W	0.5 W
June, July, August:							
Latitude 37° to 49°	4.1 W	5.6 W	5.6 W	4.6 W	3.0 W	1.4 W	0.6 W
Latitude 25° to 37°	2.8 W	3.2 W	3.1 W	2.4 W	1.5 W	0.8 W	0.4 W
September, October, November:							
Latitude 37° to 49°	3.3 W	4.0 W	3.4 W	2.3 W	1.2 W	0.6 W	0.1 W
Latitude 25° to 37°	2.1 W	2.3 W	1.9 W	1.2 W	0.7 W	0.4 W	0.2 W

[1] From "Manual of Instructions" issued by the U. S. Land Office to Surveyors General.

TABLE VI[1]

APPROXIMATE LOCAL MEAN TIMES (COUNTING FROM NOON 24 HOURS) OF THE ELONGATIONS AND CULMINATIONS OF POLARIS IN THE YEAR 1906 FOR LATITUDE 40° N.; LONGITUDE 6H W. FROM GREENWICH

DATE	EAST ELONGATION		WEST ELONGATION		UPPER CULMINATION		LOWER CULMINATION	
	h.	m.	h.	m.	h.	m.	h.	m.
Jan. 1	0	47.7	12	37.7	6	42.7	18	40.8
15	23	48.6	11	42.4	5	47.4	17	45.5
Feb. 1	22	41.5	10	35.3	4	40.3	16	38.4
15	21	46.2	9	39.8	3	45.0	15	43.1
Mar. 1	20	51.0	8	44.8	2	49.8	14	47.9
15	19	55.8	7	49.6	1	54.6	13	52.7
Apr. 1	18	48.9	6	42.7	0	47.7	12	45.8
15	17	53.7	5	47.5	23	48.7	11	50.6
May 1	16	50.9	4	44.7	22	45.9	10	47.8
15	15	56.0	3	49.8	21	51.0	9	52.9
June 1	14	49.4	2	43.2	20	44.4	8	46.3
15	13	54.5	1	48.3	19	49.5	7	51.4
July 1	12	51.9	0	45.7	18	46.9	6	48.8
15	11	57.1	23	47.1	17	52.1	5	54.0
Aug. 1	10	50.6	22	40.6	16	45.6	4	47.5
15	9	55.7	21	45.7	15	50.7	3	52.6
Sept. 1	8	49.1	20	39.1	14	44.1	2	46.0
15	7	54.2	19	44.2	13	49.2	1	51.1
Oct. 1	6	51.4	18	41.4	12	46.4	0	48.3
15	5	56.4	17	46.4	11	51.4	23	49.5
Nov. 1	4	49.6	16	39.6	10	44.6	22	42.7
15	3	54.5	15	44.5	9	49.5	21	47.6
Dec. 1	2	51.4	14	41.4	8	46.4	20	44.5
15	1	56.2	13	46.2	7	51.2	19	49.3

To refer to any calendar day other than the first and fifteenth of each month, subtract 3.94^m for every day between it and the preceding tabular day, or add 3.94^m for every day between it and the succeeding tabular day.

To find the times for the tabular dates after 1906, to the tabular value add 1.36^m for each year after 1906, less 3.9^m for each leap year. In any leap year deduction for that year is not made until March 1.

[1] From data furnished by the U. S. Coast and Geodetic Survey.

Example

For January 15, 1911, western elongation, tabular time is $11^h\ 42.4^m$. $1911 - 1906 = 5$; $5 \times 1.36 = 6.8$. One leap year has intervened, and, therefore, $6.8 - 3.9 = 2.9^m$ to be added. $11^h\ 42.4^m + 2.9^m = 11^h\ 45.3^m$.

For 1912, before March 1, add 4.2^m; on and after March 1, add $.3^m$.

To refer to any other than the tabular latitude between the limits of 25° and 50° north, *add* to the time of west elongation 0.13^m for every degree south of latitude 40°, and *subtract* from the time of west elongation 0.18^m for every degree north of 40°. Reverse these signs for corrections to the times of east elongation. For latitudes as high as 60° diminish the times of *west* elongation and increase the times of *east* elongation by 0.23^m for every degree north of latitude 40°.

TABLE VII [1]

Refraction Corrections to Declination of the Sun

The hour angle is the time either side of noon.

Latitude	Hour Angle	Declinations								
		+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°
° ′	h	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
25 00	0	0 05	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 57
	2	0 08	0 14	0 19	0 25	0 31	0 38	0 46	0 54	1 05
	3	0 12	0 18	0 24	0 30	0 37	0 44	0 53	1 04	1 18
	4	0 23	0 29	0 35	0 45	0 53	1 03	1 16	1 31	1 52
	5	0 49	0 59	1 10	1 24	1 52	2 07	2 44	3 46	5 43
27 30	0	0 08	0 13	0 18	0 24	0 30	0 36	0 44	0 52	1 02
	2	0 11	0 16	0 22	0 28	0 34	0 41	0 49	1 00	1 10
	3	0 17	0 22	0 28	0 35	0 42	0 50	1 00	1 11	1 26
	4	0 28	0 35	0 42	0 50	1 00	1 11	1 26	1 43	2 09
	5	0 54	1 05	1 18	1 34	1 54	2 24	3 11	4 38	8 15
30 00	0	0 10	0 15	0 21	0 27	0 33	0 40	0 48	0 57	1 08
	2	0 14	0 19	0 25	0 31	0 38	0 46	0 54	1 05	1 18
	3	0 20	0 26	0 32	0 39	0 47	0 55	1 06	1 19	1 36
	4	0 32	0 39	0 46	0 52	1 06	1 19	1 35	1 57	2 29
	5	1 00	1 10	1 24	1 52	2 07	2 44	3 46	5 43	13 06
32 30	0	0 13	0 18	0 24	0 30	0 36	0 44	0 52	1 02	1 14
	2	0 17	0 22	0 28	0 35	0 42	0 50	1 00	1 11	1 26
	3	0 23	0 29	0 35	0 43	0 51	1 01	1 13	1 28	1 47
	4	0 35	0 43	0 51	1 01	1 13	1 27	1 46	2 13	2 54
	5	1 03	1 15	1 31	1 53	2 20	3 05	4 25	7 36	—
35 00	0	0 15	0 21	0 27	0 33	0 40	0 48	0 57	1 08	1 21
	2	0 20	0 25	0 32	0 38	0 46	0 55	1 05	1 18	1 35
	3	0 26	0 33	0 39	0 47	0 56	1 07	1 21	1 38	2 00
	4	0 39	0 47	0 56	1 07	1 20	1 36	1 59	2 32	3 25
	5	1 07	1 20	1 38	2 00	2 34	3 29	5 14	10 16	—

[1] Computed from formula

$$C = 57'' \cot (\delta + N),$$

in which δ is the declination, plus when north, and minus when south; and N an auxiliary angle found by

$$\tan N = \cot \phi \cos t,$$

in which ϕ is the latitude of the place, and t the angle between the meridian of the place and the meridian through the sun at the given time, — called the "hour angle." The formulæ are from Chauvenet's "Spherical and Practical Astronomy," vol. I, p. 171. The table was computed by Mr. Edward W. Arms, C.E., for Messrs. W. & L. E. Gurley, of Troy, N.Y., and is used here by their permission.

TABLE VII — *Continued*

Latitude	Hour Angle	Declinations								
		+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°
° ′	h	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
37 30	0	0 18	0 24	0 30	0 36	0 44	0 52	1 02	1 14	1 29
	2	0 22	0 28	0 35	0 42	0 50	1 00	1 12	1 26	1 45
	3	0 29	0 36	0 43	0 52	1 02	1 14	1 29	1 49	2 16
	4	0 43	0 51	1 01	1 13	1 27	1 49	2 14	2 54	4 05
	5	1 11	1 26	1 44	2 10	2 49	3 55	6 15	14 58	—
40 00	0	0 21	0 27	0 33	0 40	0 48	0 57	1 08	1 21	1 03
	2	0 25	0 32	0 39	0 46	0 52	1 06	1 19	1 35	1 57
	3	0 33	0 40	0 48	0 57	1 08	1 21	1 38	2 02	2 36
	4	0 47	0 55	1 06	1 19	1 36	1 58	2 30	3 21	4 59
	5	1 15	1 31	1 51	2 20	3 05	4 25	7 34	25 18	—
42 30	0	0 24	0 30	0 36	0 44	0 52	1 02	1 14	1 29	1 49
	2	0 28	0 35	0 39	0 50	1 00	1 12	1 26	1 45	2 11
	3	0 36	0 43	0 52	1 02	1 13	1 29	1 49	2 17	2 59
	4	0 50	1 00	1 11	1 26	1 44	2 10	2 49	3 55	6 16
	5	1 16	1 36	1 58	2 30	3 22	5 00	9 24	—	—
45 00	0	0 27	0 33	0 40	0 48	0 57	1 08	1 21	1 39	2 02
	2	0 32	0 39	0 46	0 52	1 06	1 19	1 35	1 57	2 29
	3	0 40	0 47	0 56	1 07	1 21	1 38	2 00	2 34	3 29
	4	0 54	1 04	1 16	1 33	1 54	2 24	3 11	4 38	8 15
	5	1 23	1 41	2 05	2 41	3 40	5 40	12 02	—	—
47 30	0	0 30	0 36	0 44	0 52	1 02	1 14	1 29	1 49	2 18
	2	0 35	0 42	0 50	1 00	1 12	1 26	1 45	2 01	2 51
	3	0 43	0 51	1 01	1 13	1 28	1 47	2 15	2 56	4 08
	4	0 56	1 09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
	5	1 27	1 46	2 12	2 52	4 01	6 30	16 19	—	—
50 00	0	0 33	0 40	0 48	0 57	1 08	1 21	1 39	2 02	2 36
	2	0 38	0 46	0 55	1 06	1 18	1 35	1 57	2 28	3 19
	3	0 47	0 56	1 06	1 19	1 36	2 29	2 31	3 23	5 02
	4	1 02	1 14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
	5	1 30	1 51	2 19	3 04	4 22	7 28	24 10	—	—
52 30	0	0 36	0 44	0 52	1 02	1 14	1 29	1 49	2 18	3 05
	2	0 43	0 50	0 59	1 11	1 26	1 42	2 23	2 49	3 55
	3	0 50	1 00	1 11	1 26	1 45	2 11	2 51	2 58	6 22
	4	1 05	1 18	1 35	2 10	2 28	3 19	4 53	8 42	—
	5	1 34	1 56	2 27	3 16	4 47	8 52	—	—	—
55 00	0	0 40	0 48	0 57	1 08	1 21	1 39	2 02	2 36	3 33
	2	0 46	0 55	1 05	1 18	1 34	1 56	2 30	3 15	4 47
	3	0 55	1 06	1 19	1 35	1 58	2 30	3 21	4 58	9 19
	4	1 10	1 23	1 42	2 06	2 43	3 44	5 49	12 41	—
	5	1 37	2 01	2 34	3 28	5 15	10 18	—	—	—

TABLE VIII

Radii and their logarithms, and tangent offsets, and middle ordinates, for 100 feet chords of curves of degrees given. The degrees are given in degrees and decimals of a degree.

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
0.00	∞	∞	.000	.000	1.00	5729.65	3.75813	.873	.218
.02	286478.90	5.45709	.017	.004	.02	5617.31	.74953	.890	.223
.04	143239.45	.15606	.035	.009	.04	5509.29	.74110	.908	.227
.06	95492.97	4.97997	.052	.013	.06	5405.34	.73283	.925	.231
.08	71619.73	.85503	.070	.017	.08	5305.24	.72471	.942	.236
.10	57295.79	4.75812	.087	.022	.10	5208.79	3.71674	.960	.240
.12	47746.49	.67894	.105	.026	.12	5115.78	.70891	.977	.244
.14	40925.57	.61199	.122	.031	.14	5026.03	.70122	.995	.248
.16	35809.87	.55400	.140	.035	.16	4939.38	.69367	1.012	.253
.18	31831.00	.50285	.157	.039	.18	4855.66	.68625	.030	.257
.20	28647.90	4.45709	.175	.044	.20	4774.74	3.67895	1.047	.262
.22	26043.55	.41570	.192	.048	.22	4696.46	.67177	.065	.266
.24	23873.26	.37791	.209	.052	.24	4620.72	.66471	.082	.271
.26	22036.86	.34315	.227	.057	.26	4547.38	.65776	.100	.275
.28	20462.80	.31097	.244	.061	.28	4476.33	.65092	.117	.279
.30	19098.61	4.28100	.262	.065	.30	4407.46	3.64419	1.134	.284
.32	17904.95	.25297	.279	.070	.32	4340.69	.63756	.152	.288
.34	16851.73	.22664	.297	.074	.34	4275.90	.63103	.169	.292
.36	15915.52	.20182	.314	.079	.36	4213.02	.62459	.187	.297
.38	15077.86	.17834	.332	.083	.38	4151.97	.61825	.204	.301
.40	14323.97	4.15606	.349	.087	.40	4092.66	3.61201	1.222	.305
.42	13641.88	.13487	.367	.092	.42	4035.02	.60585	.239	.310
.44	13021.80	.11467	.384	.096	.44	3978.98	.59977	.257	.314
.46	12455.64	.09537	.401	.100	.46	3924.47	.59378	.274	.319
.48	11936.66	.07688	.419	.105	.48	3871.44	.58787	.292	.323
.50	11459.19	4.05915	.436	.109	.50	3819.83	3.58204	1.309	.327
.52	11018.46	.04212	.454	.113	.52	3769.57	.57629	.326	.332
.54	10610.37	.02573	.471	.118	.54	3720.62	.57061	.344	.336
.56	10231.43	.00994	.489	.122	.56	3672.92	.56501	.361	.340
.58	9878.62	3.99470	.506	.127	.58	3626.43	.55948	.379	.345
.60	9549.34	3.97997	.524	.131	.60	3581.10	3.55402	1.396	.349
.62	9241.30	.96573	.541	.135	.62	3536.89	.54862	.414	.353
.64	8952.51	.95194	.559	.140	.64	3493.76	.54329	.431	.358
.66	8681.26	.93858	.576	.144	.66	3451.67	.53803	.449	.362
.68	8425.90	.92562	.593	.148	.68	3410.59	.53283	.466	.367
.70	8185.16	3.91303	.611	.153	.70	3370.46	3.52769	1.483	.371
.72	7957.80	.90079	.628	.157	.72	3331.28	.52261	.501	.375
.74	7742.73	.88889	.646	.161	.74	3292.99	.51759	.518	.380
.76	7539.97	.87731	.663	.166	.76	3255.57	.51263	.536	.384
.78	7345.67	.86603	.681	.170	.78	3218.99	.50772	.553	.388
.80	7162.03	3.85504	.698	.175	.80	3183.23	3.50287	1.571	.393
.82	6987.35	.84431	.716	.179	.82	3148.25	.49807	.588	.397
.84	6820.99	.83385	.733	.183	.84	3114.03	.49332	.606	.401
.86	6662.36	.82363	.750	.188	.86	3080.55	.48863	.623	.406
.88	6510.95	.81364	.768	.192	.88	3047.78	.48398	.641	.410
.90	6366.26	3.80388	.785	.196	.90	3015.71	3.47939	1.658	.415
.92	6227.87	.79434	.803	.201	.92	2984.29	.47484	.675	.419
.94	6095.36	.78500	.820	.205	.94	2953.53	.47034	.693	.423
.96	5968.38	.77586	.838	.209	.96	2923.40	.46589	.710	.428
.98	5846.58	.76690	.855	.214	.98	2893.87	.46148	.728	.432
1.00	5729.65	3.75813	.873	.218	2.00	2864.93	3.45711	1.745	.436

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
2.00	2864.93	3.45711	1.745	.436	3.00	1910.08	3.28105	2.618	.655
.02	2836.57	.45279	.763	.441	.02	1897.43	.27817	.635	.659
.04	2808.76	.44852	.780	.445	.04	1884.95	.27530	.653	.663
.06	2781.50	.44428	.798	.449	.06	1872.63	.27245	.670	.668
.08	2754.76	.44008	.815	.454	.08	1860.48	.26962	.687	.672
.10	2728.52	3.43593	1.832	.458	.10	1848.48	3.26681	2.705	.676
.12	2702.79	.43181	.850	.463	.12	1836.63	.26402	.722	.681
.14	2677.53	.42773	.867	.467	.14	1824.93	.26125	.740	.685
.16	2652.74	.42369	.885	.471	.16	1813.39	.25849	.757	.689
.18	2628.40	.41969	.902	.476	.18	1801.99	.25575	.775	.694
.20	2604.51	3.41573	1.920	.480	.20	1790.73	3.25303	2.792	.698
.22	2581.05	.41180	.937	.484	.22	1779.61	.25032	.810	.702
.24	2558.01	.40790	.955	.489	.24	1768.62	.24764	.827	.707
.26	2535.38	.40404	.972	.493	.26	1757.78	.24496	.845	.711
.28	2513.14	.40022	.990	.497	.28	1747.06	.24231	.862	.716
.30	2491.29	3.39642	2.007	.502	.30	1736.48	3.23967	2.879	.720
.32	2469.81	.39266	.024	.506	.32	1726.02	.23705	.897	.724
.34	2448.71	.38894	.042	.511	.34	1715.69	.23444	.914	.729
.36	2427.96	.38524	.059	.515	.36	1705.48	.23185	.932	.733
.38	2407.56	.38158	.077	.519	.38	1695.39	.22927	.949	.737
.40	2387.50	3.37794	2.094	.524	.40	1685.42	3.22671	2.967	.742
.42	2367.77	.37434	.112	.528	.42	1675.56	.22416	.984	.746
.44	2348.36	.37077	.129	.532	.44	1665.83	.22163	3.002	.751
.46	2329.28	.36722	.147	.537	.46	1656.19	.21911	.019	.755
.48	2310.49	.36370	.164	.541	.48	1646.68	.21661	.036	.759
.50	2292.01	3.36022	2.181	.545	.50	1637.28	3.21412	3.054	.764
.52	2273.83	.35676	.199	.550	.52	1627.98	.21165	.071	.768
.54	2255.92	.35332	.216	.554	.54	1618.78	.20919	.089	.772
.56	2238.30	.34992	.234	.559	.56	1609.69	.20674	.106	.777
.58	2220.95	.34654	.251	.563	.58	1600.70	.20431	.124	.781
.60	2203.87	3.34319	2.269	.567	.60	1591.81	3.20189	3.141	.785
.62	2187.05	.33986	.286	.572	.62	1583.02	.19949	.159	.790
.64	2170.49	.33656	.304	.576	.64	1574.32	.19709	.176	.794
.66	2154.17	.33328	.321	.580	.66	1565.72	.19472	.193	.799
.68	2138.10	.33003	.339	.585	.68	1557.22	.19235	.211	.803
.70	2122.26	3.32681	2.356	.589	.70	1548.80	3.19000	3.228	.807
.72	2106.66	.32359	.373	.593	.72	1540.48	.18766	.246	.812
.74	2091.29	.32041	.391	.598	.74	1532.24	.18533	.263	.816
.76	2076.13	.31726	.408	.602	.76	1524.10	.18301	.281	.820
.78	2061.20	.31412	.426	.607	.78	1516.14	.18071	.298	.825
.80	2046.48	3.31101	2.443	.611	.80	1508.06	3.17842	3.316	.829
.82	2031.97	.30792	.460	.615	.82	1500.17	.17614	.333	.833
.84	2017.66	.30485	.478	.620	.84	1492.36	.17387	.351	.838
.86	2003.56	.30180	.496	.624	.86	1484.63	.17162	.368	.842
.88	1989.65	.29878	.513	.628	.88	1476.98	.16937	.385	.847
.90	1975.93	3.29577	2.530	.633	.90	1469.41	3.16714	3.403	.851
.92	1962.40	.29279	.548	.637	.92	1461.91	.16492	.420	.855
.94	1949.05	.28982	.565	.641	.94	1454.49	.16271	.438	.860
.96	1935.88	.28688	.583	.646	.96	1447.15	.16051	.455	.864
.98	1922.89	.28396	.600	.650	.98	1439.88	.15833	.473	.868
3.00	1910.08	3.28105	2.618	.655	4.00	1432.69	3.15615	3.490	.873

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
4.00	1432.69	3.15615	3.490	.873	5.00	1146.28	3.05929	4.362	1.091
.02	1425.56	.15399	.507	.877	.02	1141.72	.05756	.379	.095
.04	1418.51	.15183	.525	.881	.04	1137.19	.05583	.397	.100
.06	1411.52	.14969	.542	.886	.06	1132.70	.05411	.414	.104
.08	1404.60	.14755	.560	.890	.08	1128.24	.05240	.432	.108
.10	1397.76	3.14543	3.577	.895	.10	1123.82	3.05070	4.449	1.113
.12	1390.98	.14332	.595	.899	.12	1119.43	.04900	.467	.117
.14	1384.26	.14122	.612	.903	.14	1115.08	.04731	.484	.122
.16	1377.61	.13912	.629	.908	.16	1111.00	.04571	.501	.126
.18	1371.02	.13704	.647	.912	.18	1106.47	.04304	.519	.130
.20	1364.49	3.13497	3.664	.916	.20	1102.22	3.04227	4.536	1.135
.22	1358.03	.13291	.682	.921	.22	1098.00	.04060	.554	.139
.24	1351.31	.13076	.699	.925	.24	1093.81	.03894	.571	.143
.26	1345.28	.12881	.717	.929	.26	1089.66	.03729	.589	.148
.28	1339.00	.12678	.734	.934	.28	1085.53	.03564	.606	.152
.30	1332.77	3.12476	3.752	.938	.30	1081.44	3.03400	4.623	1.156
.32	1326.61	.12274	.769	.943	.32	1077.38	.03237	.641	.161
.34	1320.49	.12074	.786	.947	.34	1073.34	.03074	.658	.165
.36	1314.44	.11874	.804	.951	.36	1069.34	.02912	.676	.170
.38	1308.44	.11675	.821	.956	.38	1065.37	.02750	.693	.174
.40	1302.50	3.11478	3.839	.960	.40	1061.43	3.02589	4.711	1.178
.42	1296.61	.11281	.856	.964	.42	1057.51	.02429	.728	.183
.44	1290.77	.11085	.874	.969	.44	1053.63	.02269	.746	.187
.46	1284.98	.10890	.891	.973	.46	1049.77	.02109	.763	.191
.48	1279.25	.10696	.909	.978	.48	1045.94	.01951	.780	.196
.50	1273.57	3.10502	3.926	.982	.50	1042.14	3.01793	4.798	1.200
.52	1267.93	.10310	.943	.986	.52	1038.37	.01635	.815	.205
.54	1262.35	.10118	.961	.991	.54	1034.62	.01478	.833	.209
.56	1256.82	.09927	.978	.995	.56	1030.90	.01322	.850	.213
.58	1251.33	.09737	.996	.999	.58	1027.21	.01166	.868	.218
.60	1245.89	3.09548	4.013	1.004	.60	1023.55	3.01011	4.885	1.222
.62	1240.50	.09360	.031	.008	.62	1019.91	.00856	.902	.226
.64	1235.16	.09172	.048	.012	.64	1016.29	.00702	.920	.231
.66	1229.86	.08986	.065	.017	.66	1012.70	.00548	.937	.235
.68	1224.61	.08800	.083	.021	.68	1009.14	.00395	.955	.239
.70	1219.40	3.08615	4.100	1.026	.70	1005.60	3.00243	4.972	1.244
.72	1214.24	.08430	.118	.030	.72	1002.09	.00091	.990	.248
.74	1209.12	.08247	.135	.034	.74	998.60	2.99939	5.007	.253
.76	1204.04	.08064	.153	.039	.76	995.14	.99788	.024	.257
.78	1199.00	.07882	.170	.043	.78	991.68	.99638	.042	.261
.80	1194.01	3.07701	4.188	1.047	.80	988.28	2.99488	5.059	1.266
.82	1189.06	.07520	.205	.052	.82	984.89	.99339	.077	.270
.84	1184.15	.07341	.222	.056	.84	981.52	.99190	.094	.274
.86	1179.28	.07162	.240	.060	.86	978.17	.99041	.112	.279
.88	1174.45	.06983	.257	.065	.88	974.66	.98885	.129	.283
.90	1169.66	3.06806	4.275	1.069	.90	971.54	2.98746	5.146	1.287
.92	1164.91	.06629	.292	.074	.92	968.26	.98599	.164	.292
.94	1160.19	.06453	.310	.078	.94	965.01	.98453	.181	.296
.96	1155.52	.06278	.327	.082	.96	961.77	.98307	.199	.301
.98	1150.88	.06103	.345	.087	.98	958.56	.98162	.216	.305
5.00	1146.28	3.05929	4.362	1.091	6.00	955.37	2.98017	5.234	1.309

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
6.00	955.37	2.98017	5.234	1.309	7.00	819.02	2.91329	6.105	1.528
.02	952.20	.97873	.251	.314	.02	816.69	.91206	.122	.532
.04	949.05	.97729	.268	.318	.04	814.37	.91082	.140	.536
.06	945.92	.97585	.286	.322	.06	812.07	.90959	.157	.541
.08	942.81	.97442	.303	.327	.08	809.78	.90837	.175	.545
.10	939.72	2.97300	5.321	1.331	.10	807.50	2.90714	6.192	1.550
.12	936.65	.97158	.338	.335	.12	805.23	.90592	.209	.554
.14	933.60	.97016	.356	.340	.14	802.98	.90471	.227	.558
.16	930.57	.96875	.373	.344	.16	800.74	.90349	.244	.563
.18	927.58	.96734	.390	.349	.18	798.51	.90228	.262	.567
.20	924.58	2.96594	5.408	1.353	.20	796.30	2.90108	6.279	1.572
.22	921.61	.96455	.425	.357	.22	794.10	.89987	.296	.576
.24	918.66	.96315	.443	.362	.24	791.90	.89867	.314	.580
.26	915.72	.96176	.460	.366	.26	789.73	.89748	.331	.584
.28	912.81	.96038	.478	.370	.28	787.56	.89628	.349	.589
.30	909.92	2.95900	5.495	1.375	.30	785.40	2.89509	6.366	1.593
.32	906.48	.95736	.512	.379	.32	783.26	.89391	.384	.598
.34	904.18	.95625	.530	.384	.34	781.13	.89272	.401	.602
.36	901.34	.95489	.547	.389	.36	779.01	.89154	.418	.606
.38	898.52	.95353	.565	.392	.38	776.90	.89037	.436	.611
.40	895.71	2.95217	5.582	1.397	.40	774.81	2.88919	6.453	1.615
.42	892.92	.95081	.600	.401	.42	772.72	.88802	.471	.619
.44	890.15	.94947	.617	.405	.44	770.65	.88685	.488	.624
.46	887.40	.94812	.634	.410	.46	768.58	.88569	.505	.628
.48	884.67	.94678	.652	.414	.48	766.53	.88453	.523	.632
.50	881.95	2.94544	5.669	1.418	.50	764.49	2.88337	6.540	1.637
.52	879.24	.94411	.687	.423	.52	762.46	.88222	.558	.641
.54	876.56	.94278	.704	.427	.54	760.44	.88106	.575	.646
.56	873.89	.94146	.722	.432	.56	758.45	.87993	.503	.650
.58	871.24	.94014	.739	.436	.58	756.43	.87877	.610	.654
.60	868.60	2.93882	5.756	1.440	.60	754.44	2.87763	6.627	1.659
.62	865.98	.93751	.774	.445	.62	752.47	.87649	.645	.663
.64	863.37	.93620	.791	.449	.64	750.50	.87535	.662	.667
.66	860.78	.93489	.809	.453	.66	748.54	.87422	.680	.672
.68	858.21	.93359	.826	.458	.68	746.60	.87309	.607	.676
.70	855.65	2.93230	5.844	1.462	.70	744.66	2.87196	6.714	1.680
.72	853.10	.93100	.861	.466	.72	742.74	.87083	.732	.685
.74	850.58	.92971	.878	.471	.74	740.82	.86971	.749	.689
.76	848.06	.92843	.896	.475	.76	738.91	.86859	.767	.694
.78	845.56	.92715	.913	.480	.78	737.02	.86748	.784	.698
.80	843.08	2.92587	5.931	1.484	.80	735.13	2.86636	6.802	1.702
.82	840.61	.92459	.948	.488	.82	733.25	.86525	.819	.707
.84	838.16	.92332	.965	.493	.84	731.38	.86415	.836	.711
.86	835.71	.92206	.983	.497	.86	729.53	.86304	.854	.715
.88	833.29	.92080	6.000	1.501	.88	727.68	.86194	.871	.720
.90	830.88	2.91954	6.018	1.505	.90	725.85	2.86084	6.889	1.724
.92	828.48	.91828	.035	.510	.92	724.01	.85974	.906	.729
.94	826.09	.91703	.053	.515	.94	722.19	.85865	.923	.733
.96	823.72	.91578	.070	.519	.96	720.38	.85756	.941	.737
.98	821.36	.91454	.087	.523	.98	718.57	.85647	.958	.742
7.00	819.02	2.91329	6.105	1.528	8.00	716.78	2.85539	6.976	1.746

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
8.00	716.78	2.85539	6.976	1.746	9.00	637.27	2.80433	7.846	1.965
.02	714.99	.85430	.993	.750	.02	635.86	.80336	.863	.969
.04	713.22	.85322	7.010	.755	.04	634.46	.80240	.881	.973
.06	711.45	.85215	.028	.759	.06	633.06	.80145	.898	.978
.08	709.69	.85107	.045	.764	.08	631.67	.80049	.916	.982
.10	707.94	2.85000	7.063	1.768	.10	630.29	2.79954	7.933	1.987
.12	706.20	.84893	.080	.772	.12	628.91	.79859	.950	.991
.14	704.47	.84786	.098	.777	.14	627.53	.79764	.968	.995
.16	702.75	.84680	.115	.781	.16	626.17	.79669	.985	.999
.18	701.03	.84574	.132	.785	.18	624.81	.79574	8.002	2.004
.20	699.33	2.84468	7.150	1.790	.20	623.45	2.79480	8.020	2.008
.22	697.63	.84362	.167	.794	.22	622.10	.79386	.037	.013
.24	695.94	.84257	.185	.798	.24	620.76	.79292	.055	.017
.26	694.25	.84152	.202	.803	.26	619.42	.79198	.072	.021
.28	692.58	.84047	.219	.807	.28	618.09	.79105	.090	.026
.30	690.91	2.83942	7.237	1.811	.30	616.76	2.79012	8.107	2.030
.32	689.26	.83838	.254	.816	.32	615.44	.78919	.124	.034
.34	687.61	.83734	.272	.820	.34	614.12	.78826	.142	.039
.36	685.96	.83630	.289	.825	.36	612.82	.78733	.159	.043
.38	684.33	.83527	.306	.829	.38	611.51	.78640	.176	.048
.40	682.70	2.83423	7.324	1.833	.40	610.21	2.78548	8.194	2.052
.42	681.09	.83320	.341	.838	.42	608.92	.78456	.211	.056
.44	679.47	.83217	.359	.842	.44	607.63	.78364	.229	.061
.46	677.87	.83115	.376	.847	.46	606.35	.78272	.246	.065
.48	676.27	.83012	.393	.851	.48	605.08	.78181	.263	.069
.50	674.69	2.82910	7.411	1.855	.50	603.80	2.78090	8.281	2.074
.52	673.11	.82808	.428	.860	.52	602.54	.77999	.298	.078
.54	671.53	.82707	.446	.864	.54	601.28	.77908	.316	.083
.56	669.97	.82605	.463	.868	.56	600.02	.77817	.333	.087
.58	668.41	.82504	.480	.873	.58	598.77	.77726	.350	.091
.60	666.86	2.82403	7.498	1.877	.60	597.53	2.77636	8.368	2.096
.62	665.33	.82303	.515	.881	.62	596.29	.77546	.385	.100
.64	663.77	.82202	.533	.886	.64	595.06	.77456	.403	.104
.66	662.24	.82102	.550	.890	.66	593.83	.77366	.420	.109
.68	660.72	.82002	.567	.895	.68	592.60	.77276	.437	.113
.70	659.21	2.81902	7.585	1.899	.70	591.38	2.77187	8.455	2.117
.72	657.70	.81803	.602	.903	.72	590.17	.77098	.472	.121
.74	656.19	.81703	.620	.908	.74	588.96	.77009	.490	.126
.76	654.70	.81604	.637	.912	.76	587.76	.76920	.507	.131
.78	653.21	.81505	.655	.916	.78	586.56	.76831	.524	.135
.80	651.73	2.81407	7.672	1.921	.80	585.36	2.76743	8.542	2.139
.82	650.25	.81308	.689	.925	.82	584.17	.76654	.559	.144
.84	648.79	.81210	.707	.930	.84	582.99	.76566	.576	.148
.86	647.32	.81112	.724	.934	.86	581.81	.76478	.594	.152
.88	645.87	.81014	.742	.938	.88	580.64	.76390	.611	.157
.90	644.42	2.80917	7.759	1.943	.90	579.46	2.76303	8.629	2.161
.92	642.98	.80820	.776	.947	.92	578.30	.76215	.646	.166
.94	641.54	.80723	.794	.951	.94	577.14	.76128	.663	.170
.96	640.11	.80626	.811	.956	.96	575.99	.76041	.681	.174
.98	638.69	.80529	.829	.960	.98	574.83	.75954	.698	.179
9.00	637.27	2.80433	7.846	1.965	10.00	573.69	2.75867	8.716	2.183

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
10.0	573.69	2.75867	8.716	2.183	15.0	383.06	2.58327	13.053	3.277
.1	568.02	.75436	.803	.205	.1	380.54	.58040	.139	.299
.2	562.47	.75010	.889	.227	.2	378.05	.57755	.226	.321
.3	557.02	.74587	.976	.234	.3	375.60	.57472	.312	.343
.4	551.68	.74169	9.063	.270	.4	373.17	.57191	.399	.365
.5	546.44	2.73754	9.150	2.293	.5	370.78	2.56912	13.485	3.387
.6	541.30	.73344	.237	.314	.6	368.42	.56634	.572	.409
.7	536.25	.72937	.324	.336	.7	366.09	.56358	.658	.431
.8	531.30	.72534	.411	.358	.8	363.78	.56084	.744	.452
.9	526.44	.72135	.498	.380	.9	361.51	.55812	.831	.474
11.0	521.67	2.71740	9.585	2.402	16.0	359.26	2.55541	13.917	3.496
.1	516.99	.71348	.671	.423	.1	357.05	.55273	14.004	.518
.2	512.38	.70960	.758	.445	.2	354.86	.55006	.090	.540
.3	507.86	.70575	.845	.467	.3	352.70	.54740	.177	.562
.4	503.42	.70193	.932	.489	.4	350.56	.54476	.263	.584
.5	499.06	2.69815	10.019	2.511	.5	348.45	2.54214	14.349	3.606
.6	494.77	.69441	.106	.533	.6	346.37	.53953	.436	.628
.7	490.56	.69069	.192	.555	.7	344.31	.53694	.522	.650
.8	486.42	.68701	.279	.577	.8	342.27	.53437	.608	.672
.9	482.34	.68336	.366	.598	.9	340.26	.53181	.695	.694
12.0	478.34	2.67974	10.453	2.620	17.0	338.27	2.52927	14.781	3.716
.1	474.40	.67614	.540	.642	.1	336.31	.52674	.867	.738
.2	470.53	.67258	.626	.664	.2	334.37	.52423	.954	.760
.3	466.72	.66905	.713	.686	.3	332.45	.52173	15.040	.781
.4	462.96	.66555	.800	.708	.4	330.56	.51924	.126	.803
.5	459.28	2.66207	10.887	2.730	.5	328.69	2.51677	15.212	3.825
.6	455.65	.65863	.973	.752	.6	326.83	.51432	.299	.847
.7	452.07	.65521	11.060	.774	.7	325.00	.51188	.385	.869
.8	448.56	.65182	.147	.795	.8	323.18	.50945	.471	.891
.9	445.09	.64845	.234	.817	.9	321.39	.50704	.557	.913
13.0	441.68	2.64511	11.320	2.839	18.0	319.62	2.50464	15.643	3.935
.1	438.33	.64180	.407	.861	.1	317.87	.50225	.730	.957
.2	435.02	.63851	.494	.883	.2	316.14	.49988	.816	.979
.3	431.76	.63525	.580	.905	.3	314.43	.49752	.902	4.001
.4	428.56	.63201	.667	.927	.4	312.73	.49517	.988	.023
.5	425.40	2.62879	11.754	2.949	.5	311.06	2.49284	16.074	4.045
.6	422.28	.62560	.840	.971	.6	309.40	.49052	.160	.067
.7	419.22	.62244	.927	.992	.7	307.76	.48821	.246	.089
.8	416.19	.61929	12.014	3.014	.8	306.14	.48591	.333	.111
.9	413.21	.61617	.100	.036	.9	304.53	.48363	.419	.133
14.0	410.28	2.61308	12.187	3.058	19.0	302.94	2.48136	16.505	4.155
.1	407.38	.61000	.274	.080	.1	301.37	.47910	.591	.177
.2	404.53	.60695	.360	.102	.2	299.82	.47686	.677	.199
.3	401.71	.60391	.447	.124	.3	298.28	.47462	.763	.221
.4	398.94	.60090	.533	.146	.4	296.75	.47240	.849	.243
.5	396.20	2.59791	12.620	3.168	.5	295.25	2.47019	16.935	4.265
.6	393.50	.59495	.706	.190	.6	293.76	.46799	17.021	.287
.7	390.84	.59200	.793	.211	.7	292.28	.46580	.107	.308
.8	388.21	.58907	.880	.233	.8	290.82	.46362	.193	.330
.9	385.62	.58616	.966	.255	.9	289.37	.46145	.279	.352
15.0	383.06	2.58327	13.053	3.277	20.0	287.94	2.45930	17.365	4.374

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
20.0	287.94	2.45930	17.365	4.374	25.0	231.01	2.36363	21.644	5.476
.1	286.52	.45716	.451	.396	.1	230.11	.36193	.729	.498
.2	285.12	.45502	.537	.418	.2	229.21	.36023	.814	.520
.3	283.73	.45290	.623	.440	.3	228.32	.35854	.899	.542
.4	282.35	.45079	.708	.462	.4	227.43	.35685	.985	.564
.5	280.99	2.44869	17.794	4.484	.5	226.55	2.35517	22.070	5.586
.6	279.64	.44660	.880	.506	.6	225.68	.35350	.155	.608
.7	278.30	.44452	.966	.528	.7	224.82	.35184	.240	.630
.8	276.98	.44245	18.052	.550	.8	223.96	.35018	.325	.653
.9	275.67	.44039	.138	.572	.9	223.11	.34853	.410	.675
21.0	274.37	2.43834	18.224	4.594	26.0	222.27	2.34688	22.495	5.697
.1	273.08	.43630	.309	.616	.1	221.43	.34524	.580	.719
.2	271.81	.43427	.395	.638	.2	220.60	.34361	.665	.741
.3	270.55	.43225	.481	.660	.3	219.78	.34199	.750	.763
.4	269.30	.43024	.567	.682	.4	218.96	.34037	.835	.785
.5	268.06	2.42824	18.652	4.704	.5	218.15	2.33875	22.920	5.807
.6	266.84	.42624	.738	.726	.6	217.34	.33715	23.005	.829
.7	265.62	.42426	.824	.748	.7	216.54	.33555	.090	.852
.8	264.42	.42229	.910	.770	.8	215.75	.33395	.175	.874
.9	263.22	.42033	.995	.792	.9	214.96	.33237	.260	.896
22.0	262.04	2.41837	19.081	4.814	27.0	214.18	2.33078	23.345	5.918
.1	260.87	.41643	.167	.836	.1	213.41	.32921	.429	.940
.2	259.71	.41449	.252	.858	.2	212.64	.32764	.514	.962
.3	258.56	.41256	.338	.881	.3	211.87	.32608	.599	.984
.4	257.42	.41064	.423	.903	.4	211.11	.32452	.684	6.006
.5	256.29	2.40873	19.509	4.925	.5	210.36	2.32297	23.769	6.029
.6	255.17	.40683	.595	.947	.6	209.61	.32142	.853	.051
.7	254.06	.40494	.680	.969	.7	208.87	.31988	.938	.073
.8	252.96	.40306	.766	.991	.8	208.14	.31835	24.023	.095
.9	251.87	.40118	.851	5.013	.9	207.40	.31682	.108	.117
23.0	250.79	2.39931	19.937	5.035	28.0	206.68	2.31529	24.192	6.139
.1	249.72	.39746	20.022	.057	.1	205.96	.31378	.277	.161
.2	248.66	.39561	.108	.079	.2	205.24	.31227	.362	.184
.3	247.61	.39376	.193	.101	.3	204.53	.31076	.446	.206
.4	246.56	.39193	.279	.123	.4	203.83	.30926	.531	.228
.5	245.53	2.39010	20.364	5.145	.5	203.13	2.30776	24.615	6.250
.6	244.50	.38829	.450	.167	.6	202.43	.30627	.700	.272
.7	243.49	.38647	.535	.189	.7	201.74	.30479	.784	.294
.8	242.48	.38467	.620	.211	.8	201.05	.30331	.869	.316
.9	241.48	.38288	.706	.233	.9	200.37	.30184	.954	.339
24.0	240.49	2.38109	20.791	5.255	29.0	199.70	2.30037	25.038	6.361
.1	239.50	.37931	.877	.278	.1	199.02	.29891	.122	.383
.2	238.53	.37754	.962	.299	.2	198.36	.29745	.207	.405
.3	237.56	.37578	21.047	.321	.3	197.70	.29600	.291	.427
.4	236.60	.37402	.132	.343	.4	197.04	.29455	.376	.450
.5	235.65	2.37227	21.218	5.366	.5	196.38	2.29311	25.460	6.472
.6	234.71	.37053	.303	.388	.6	195.74	.29167	.545	.494
.7	233.77	.36879	.388	.410	.7	195.09	.29024	.629	.516
.8	232.84	.36707	.474	.432	.8	194.45	.28881	.713	.538
.9	231.92	.36535	.559	.454	.9	193.82	.28739	.798	.560
25.0	231.01	2.36363	21.644	5.476	30.0	193.19	2.28597	25.882	6.583

Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m	Deg. D	Radius R	Logarithm Log R	Tan. Off. t	Mid. Ord. m
°					°				
30.0	193.19	2.28597	25.882	6.583	35.0	166.28	2.22083	30.071	7.696
.1	192.56	.28456	.966	.605	.1	165.82	.21963	.154	.718
.2	191.94	.28315	26.050	.627	.2	165.36	.21843	.237	.740
.3	191.32	.28175	.135	.649	.3	164.91	.21724	.320	.763
.4	190.70	.28036	.219	.671	.4	164.46	.21605	.403	.785
.5	190.09	2.27896	26.303	6.694	.5	164.01	2.21486	30.486	7.807
.6	189.49	.27757	.387	.716	.6	163.56	.21368	.570	.830
.7	188.88	.27619	.471	.738	.7	163.12	.21250	.653	.852
.8	188.28	.27481	.556	.760	.8	162.68	.21133	.736	.874
.9	187.69	.27344	.640	.782	.9	162.24	.21016	.819	.897
31.0	187.10	2.27207	26.724	6.805	36.0	161.80	2.20899	30.902	7.919
.1	186.51	.27071	.808	.827	.1	161.37	.20782	.985	.942
.2	185.93	.26935	.892	.849	.2	160.94	.20666	31.068	.964
.3	185.35	.26799	.976	.871	.3	160.51	.20550	.151	.986
.4	184.77	.26664	27.060	.894	.4	160.08	.20435	.233	8.009
.5	184.20	2.26530	27.144	6.916	.5	159.66	2.20320	31.316	8.031
.6	183.63	.26395	.228	.938	.6	159.24	.20205	.399	.053
.7	183.07	.26262	.312	.960	.7	158.82	.20091	.482	.076
.8	182.51	.26128	.396	.983	.8	158.40	.19977	.565	.098
.9	181.95	.25996	.480	7.005	.9	157.99	.19863	.648	.121
32.0	181.40	2.25863	27.564	7.027	37.0	157.58	2.19749	31.730	8.143
.1	180.85	.25731	.648	.049	.1	157.17	.19636	.813	.165
.2	180.30	.25600	.731	.072	.2	156.76	.19523	.896	.188
.3	179.76	.25469	.815	.094	.3	156.35	.19411	.979	.210
.4	179.22	.25338	.899	.116	.4	155.95	.19299	32.061	.233
.5	178.68	2.25208	27.983	7.138	.5	155.55	2.19187	32.144	8.255
.6	178.15	.25078	28.067	.161	.6	155.15	.19076	.227	.277
.7	177.62	.24949	.150	.183	.7	154.75	.18964	.309	.300
.8	177.09	.24820	.234	.205	.8	154.36	.18854	.392	.322
.9	176.57	.24691	.318	.227	.9	153.97	.18743	.474	.345
33.0	176.05	2.24563	28.402	7.250	38.0	153.58	2.18633	32.557	8.367
.1	175.53	.24435	.485	.272	.1	153.19	.18523	.639	.390
.2	175.02	.24308	.569	.294	.2	152.80	.18413	.722	.412
.3	174.51	.24181	.652	.316	.3	152.42	.18304	.804	.434
.4	174.00	.24054	.736	.339	.4	152.04	.18195	.887	.457
.5	173.49	2.23928	28.820	7.361	.5	151.66	2.18086	32.969	8.479
.6	172.99	.23802	.903	.383	.6	151.28	.17978	33.051	.502
.7	172.49	.23677	.987	.406	.7	150.90	.17870	.134	.524
.8	172.00	.23552	29.070	.428	.8	150.53	.17762	.216	.547
.9	171.50	.23428	.154	.450	.9	150.16	.17655	.298	.569
34.0	171.02	2.23303	29.237	7.473	39.0	149.79	2.17547	33.381	8.592
.1	170.53	.23180	.321	.495	.1	149.42	.17441	.463	.614
.2	170.04	.23056	.404	.517	.2	149.05	.17334	.545	.636
.3	169.56	.22933	.487	.539	.3	148.69	.17228	.627	.659
.4	169.09	.22811	.571	.562	.4	148.33	.17122	.710	.681
.5	168.61	2.22688	29.654	7.584	.5	147.97	2.17016	33.792	8.704
.6	168.14	.22567	.737	.606	.6	147.61	.16911	.874	.726
.7	167.67	.22445	.821	.629	.7	147.25	.16805	.956	.749
.8	167.20	.22324	.904	.651	.8	146.89	.16701	34.038	.771
.9	166.74	.22203	.987	.673	.9	146.54	.16596	.120	.794
35.0	166.28	2.22083	30.071	7.696	40.0	146.19	2.16492	34.202	8.816

TABLE IX

Angular Convergences and Distances between Meridians

1. Angular convergence of meridians per mile of easting or westing.
2. Distance between meridians converging by one minute.

Latitude	Angular Convergence per Mile Minutes	Distance for Convergence of 1′ Feet	Latitude	Angular Convergence per Mile Minutes	Distance for Convergence of 1′ Feet
°			°		
1	0.015	348733	31	0.521	10140
2	.030	174314	32	.542	9751
3	.045	116150	33	.563	9382
4	.061	87052	34	.585	9034
5	.076	69578	35	.607	8703
6	.091	57917	36	.630	8387
7	.107	49578	37	.653	8087
8	.122	43337	38	.677	7801
9	.137	38436	39	.702	7527
10	.153	34525	40	.727	7265
11	.169	31320	41	.753	7013
12	.184	28642	42	.780	6770
13	.200	26371	43	.808	6537
14	.216	24419	44	.836	6313
15	.232	22723	45	.866	6097
16	.249	21234	46	.897	5888
17	.265	19916	47	.929	5686
18	.282	18740	48	.962	5491
19	.299	17685	49	.998	5301
20	.316	16731	50	1.032	5118
21	.333	15864	51	1.069	4940
22	.350	15073	52	1.108	4766
23	.368	14348	53	1.149	4597
24	.386	13680	54	1.191	4433
25	.404	13062	55	1.236	4271
26	.423	12488	56	1.283	4115
27	.442	11955	57	1.333	3962
28	.461	11457	58	1.385	3813
29	.480	10990	59	1.440	3666
30	.500	10552	60	1.499	3523

TABLE X[1]

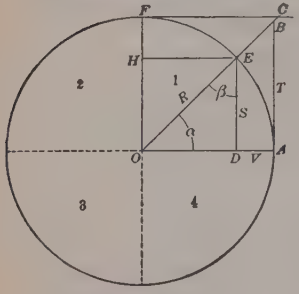
Length of One Minute of Latitude and One Minute of Longitude to the Nearest Whole Foot

Latitude	1′ Latitude Feet	1′ Longitude Feet	Latitude	1′ Latitude Feet	1′ Longitude Feet
°			°		
1	6046	6086	31	6062	5222
2	6046	6083	32	6063	5167
3	6046	6079	33	6064	5110
4	6046	6072	34	6065	5052
5	6046	6064	35	6066	4992
6	6047	6054	36	6067	4930
7	6047	6042	37	6068	4867
8	6047	6028	38	6069	4803
9	6047	6013	39	6070	4737
10	6048	5995	40	6071	4670
11	6048	5976	41	6072	4601
12	6049	5955	42	6074	4531
13	6049	5932	43	6075	4459
14	6050	5908	44	6076	4386
15	6050	5881	45	6077	4312
16	6051	5853	46	6078	4236
17	6051	5823	47	6079	4159
18	6052	5791	48	6080	4081
19	6052	5758	49	6081	4001
20	6053	5722	50	6082	3921
21	6054	5685	51	6083	3839
22	6055	5647	52	6084	3756
23	6055	5606	53	6085	3671
24	6056	5564	54	6086	3586
25	6057	5520	55	6087	3499
26	6058	5475	56	6088	3412
27	6059	5427	57	6089	3323
28	6059	5379	58	6090	3234
29	6060	5328	59	6091	3143
30	6061	5276	60	6092	3051

[1] Abbreviated from the Smithsonian Geographical Tables.

TABLE XI

Trigonometric Functions and Formulas. Solution of Triangles



By definition, if $R = 1$,

ED = sine α.
OD = cosine α.
DA = versed sine α.
HF = coversed sine α.
BA = tangent α.
FC = cotangent α.
OB = secant α.
OC = cosecant α.

If R is other than 1, it follows from the above definitions and the proportionality of similar figures, that

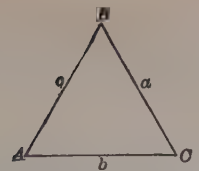
1. $ED = R \sin \alpha$.
2. $OD = R \cos \alpha$.
3. $DA = R \operatorname{versin} \alpha$.
4. $HF = R \operatorname{coversin} \alpha$.
5. $BA = R \tan \alpha$.
6. $FC = R \cot \alpha$.
7. $OB = R \sec \alpha$.
8. $OC = R \operatorname{cosec} \alpha$.

from which also in *any right triangle* of angles α and β, if o be the side *opposite* the angle α, a the side *adjacent* thereto, and h the hypotenuse,

9. $\sin \alpha = \dfrac{o}{h} = \cos \beta$.
10. $\cos \alpha = \dfrac{a}{h} = \sin \beta$.
11. $\tan \alpha = \dfrac{o}{a} = \cot \beta$.
12. $\cot \alpha = \dfrac{a}{o} = \tan \beta$.
13. $\sec \alpha = \dfrac{h}{a} = \operatorname{cosec} \beta$.
14. $\operatorname{cosec} \alpha = \dfrac{h}{o} = \sec \beta$.
15. $\operatorname{vers} \alpha = \dfrac{h-a}{h} = \operatorname{covers} \beta$.
16. $\operatorname{covers} \alpha = \dfrac{h-o}{h} = \operatorname{vers} \beta$.

Hence,

17. $\begin{cases} o = h \sin \alpha = h \cos \beta. \\ h = \dfrac{o}{\sin \alpha} = \dfrac{o}{\cos \beta}. \end{cases}$
18. $\begin{cases} a = h \cos \alpha = h \sin \beta. \\ h = \dfrac{a}{\cos \alpha} = \dfrac{a}{\sin \beta}. \end{cases}$
19. $\begin{cases} o = a \tan \alpha = a \cot \beta. \\ a = \dfrac{o}{\tan \alpha} = \dfrac{o}{\cot \beta}. \end{cases}$
20. $\begin{cases} a = o \cot \alpha = o \tan \beta. \\ o = \dfrac{a}{\cot \alpha} = \dfrac{a}{\tan \beta}. \end{cases}$
21. $\begin{cases} h = a \sec \alpha = a \operatorname{cosec} \beta. \\ a = \dfrac{h}{\sec \alpha} = \dfrac{h}{\operatorname{cosec} \beta}. \end{cases}$
22. $\begin{cases} h = o \operatorname{cosec} \alpha = o \sec \beta. \\ o = \dfrac{h}{\operatorname{cosec} \alpha} = \dfrac{h}{\sec \beta}. \end{cases}$
23. $o = \sqrt{h^2 - a^2} = \sqrt{(h+a)(h-a)}$.
24. $a = \sqrt{h^2 - o^2} = \sqrt{(h+o)(h-o)}$.
25. $h = \sqrt{o^2 + a^2}$.
26. Area $= \dfrac{oa}{2}$.



Oblique triangles may be solved by some one of the following formulas:

GIVEN	SOUGHT	FORMULAS
27. $A, B, a,$	$C, b, c,$	$C = 180° - (A + B)$, $b = \frac{a}{\sin A} \sin B$, $c = \frac{a}{\sin A} \sin (A + B)$.
28. $A, a, b,$	$B, C, c,$	$\sin B = \frac{\sin A}{a} b$, $C = 180° - (A + B)$, $c = \frac{a}{\sin A} \sin C$.
29. $C, a, b,$	$\frac{1}{2}(A + B),$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C.$
30.	$\frac{1}{2}(A - B),$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B).$
31.	$A, B,$	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$; $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B).$
32.	$c,$	$c = (a + b) \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}.$
33.	Area,	Area $= \frac{1}{2} ab \sin C.$
34. $a, b, c,$	$A,$	If $s = \frac{1}{2}(a + b + c)$, $\sin \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}}$, $\cos \frac{1}{2} A = \sqrt{\frac{s(s - a)}{bc}}$, $\tan \frac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}}$, $\sin A = \frac{2\sqrt{(s - a)(s - b)(s - c)}}{bc}$, $\text{vers } A = \frac{2(s - b)(s - c)}{bc}.$
35.	Area,	Area $= \sqrt{s(s - a)(s - b)(s - c)}.$
36. $A, B, C, a,$	Area,	Area $= \frac{a^2 \sin B \sin C}{2 \sin A}.$

From the definitions of the trigonometric functions, the geometrical properties of right triangles and in some cases algebraic transformations, it may be shown that if A is any angle and B any other angle,

37. $\sin^2 A + \cos^2 A = 1.$

38. $\sin A = \dfrac{1}{\operatorname{cosec} A} = \sqrt{1 - \cos^2 A} = \tan A \cos A$
$= 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \operatorname{vers} A \cot \tfrac{1}{2} A$
$= \sqrt{\tfrac{1}{2} \operatorname{vers} 2A} = \sqrt{\tfrac{1}{2}(1 - \cos 2A)}.$

39. $\cos A = \dfrac{1}{\sec A} = \sqrt{1 - \sin^2 A} = \cot A \sin A$
$= 1 - \operatorname{vers} A = 2 \cos^2 \tfrac{1}{2} A - 1 = 1 - 2 \sin^2 \tfrac{1}{2} A$
$= \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = \sqrt{\tfrac{1}{2} + \tfrac{1}{2} \cos 2A}.$

40. $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{1}{\cot A} = \sqrt{\sec^2 A - 1}$
$= \sqrt{\dfrac{1}{\cos^2 A} - 1} = \dfrac{\sqrt{1 - \cos^2 A}}{\cos A} = \dfrac{\sin 2A}{1 + \cos 2A}$
$= \dfrac{1 - \cos 2A}{\sin 2A} = \dfrac{\operatorname{vers} 2A}{\sin 2A} = \cot \tfrac{1}{2} A \, (\sec A - 1).$

41. $\cot A = \dfrac{\cos A}{\sin A} = \dfrac{1}{\tan A} = \sqrt{\operatorname{cosec}^2 A - 1}$
$= \dfrac{\sin 2A}{1 - \cos 2A} = \dfrac{\sin 2A}{\operatorname{vers} 2A} = \dfrac{1 + \cos 2A}{\sin 2A} = \dfrac{\tan \tfrac{1}{2} A}{\sec A - 1}.$

42. $\operatorname{vers} A = 1 - \cos A = \sin A \tan \tfrac{1}{2} A = 2 \sin^2 \tfrac{1}{2} A = \cos A \, (\sec A - 1).$

43. $\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A.$

44. $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B.$

45. $\sin \tfrac{1}{2} A = \sqrt{\dfrac{1 - \cos A}{2}} = \sqrt{\dfrac{\operatorname{vers} A}{2}}.$

46. $\sin 2A = 2 \sin A \cos A.$

47. $\cos \tfrac{1}{2} A = \sqrt{\dfrac{1 + \cos A}{2}}.$

48. $\cos 2A = 2 \cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A.$

49. $\tan \tfrac{1}{2} A = \dfrac{\tan A}{1 + \sec A} = \operatorname{cosec} A - \cot A = \dfrac{1 - \cos A}{\sin A} = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}.$

50. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}.$

51. $\cot \frac{1}{2} A = \dfrac{\sin A}{\operatorname{vers} A} = \dfrac{1 + \cos A}{\sin A} = \dfrac{1}{\operatorname{cosec} A - \cot A}$.

52. $\cot 2A = \dfrac{\cot^2 A - 1}{2 \cot A}$.

53. $\operatorname{vers} \frac{1}{2} A = \dfrac{\frac{1}{2} \operatorname{vers} A}{1 + \sqrt{1 - \frac{1}{2} \operatorname{vers} A}} = \dfrac{1 - \cos A}{2 + \sqrt{2(1 + \cos A)}}$.

54. $\operatorname{vers} 2A = 2 \sin^2 A$.

55. $\sin A + \sin B = 2 \sin \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$.

56. $\sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$.

57. $\cos A + \cos B = 2 \cos \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$.

58. $\cos B - \cos A = 2 \sin \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$.

59. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - B)$.

60. $\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$.

61. $\tan A + \tan B = \dfrac{\sin (A + B)}{\cos A \cos B}$.

62. $\tan A - \tan B = \dfrac{\sin (A - B)}{\cos A \cos B}$.

TABLE XII

Lengths of Circular Arcs of Radius 1, and Various Circular Measures

No.	Degrees.	Minutes.	Seconds.	No.	Degrees.	Minutes.	Seconds.
1	.0174533	.0002909	.0000048	6	.1047198	.0017453	.0000291
2	.0349066	.0005818	.0000097	7	.1221730	.0020362	.0000339
3	.0523599	.0008727	.0000145	8	.1396263	.0023271	.0000388
4	.0698132	.0011636	.0000194	9	.1570796	.0026180	.0000436
5	.0872665	.0014544	.0000242	10	.1745329	.0029089	.0000485

Degrees in arc of length equal to radius, 57.295 780.
Degrees in arc of length equal to π, 180.
Circumference $= 2\pi r =$ 360°.
Area $= \pi r^2$.

If l = length of circular arc
d = number of degrees in same
r = radius of same
c = chord of same
m = middle ordinate

$$d = \frac{l}{r} \cdot \frac{180^\circ}{\pi} = \frac{l}{r} 57.3^\circ \text{ approx.}$$

$$r = \frac{l}{d} \cdot \frac{180^\circ}{\pi} = \frac{l}{d} 57.3^\circ \text{ approx.}$$

$$l = \frac{d}{180^\circ} \pi r = \frac{d}{57.3^\circ} r \text{ approx.}$$

$$\text{Area of sector} = \tfrac{1}{2} lr.$$

$$\text{Area of sector} = \frac{d}{360} \pi r^2.$$

$$\text{Approximate area of segment} = \tfrac{2}{3}\, cm.$$

TABLE XIII

Linear Transformations

1. *Gunter's Chains to Feet*

Chains	0.0	0.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0		.66	1.32	1.98	2.64	3.30	3.96	4.62	5.28	5.94
.1	6.60	7.26	7.92	8.58	9.24	9.90	10.56	11.22	11.88	12.54
.2	13.20	13.86	14.52	15.18	15.84	16.50	17.16	17.82	18.48	19.14
.3	19.80	20.46	21.12	21.78	22.44	23.10	23.76	24.42	25.08	25.74
.4	26.40	27.06	27.72	28.38	29.04	29.70	30.36	31.02	31.68	32.34
.5	33.00	33.66	34.32	34.98	35.64	36.30	36.96	37.62	38.28	38.94
.6	39.60	40.26	40.92	41.58	42.24	42.90	43.56	44.22	44.88	45.54
.7	46.20	46.86	47.52	48.18	48.84	49.50	50.16	50.82	51.48	52.14
.8	52.80	53.46	54.12	54.78	55.44	56.10	56.76	57.42	58.08	58.74
.9	59.40	60.06	60.72	61.38	62.04	62.70	63.36	64.02	64.68	65.34

	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
0.0		66	132	198	264	330	396	462	528	594
10.0	660	726	792	858	924	990	1056	1122	1188	1254
20.0	1320	1386	1452	1518	1584	1650	1716	1782	1848	1914
30.0	1980	2046	2112	2178	2244	2310	2376	2442	2508	2574
40.0	2640	2706	2772	2838	2904	2970	3036	3102	3168	3234
50.0	3300	3366	3432	3498	3564	3630	3696	3762	3828	3894
60.0	3960	4026	4092	4158	4224	4290	4356	4422	4488	4554
70.0	4620	4686	4752	4818	4884	4950	5016	5082	5148	5214
80.0	5280	5346	5412	5478	5544	5610	5676	5742	5808	5874

Chains	0.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
0.0	—	.2012	.4023	.6035	.8047	1.0058	1.2070	1.4082	1.6093	1.8105
.1	2.0117	2.2128	2.4140	2.6152	2.8163	3.0175	3.2187	3.4198	3.6210	3.8222
.2	4.0233	4.2245	4.4257	4.6268	4.8280	5.0292	5.2303	5.4315	5.6326	5.8338
.3	6.0350	6.2361	6.4373	6.6385	6.8396	7.0408	7.2420	7.4431	7.6443	7.8455
.4	8.0466	8.2478	8.4490	8.6501	8.8513	9.0525	9.2536	9.4548	9.6560	9.8571
.5	10.0583	10.2595	10.4606	10.6618	10.8630	11.0641	11.2653	11.4665	11.6676	11.8688
.6	12.0700	12.2711	12.4723	12.6735	12.8746	13.0758	13.2770	13.4781	13.6793	13.8805
.7	14.0816	14.2828	14.4840	14.6851	14.8863	15.0875	15.2886	15.4898	15.6910	15.8921
.8	16.0933	16.2945	16.4956	16.6968	16.8980	17.0991	17.3003	17.5015	17.7026	17.9038
.9	18.1050	18.3061	18.5073	18.7084	18.9096	19.1108	19.3119	19.5131	19.7143	19.9154
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
0.0	—	20.1166	40.2332	60.3499	80.4665	100.5831	120.6996	140.8163	160.9329	181.0495
10.0	201.1662	221.2828	241.3994	261.5160	281.6326	301.7493	321.8659	341.9825	362.0991	382.2157
20.0	402.3323	422.4489	442.5656	462.6822	482.7988	502.9154	523.0320	543.1487	563.2653	583.3819
30.0	603.4985	623.6151	643.7317	663.8483	683.9649	704.0816	724.1982	744.3148	764.4314	784.5480
40.0	804.6646	824.7812	844.8979	865.0145	885.1311	905.2477	925.3643	945.4810	965.5976	985.7142
50.0	1005.8308	1025.9474	1046.0640	1066.1806	1086.2973	1106.4139	1126.5305	1146.6471	1166.7637	1186.8803
60.0	1206.9969	1227.1135	1247.2302	1267.3468	1287.4634	1307.5800	1327.6966	1347.8133	1367.9299	1388.0465
70.0	1408.1631	1428.2797	1448.3963	1468.5129	1488.6296	1508.7462	1528.8628	1548.9794	1569.0960	1589.2126
80.0	1609.3292	1629.4459	1649.5625	1669.6791	1689.7957	1709.9123	1730.0290	1750.1456	1770.2622	1790.3788

3. *Feet to Meters*

Feet	0	1	2	3	4	5	6	7	8	9
	Meters	Meters	Meters	Meters	Meters	Meters	Meters	Meters	Meters	Meters
0	0.000	0.305	0.610	0.914	1.219	1.524	1.829	2.134	2.438	2.743
10	3.048	3.353	3.658	3.962	4.267	4.572	4.877	5.182	5.486	5.791
20	6.036	6.401	6.706	7.010	7.315	7.620	7.925	8.229	8.534	8.839
30	9.144	9.449	9.753	10.058	10.363	10.668	10.972	11.277	11.582	11.887
40	12.192	12.496	12.801	13.106	13.411	13.716	14.020	14.325	14.630	14.935
50	15.239	15.544	15.849	16.154	16.459	16.763	17.068	17.373	17.678	17.983
60	18.287	18.592	18.897	19.202	19.507	19.811	20.116	20.421	20.726	21.031
70	21.335	21.640	21.945	22.250	22.555	22.859	23.164	23.469	23.774	24.079
80	24.383	24.688	24.993	25.298	25.602	25.907	26.212	26.517	26.822	27.126
90	27.431	27.736	28.041	28.346	28.651	28.955	29.260	29.565	29.870	30.174
100	30.479	30.784	31.089	31.394	31.698	32.003	32.308	32.613	32.918	33.222

4. *Meters to Feet*

Meters	0	1	2	3	4	5	6	7	8	9
	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
0	0.00	3.28	6.56	9.84	13.12	16.40	19.69	22.97	26.25	29.53
10	32.81	36.09	39.37	42.65	45.93	49.21	52.49	55.78	59.06	62.34
20	65.62	68.90	72.18	75.46	78.74	82.02	85.30	88.58	91.87	95.15
30	98.43	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.96
40	131.24	134.52	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.33	170.61	173.89	177.17	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.42	206.70	209.98	213.26	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.51	242.79	246.07	249.35	252.63	255.91	259.19
80	262.47	265.75	269.03	272.31	275.60	278.88	282.16	285.44	288.72	292.00
90	295.28	298.56	391.84	305.12	308.40	311.69	314.97	318.25	321.53	324.81
100	328.09	331.37	334.65	337.93	341.21	344.49	347.78	351.06	354.34	357.62

1 statute mile = 1.6093 kilometers
1 kilometer = 0.6214 statute mile

TABLE XIV — Stadia Reductions for 100 Unit Readings and Various Vertical Angles

M.	0°		1°		2°		3°		4°		5°		6°		7°		8°		9°		10°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45	96.98	17.10
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15	98.05	13.84	97.53	15.51	96.96	17.16
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21	98.03	13.89	97.52	15.56	96.94	17.21
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26	98.01	13.95	97.50	15.62	96.92	17.26
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32	98.00	14.01	97.48	16.67	96.90	17.32
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38	97.98	14.06	97.46	15.73	96.88	17.37
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43	97.97	14.12	97.44	15.78	96.86	17.43
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49	97.95	14.17	97.43	15.84	96.84	17.48
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55	97.93	14.23	97.41	15.89	96.82	17.54
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60	97.92	14.28	97.39	15.95	96.80	17.59
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66	97.90	14.34	97.37	16.00	96.78	17.65
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72	97.88	14.40	97.35	16.06	96.76	17.70
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77	97.87	14.45	97.33	16.11	96.74	17.76
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83	97.85	14.51	97.31	16.17	96.72	17.81
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88	97.83	14.56	97.29	16.22	96.70	17.86
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94	97.82	14.62	97.28	16.28	96.68	17.92
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00	97.80	14.67	97.26	16.33	96.66	17.97
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05	97.78	14.73	97.24	16.39	96.64	18.03
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11	97.76	14.79	97.22	16.44	96.62	18.08
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17	97.75	14.84	97.20	16.50	96.60	18.14
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22	97.73	14.90	97.18	16.55	96.57	18.19
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28	97.71	14.95	97.16	16.61	96.55	18.24
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33	97.69	15.01	97.14	16.66	96.53	18.30
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39	97.68	15.06	97.12	16.72	96.51	18.35
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45	97.66	15.12	97.10	16.77	96.49	18.41
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50	97.64	15.17	97.08	16.83	96.47	18.46
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56	97.62	15.23	97.06	16.88	96.45	18.51
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61	97.61	15.28	97.04	16.94	96.42	18.57
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67	97.59	15.34	97.02	16.99	96.40	18.62
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73	97.57	15.40	97.00	17.05	96.38	18.68
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
$c = 0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10	0.74	0.11	0.74	0.12	0.74	0.14
$c = 1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06	1.00	0.08	0.99	0.90	0.99	0.11	0.99	0.13	0.99	0.15	0.99	0.16	0.98	0.18
$c = 1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16	1.23	0.18	1.23	0.21	1.23	0.23

TABLE XIV—*Continued*

M	11°		12°		13°		14°		15°		16°		17°		18°		19°		20°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	96.36	18.73	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
2	96.34	18.78	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83	88.26	32.18
4	96.32	18.84	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87	88.23	32.23
6	96.29	18.89	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92	88.19	32.27
8	96.27	18.95	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97	88.15	32.32
10	96.25	19.00	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01	88.11	32.36
12	96.23	19.05	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06	88.08	32.41
14	96.21	19.11	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10	88.04	32.45
16	96.18	19.16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15	88.00	32.49
18	96.16	19.21	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19	87.96	32.54
20	96.14	19.27	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24	87.93	32.58
22	96.12	19.32	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28	87.89	32.63
24	96.09	19.38	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33	87.85	32.67
26	96.07	19.43	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38	87.81	32.72
28	96.05	19.48	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42	87.77	32.76
30	96.03	19.54	95.32	21.13	94.55	22.70	93.73	24.24	92.86	27.75	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47	87.74	32.80
32	96.00	19.59	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51	87.70	32.85
34	95.98	19.64	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56	87.66	32.89
36	95.96	19.70	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.00	87.62	32.93
38	95.93	19.75	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65	87.58	32.98
40	95.91	19.80	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69	87.54	33.02
42	95.89	19.86	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74	87.51	33.07
44	95.86	19.91	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78	87.47	33.11
46	95.84	19.96	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83	87.43	33.15
48	95.82	20.02	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87	87.39	33.20
50	95.79	20.07	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92	87.35	33.24
52	95.77	20.12	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96	87.31	33.28
54	95.75	20.18	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01	87.27	33.33
56	95.72	20.23	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05	87.24	33.37
58	95.70	20.28	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09	87.20	33.41
60	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14	87.16	33.46
$c = 0.75$	0.73	0.15	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25	0.70	0.26
$c = 1.00$	0.98	0.20	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33	0.94	0.35
$c = 1.25$	1.22	0.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34	1.20	0.36	1.19	0.38	1.19	0.40	1.18	0.42	1.17	0.44

TABLE XIV — *Concluded*

M.	21°		22°		23°		24°		25°		26°		27°		28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30
2	87.12	33.50	85.93	34.77	84.69	36.01	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49	77.91	41.48	76.45	42.43	74.95	43.33
4	87.08	33.54	85.89	34.82	84.65	36.05	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52	77.86	41.52	76.40	42.46	74.90	43.36
6	87.04	33.59	85.85	34.86	84.61	36.09	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55	77.81	41.55	76.35	42.49	74.85	43.39
8	87.00	33.63	85.80	34.90	84.57	36.13	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59	77.77	41.58	76.30	42.53	74.80	43.42
10	86.96	33.67	85.76	34.94	84.52	36.17	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62	77.72	41.61	76.25	42.56	74.75	43.45
12	86.92	33.72	85.72	34.98	84.48	36.21	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66	77.67	41.65	76.20	42.59	74.70	43.47
14	86.88	33.76	85.68	35.02	84.44	36.25	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69	77.62	41.68	76.15	42.62	74.65	43.50
16	86.84	33.80	85.64	35.07	84.40	36.29	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72	77.57	41.71	76.10	42.65	74.60	43.53
18	86.80	33.84	85.60	35.11	84.35	36.33	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76	77.52	41.74	76.05	42.68	74.55	43.56
20	86.77	33.89	85.56	35.15	84.31	36.37	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79	77.48	41.77	76.00	42.71	74.49	43.59
22	86.73	33.93	85.52	35.19	84.27	36.41	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82	77.42	41.81	75.95	42.74	74.44	43.62
24	86.69	33.97	85.48	35.23	84.23	36.45	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86	77.38	41.84	75.90	42.77	74.39	43.65
26	86.65	34.01	85.44	35.27	84.18	36.49	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89	77.33	41.87	75.85	42.80	74.34	43.67
28	86.61	34.06	85.40	35.31	84.14	36.53	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92	77.28	41.90	75.80	42.83	74.29	43.70
30	86.57	34.10	85.36	35.36	84.10	36.57	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96	77.23	41.93	75.75	42.86	74.24	43.73
32	86.53	34.14	85.31	35.40	84.06	36.61	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99	77.18	41.97	75.70	42.89	74.19	43.76
34	86.49	34.18	85.27	35.44	84.01	36.65	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02	77.13	42.00	75.65	42.92	74.14	43.79
36	86.45	34.23	85.23	35.48	83.97	36.69	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06	77.09	42.03	75.60	42.95	74.09	43.82
38	86.41	34.27	85.19	35.52	83.93	36.73	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09	77.04	42.06	75.55	42.98	74.04	43.84
40	86.37	34.31	85.15	35.56	83.89	36.77	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12	76.99	42.09	75.50	43.01	73.99	43.87
42	86.33	34.35	85.11	35.60	83.84	36.80	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16	76.94	42.12	75.45	43.04	73.93	43.90
44	86.29	34.40	85.07	35.64	83.80	36.84	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19	76.89	42.15	75.40	43.07	73.88	43.93
46	86.25	34.44	85.02	35.68	83.76	36.88	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22	76.84	42.19	75.35	43.10	73.83	43.95
48	86.21	34.48	84.98	35.72	83.72	36.92	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26	76.79	42.22	75.30	43.13	73.78	43.98
50	86.17	34.52	84.94	35.76	83.67	36.96	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29	76.74	42.25	75.25	43.16	73.73	44.01
52	86.13	34.57	84.90	35.80	83.63	37.00	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32	76.69	42.28	75.20	43.18	73.68	44.04
54	86.09	34.61	84.86	35.85	83.59	37.04	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35	76.64	42.31	75.15	43.21	73.63	44.07
56	86.05	34.65	84.82	35.89	83.54	37.08	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39	76.59	42.34	75.10	43.24	73.58	44.09
58	86.01	34.69	84.77	35.93	83.50	37.12	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42	76.55	42.37	75.05	43.27	73.52	44.12
60	85.97	34.73	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30	73.47	44.15
$e = 0.75$	0.70	0.27	0.69	0.29	0.69	0.30	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35	0.66	0.36	0.65	0.37	0.65	0.38
$e = 1.00$	0.93	0.37	0.92	0.38	0.92	0.40	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46	0.88	0.48	0.87	0.49	0.86	0.51
$e = 1.25$	1.16	0.46	1.15	0.48	1.15	0.50	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58	1.10	0.60	1.09	0.62	1.08	0.64

TABLE XV[1]

Volume in cubic feet per second discharging over a thin plate weir one foot in length without end contractions according to Francis' Formula $Q = 3.33\ L\sqrt{H^3}$ [2].

Head, H, in ft.	Cu. ft. per sec.	Head, H, in ft.	Cu. ft. per sec.	Head, H, in ft.	Cu. ft. per sec.	Head, H, in ft.	Cu. ft. per sec.	Head, H, in ft.	Cu. ft. per sec.
.01	0.003	.51	1.213	1.01	3.380	1.51	6.179	2.01	9.489
.02	0.009	.52	1.249	1.02	3.430	1.52	6.240	2.02	9.560
.03	0.017	.53	1.285	1.03	3.481	1.53	6.302	2.03	9.631
.04	0.027	.54	1.321	1.04	3.532	1.54	6.364	2.04	9.703
.05	0.037	.55	1.358	1.05	3.583	1.55	6.426	2.05	9.774
.06	0.049	.56	1.395	1.06	3.634	1.56	6.488	2.06	9.846
.07	0.062	.57	1.433	1.07	3.686	1.57	6.551	2.07	9.917
.08	0.075	.58	1.471	1.08	3.737	1.58	6.613	2.08	9.989
.09	0.090	.59	1.509	1.09	3.790	1.59	6.676	2.09	10.062
.10	0.105	.60	1.548	1.10	3.842	1.60	6.739	2.10	10.134
.11	0.121	.61	1.586	1.11	3.894	1.61	6.803	2.11	10.206
.12	0.138	.62	1.626	1.12	3.947	1.62	6.866	2.12	10.279
.13	0.156	.63	1.665	1.13	4.000	1.63	6.930	2.13	10.352
.14	0.174	.64	1.705	1.14	4.053	1.64	6.994	2.14	10.425
.15	0.193	.65	1.745	1.15	4.107	1.65	7.058	2.15	10.498
.16	0.213	.66	1.786	1.16	4.160	1.66	7.122	2.16	10.571
.17	0.233	.67	1.826	1.17	4.214	1.67	7.187	2.17	10.645
.18	0.254	.68	1.867	1.18	4.268	1.68	7.251	2.18	10.718
.19	0.276	.69	1.909	1.19	4.323	1.69	7.316	2.19	10.792
.20	0.298	.70	1.950	1.20	4.377	1.70	7.381	2.20	10.866
.21	0.320	.71	1.992	1.21	4.432	1.71	7.446	2.21	10.940
.22	0.344	.72	2.034	1.22	4.487	1.72	7.512	2.22	11.015
.23	0.367	.73	2.077	1.23	4.543	1.73	7.577	2.23	11.089
.24	0.392	.74	2.120	1.24	4.598	1.74	7.643	2.24	11.164
.25	0.416	.75	2.163	1.25	4.654	1.75	7.709	2.25	11.239
.26	0.441	.76	2.206	1.26	4.710	1.76	7.775	2.26	11.314
.27	0.467	.77	2.250	1.27	4.766	1.77	7.842	2.27	11.389
.28	0.493	.78	2.294	1.28	4.822	1.78	7.908	2.28	11.464
.29	0.520	.79	2.338	1.29	4.879	1.79	7.975	2.29	11.540
.30	0.547	.80	2.383	1.30	4.936	1.80	8.042	2.30	11.615
.31	0.575	.81	2.428	1.31	4.993	1.81	8.109	2.31	11.691
.32	0.603	.82	2.473	1.32	5.050	1.82	8.176	2.32	11.767
.33	0.631	.83	2.518	1.33	5.108	1.83	8.244	2.33	11.843
.34	0.660	.84	2.564	1.34	5.165	1.84	8.311	2.34	11.920
.35	0.690	.85	2.610	1.35	5.223	1.85	8.379	2.35	11.996
.36	0.719	.86	2.656	1.36	5.281	1.86	8.447	2.36	12.073
.37	0.749	.87	2.702	1.37	5.340	1.87	8.515	2.37	12.150
.38	0.780	.88	2.749	1.38	5.398	1.88	8.584	2.38	12.227
.39	0.811	.89	2.796	1.39	5.457	1.89	8.652	2.39	12.304
.40	0.842	.90	2.843	1.40	5.516	1.90	8.721	2.40	12.381
.41	0.874	.91	2.891	1.41	5.575	1.91	8.790	2.41	12.459
.42	0.906	.92	2.939	1.42	5.635	1.92	8.859	2.42	12.536
.43	0.939	.93	2.987	1.43	5.094	1.93	8.929	2.43	12.614
.44	0.972	.94	3.035	1.44	5.754	1.94	8.998	2.44	12.692
.45	1.005	.95	3.083	1.45	5.814	1.95	9.068	2.45	12.770
.46	1.039	.96	3.132	1.46	5.875	1.96	9.138	2.46	12.848
.47	1.073	.97	3.181	1.47	5.935	1.97	9.208	2.47	12.927
.48	1.107	.98	3.231	1.48	5.996	1.98	9.278	2.48	13.005
.49	1.142	.99	3.280	1.49	6.057	1.99	9.348	2.49	13.084
.50	1.177	1.00	3.330	1.50	6.118	2.00	9.419	2.50	13.163

[1] From Trautwine's Engineers' Pocketbook.

[2] The table values are not seriously in error when there are end contractions provided L is at least 10 H. The original formula was limited to heads H between $\frac{1}{2}$ foot and 2 feet. The tabular values are probably not seriously in error for the range given. For any weir of length L feet multiply the tabular values by L.

TABLE XVI

Natural Sines and Cosines

Natural Sines

Angle	0′	10′	20′	30′	40′	50′	60′	Angle	Prop. Parts for 1′
0°	.0000 00	.0029 09	.0058 18	.0087 27	.0116 35	.0145 44	.0174 52	**89°**	2.9
1	.0174 52	.0203 6	.0232 7	.0261 8	.0290 8	.0319 9	.0349 0	88	2.9
2	.0349 0	.0378 1	.0407 1	.0436 2	.0465 3	.0494 3	.0523 4	87	2.9
3	.0523 4	.0552 4	.0581 4	.0610 5	.0639 5	.0668 5	.0697 6	86	2.9
4	.0697 6	.0726 6	.0755 6	.0784 6	.0813 6	.0842 6	.0871 6	85	2.9
5	.0871 6	.0900 5	.0929 5	.0958 5	.0987 4	.1016 4	.1045 3	**84**	2.9
6	.1045 3	.1074 2	.1103 1	.1132 0	.1160 9	.1189 8	.1218 7	83	2.9
7	.1218 7	.1247 6	.1276 4	.1305 3	.1334	.1363	.1392	82	2.9
8	.1392	.1421	.1449	.1478	.1507	.1536	.1564	81	2.9
9	.1564	.1593	.1622	.1650	.1679	.1708	.1736	80	2.9
10	.1736	.1765	.1794	.1822	.1851	.1880	.1908	**79**	2.9
11	.1908	.1937	.1965	.1994	.2022	.2051	.2079	78	2.9
12	.2079	.2108	.2136	.2164	.2193	.2221	.2250	77	2.8
13	.2250	.2278	.2306	.2334	.2363	.2391	.2419	76	2.8
14	.2419	.2447	.2476	.2504	.2532	.2560	.2588	75	2.8
15	.2588	.2616	.2644	.2672	.2700	.2728	.2756	**74**	2.8
16	.2756	.2784	.2812	.2840	.2868	.2896	.2924	73	2.8
17	.2924	.2952	.2979	.3007	.3035	.3062	.3090	72	2.8
18	.3090	.3118	.3145	.3173	.3201	.3228	.3256	71	2.8
19	.3256	.3283	.3311	.3338	.3365	.3393	.3420	70	2.7
20	.3420	.3448	.3475	.3502	.3529	.3557	.3584	**69**	2.7
21	.3584	.3611	.3638	.3665	.3692	.3719	.3746	68	2.7
22	.3746	.3773	.3800	.3827	.3854	.3881	.3907	67	2.7
23	.3907	.3934	.3961	.3987	.4014	.4041	.4067	66	2.7
24	.4067	.4094	.4120	.4147	.4173	.4200	.4226	65	2.7
25	.4226	.4253	.4279	.4305	.4331	.4358	.4384	**64**	2.6
26	.4384	.4410	.4436	.4462	.4488	.4514	.4540	63	2.6
27	.4540	.4566	.4592	.4617	.4643	.4669	.4695	62	2.6
28	.4695	.4720	.4746	.4772	.4797	.4823	.4848	61	2.6
29	.4848	.4874	.4899	.4924	.4950	.4975	.5000	60	2.5
30	.5000	.5025	.5050	.5075	.5100	.5125	.5150	**59**	2.5
31	.5150	.5175	.5200	.5225	.5250	.5275	.5299	58	2.5
32	.5299	.5324	.5348	.5373	.5398	.5422	.5446	57	2.5
33	.5446	.5471	.5495	.5519	.5544	.5568	.5592	56	2.4
34	.5592	.5616	.5640	.5664	.5688	.5712	.5736	55	2.4
35	.5736	.5760	.5783	.5807	.5831	.5854	.5878	**54**	2.4
36	.5878	.5901	.5925	.5948	.5972	.5995	.6018	53	2.3
37	.6018	.6041	.6065	.6088	.6111	.6134	.6157	52	2.3
38	.6157	.6180	.6202	.6225	.6248	.6271	.6293	51	2.3
39	.6293	.6316	.6338	.6361	.6383	.6406	.6428	50	2.3
40	.6428	.6450	.6472	.6494	.6517	.6539	.6561	**49**	2.2
41	.6561	.6583	.6604	.6626	.6648	.6670	.6691	48	2.2
42	.6691	.6713	.6734	.6756	.6777	.6799	.6820	47	2.2
43	.6820	.6841	.6862	.6884	.6905	.6926	.6947	46	2.1
44	.6947	.6967	.6988	.7009	.7030	.7050	.7071	45	2.1
	60′	50′	40′	30′	20′	10′	0′	Angle	

Natural Cosines

TABLE XVI

Natural Sines and Cosines — *Continued*

Natural Sines

Angle	0′	10′	20′	30′	40′	50′	60′	Angle	Prop. Parts for 1′
45°	.7071	.7092	.7112	.7133	.7153	.7173	.7193	**44°**	2.0
46	.7193	.7214	.7234	.7254	.7274	.7294	.7314	43	2.0
47	.7314	.7333	.7353	.7373	.7392	.7412	.7431	42	2.0
48	.7431	.7451	.7470	.7490	.7509	.7528	.7547	41	1.9
49	.7547	.7566	.7585	.7604	.7623	.7642	.7660	40	1.9
50	.7660	.7679	.7698	.7716	.7735	.7753	.7771	**39**	1.9
51	.7771	.7790	.7808	.7826	.7844	.7862	.7880	38	1.8
52	.7880	.7898	.7916	.7934	.7951	.7969	.7986	37	1.8
53	.7986	.8004	.8021	.8039	.8056	.8073	.8090	36	1.7
54	.8090	.8107	.8124	.8141	.8158	.8175	.8192	35	1.7
55	.8192	.8208	.8225	.8241	.8258	.8274	.8290	**34**	1.6
56	.8290	.8307	.8323	.8339	.8355	.8371	.8387	33	1.6
57	.8387	.8403	.8418	.8434	.8450	.8465	.8480	32	1.6
58	.8480	.8496	.8511	.8526	.8542	.8557	.8572	31	1.5
59	.8572	.8587	.8601	.8616	.8631	.8646	.8660	30	1.5
60	.8660	.8675	.8689	.8704	.8718	.8732	.8746	**29**	1.4
61	.8746	.8760	.8774	.8788	.8802	.8816	.8829	28	1.4
62	.8829	.8843	.8857	.8870	.8884	.8897	.8910	27	1.4
63	.8910	.8923	.8936	.8949	.8962	.8975	.8988	26	1.3
64	.8988	.9001	.9013	.9026	.9038	.9051	.9063	25	1.3
65	.9063	.9075	.9088	.9100	.9112	.9124	.9135	**24**	1.2
66	.9135	.9147	.9159	.9171	.9182	.9194	.9205	23	1.2
67	.9205	.9216	.9228	.9239	.9250	.9261	.9272	22	1.1
68	.9272	.9283	.9293	.9304	.9315	.9325	.9336	21	1.1
69	.9336	.9346	.9356	.9367	.9377	.9387	.9397	20	1.0
70	.9397	.9407	.9417	.9426	.9436	.9446	.9455	**19**	1.0
71	.9455	.9465	.9474	.9483	.9492	.9502	.9511	18	0.9
72	.9511	.9520	.9528	.9537	.9546	.9555	.9563	17	0.9
73	.9563	.9572	.9580	.9588	.9596	.9605	.9613	16	0.8
74	.9613	.9621	.9628	.9636	.9644	.9652	.9659	15	0.8
75	.9659	.9667	.9674	.9681	.9689	.9696	.9703	**14**	0.7
76	.9703	.9710	.9717	.9724	.9730	.9737	.9744	13	0.7
77	.9744	.9750	.9757	.9763	.9769	.9775	.9781	12	0.6
78	.9781	.9787	.9793	.9799	.9805	.9811	.9816	11	0.6
79	.9816	.9822	.9827	.9833	.9838	.9843	.9848	10	0.5
80	.9848	.9853	.9858	.9863	.9868	.9872	.9877	**9**	0.5
81	.9877	.9881	.9886	.9890	.9894	.9899	.9903	8	0.4
82	.9903	.9907	.9911	.9914	.9918	.9922	.9925	7	0.4
83	.9925	.9929	.9932	.9936	.9939	.9942	.9945	6	0.3
84	.9945	.9948	.9951	.9954	.9957	.9959	.9962	5	0.3
85	.9962	.9964	.9967	.9969	.9971	.9974	.9976	**4**	0.2
86	.9976	.9978	.9980	.9981	.9983	.9985	.9986	3	0.2
87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	0.1
88	.9994	.9995	.9996	.9997	.9997	.9998	.9998	1	0.1
89	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	0	0.0
	60′	50′	40′	30′	20′	10′	0′	Angle	

Natural Cosines

TABLE XVII

Natural Tangents and Cotangents

Natural Tangents

Angle	0′	10′	20′	30′	40′	50′	60′	Angle	Prop. Parts for 1′
0°	.0000 0	.0029 1	.0058 2	.0087 3	.0116 4	.0145 5	.0174 6	**89°**	2.9
1	.0174 6	.0203 6	.0232 8	.0261 9	.0291 0	.0320 1	.0349 2	88	2.9
2	.0349 2	.0378 3	.0407 5	.0436 6	.0465 8	.0494 9	.0524 1	87	2.9
3	.0524 1	.0553 3	.0582 4	.0611 6	.0640 8	.0670 0	.0699 3	86	2.9
4	.0699 3	.0728 5	.0757 8	.0787 0	.0816 3	.0845 6	.0874 9	85	2.9
5	.0874 9	.0904 2	.0933 5	.0962 9	.0992 3	.1021 6	.1051 0	**84**	2.9
6	.1051 0	.1080 5	.1109 9	.1139 4	.1168 8	.1198 3	.1227 8	83	2.9
7	.1227 8	.1257 4	.1286 9	.1316 5	.1346	.1376	.1405	82	3.0
8	.1405	.1435	.1465	.1495	.1524	.1554	.1584	81	3.0
9	.1584	.1614	.1644	.1673	.1703	.1733	.1763	80	3.0
10	.1763	.1793	.1823	.1853	.1883	.1914	.1944	**79**	3.0
11	.1944	.1974	.2004	.2035	.2065	.2095	.2126	78	3.0
12	.2126	.2156	.2186	.2217	.2247	.2278	.2309	77	3.1
13	.2309	.2339	.2370	.2401	.2432	.2462	.2493	76	3.1
14	.2493	.2524	.2555	.2586	.2617	.2648	.2679	75	3.1
15	.2679	.2711	.2742	.2773	.2805	.2836	.2867	**74**	3.1
16	.2867	.2899	.2931	.2962	.2994	.3026	.3057	73	3.2
17	.3057	.3089	.3121	.3153	.3185	.3217	.3249	72	3.2
18	.3249	.3281	.3314	.3346	.3378	.3411	.3443	71	3.2
19	.3443	.3476	.3508	.3541	.3574	.3607	.3640	70	3.3
20	.3640	.3673	.3706	.3739	.3772	.3805	.3839	**69**	3.3
21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	3.4
22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	3.4
23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	3.5
24	.4452	.4487	.4522	.4557	.4592	.4628	.4663	65	3.5
25	.4663	.4699	.4734	.4770	.4806	.4841	.4877	**64**	3.6
26	.4877	.4913	.4950	.4986	.5022	.5059	.5095	63	3.6
27	.5095	.5132	.5169	.5206	.5243	.5280	.5317	62	3.7
28	.5317	.5354	.5392	.5430	.5467	.5505	.5543	61	3.8
29	.5543	.5581	.5619	.5658	.5696	.5735	.5774	60	3.8
30	.5774	.5812	.5851	.5890	.5930	.5969	.6009	**59**	3.9
31	.6009	.6048	.6088	.6128	.6168	.6208	.6249	58	4.0
32	.6249	.6289	.6330	.6371	.6412	.6453	.6494	57	4.1
33	.6494	.6536	.6577	.6619	.6661	.6703	.6745	56	4.2
34	.6745	.6787	.6830	.6873	.6916	.6959	.7002	55	4.3
35	.7002	.7046	.7089	.7133	.7177	.7221	.7265	**54**	4.4
36	.7265	.7310	.7355	.7400	.7445	.7490	.7536	53	4.5
37	.7536	.7581	.7627	.7673	.7720	.7766	.7813	52	4.6
38	.7813	.7860	.7907	.7954	.8002	.8050	.8098	51	4.7
39	.8098	.8146	.8195	.8243	.8292	.8342	.8391	50	4.9
40	.8391	.8441	.8491	.8541	.8591	.8642	.8693	**49**	5.0
41	.8693	.8744	.8796	.8847	.8899	.8952	.9004	48	5.2
42	.9004	.9057	.9110	.9163	.9217	.9271	.9325	47	5.4
43	.9325	.9380	.9435	.9490	.9545	.9601	.9657	46	5.5
44	.9657	.9713	.9770	.9827	.9884	.9942	1.0000	45	5.7
	60′	50′	40′	30′	20′	10′	0′	Angle	

Natural Cotangents

TABLE XVII

Natural Tangents and Cotangents — *Continued*

Natural Tangents

Angle	0′	10′	20′	30′	40′	50′	60′	Angle	Prop. Parts for 1′
45°	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	**44°**	5.9
46	1.0355	1.0416	1.0477	1.0538	1.0599	1.0661	1.0724	43	6.1
47	1.0724	1.0786	1.0850	1.0913	1.0977	1.1041	1.1106	42	6.4
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436	1.1504	41	6.6
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847	1.1918	40	6.9
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276	1.2349	**39**	7.2
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723	1.2799	38	7.5
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37	7.9
53	1.3270	1.3351	1.3432	1.3514	1.3597	1.3680	1.3764	36	8.2
54	1.3764	1.3848	1.3934	1.4019	1.4106	1.4193	1.4281	35	8.6
55	1.4281	1.4370	1.4460	1.4550	1.4641	1.4733	1.4826	**34**	9.1
56	1.4826	1.4919	1.5013	1.5108	1.5204	1.5301	1.5399	33	9.6
57	1.5399	1.5497	1.5597	1.5697	1.5798	1.5900	1.6003	32	10.1
58	1.6003	1.6107	1.6212	1.6319	1.6426	1.6534	1.6643	31	10.7
59	1.6643	1.6753	1.6864	1.6977	1.7090	1.7205	1.7321	30	11.3
60	1.7321	1.7437	1.7556	1.7675	1.7796	1.7917	1.8040	**29**	12.0
61	1.8040	1.8165	1.8291	1.8418	1.8546	1.8676	1.8807	28	12.8
62	1.8807	1.8940	1.9074	1.9210	1.9347	1.9486	1.9626	27	13.6
63	1.9626	1.9768	1.9912	2.0057	2.0204	2.0353	2.0503	26	14.6
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25	15.7
65	2.1445	2.1609	2.1775	2.1943	2.2113	2.2286	2.2460	**24**	16.9
66	2.2460	2.2637	2.2817	2.2998	2.3183	2.3369	2.3559	23	18.3
67	2.3559	2.3750	2.3945	2.4142	2.4342	2.4545	2.4751	22	19.9
68	2.4751	2.4960	2.5172	2.5386	2.5605	2.5826	2.6051	21	21.7
69	2.6051	2.6279	2.6511	2.6746	2.6985	2.7228	2.7475	20	23.7
70	2.7475	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	**19**	
71	2.9042	2.9319	2.9600	2.9887	3.0178	3.0475	3.0777	18	
72	3.0777	3.1084	3.1397	3.1716	3.2041	3.2371	3.2709	17	
73	3.2709	3.3052	3.3402	3.3759	3.4124	3.4495	3.4874	16	
74	3.4874	3.5261	3.5656	3.6059	3.6470	3.6891	3.7321	15	
75	3.7321	3.7760	3.8208	3.8667	3.9136	3.9617	4.0108	**14**	
76	4.0108	4.0611	4.1126	4.1653	4.2193	4.2747	4.3315	13	
77	4.3315	4.3897	4.4494	4.5107	4.5736	4.6382	4.7046	12	
78	4.7046	4.7729	4.8430	4.9152	4.9894	5.0658	5.1446	11	
79	5.1446	5.2257	5.3093	5.3955	5.4845	5.5764	5.6713	10	
80	5.6713	5.7694	5.8708	5.9758	6.0844	6.1970	6.3138	**9**	
81	6.3138	6.4348	6.5606	6.6912	6.8269	6.9682	7.1154	8	
82	7.1154	7.2687	7.4287	7.5958	7.7704	7.9530	8.1443	7	
83	8.1443	8.3450	8.5555	8.7769	9.0098	9.2553	9.5144	6	
84	9.5144	9.7882	10.0780	10.3854	10.7119	11.0594	11.4301	5	
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	**4**	
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3	
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2	
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1	
89	57.2900	68.7501	85.9398	114.5887	171.8854	343.7737	∞	0	
	60′	50′	40′	30′	20′	10′	0′	Angle	

Natural Cotangents

TABLE XVIII

Common Logarithms of Numbers

USE OF THE TABLE OF LOGARITHMS

The logarithm of a number is the exponent denoting the power to which some fixed base number must be raised to equal the number whose logarithm is considered. There are two base numbers in use, but almost all operations are performed by one system, known as Common Logarithms, in which the base is 10. The logarithm of 1 in any system is 0 because any number raised to the 0 power is 1. Thus $\frac{x^1}{x^1} = x^{1-1} = x^0 = 1$. The logarithm of 10 in the common system is 1; of 100, 2; of 1000, 3; etc., since $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, etc. Numbers between 1 and 10, 10 and 100, 100 and 1000, etc., will have fractional logarithms. Thus, the logarithm of 8 is 0.90309; of 13 is 1.11394; of 126 is 2.10037, etc., since $10^{0.90309\pm} = 8$, $10^{1.11394\pm} = 13$, $10^{2.10037} = 126$, etc. The fraction is called the *mantissa* and is always the same for the same sequence of figures and is incommensurable, being given to 3, 5, 6, 7, or 10 decimal places according to the precision required. Thus log. 1.263 = 0.10140; log. 12.63 = 1.10140; and log. 126.3 = 2.10140, all to 5 decimal places. The whole number, called the *characteristic*, varies. The characteristic is always 1 less than the number of digits to the left of the decimal point of the number whose logarithm is being found. In the tables only the mantissas are given. The whole number must be known by the computer from the number of digits in the number whose logarithm is wanted.

To find the logarithm of a number look in the table for the mantissa and prefix the proper characteristic, determined from the number of digits to the left of the decimal point in the number in question. Thus, on page 512, line 27 we find the mantissa for the sequence 1263 in the fifth column to be .10140; then for 1.263 the log is 0.10140; for 12.63 and 126.3 as given above.

In the table the first three figures of a four-figure number appear in the first column, the fourth figure at the heads of the 10 numbered columns. The first two figures of the mantissa appear in the second column, the last three in the column under the fourth digit of the number whose logarithm is sought. If the first two figures change in going across the page, that fact is indicated by an *: which, to economize space, stands for the first two figures of the second column in the *line below the asterisk*. Thus, the logarithm of 1445 (see p. 512) is 3.15987, while that of 1446 is 3.16017. For numbers of 3 figures or less, the mantissa is found in the column headed 0, since the mantissas for 1, 10, 100, or for 2, 20, 200, or for 15, 150, 1500, etc. are respectively the same.

The logarithm of a number of more than four figures is found as indicated

in the following example: What is the log of 382.568? On page 517, line 33, columns 2 and 7, find log 382.5 to be 2.58263. Note that log 382.6 is 2.58274, a difference of 11 in the log for a difference of 1 in the last place of the sequence in the number. Assuming that the log increases in proportion to the number, the log of 382.568 may be said to be $\frac{6.8}{10.0}$ of 11 larger than log 382.5 since 382.568 is $\frac{6.8}{10.0}$ of 1 in the last place (the place of the 5) greater than 382.5. To facilitate this computation, a table of proportional parts is found alongside the log table. Thus, in the case just given, for a difference of 11 we find, from the table of proportional parts, that an increase of 0.6 of one in the last place of the number makes a difference in the log of 6.6, an increase of 0.08 in the number increases the log by 0.88, found by moving the decimal point 1 digit to the left in the tabular value for 0.8, so that an increase of 0.68 of 1 in the last place increases the log of 382.5 by 6.6 + 0.88 or 7.48; or, since we make the fifth figure the nearest whole number, we find log 382.568 = 2.58263 + 7 = 2.58270.

To find the number corresponding to a given logarithm we reverse the process. Thus, the number corresponding to the logarithm 2.58270 is found as follows: The next smaller mantissa is .58263 for the sequence 3825 and it is smaller than .58270 by 7. The difference for a whole unit in the last place is 11 and in the table of proportional parts under 11 find 6.6, corresponding to .6 of 1 in the last place of the number, as the next smaller difference to 7, leaving still a difference of 7 − 6.6 = 0.4 to be used, which, by moving the decimal point one digit to the left, is found to correspond most nearly to .04 of 1 in the last place of the number. Therefore, the whole number is 382.564. But we found that 2.58270 is the logarithm of 382.568. We see by this that this table cannot be depended on to give more than a sequence of 5 significant figures correctly. This is true of all tables of logarithms; that as many significant figures in sequence may be correctly determined as there are decimal places in the tabular mantissas. A 5-place table gives five significant figures, a 6-place table, six significant figures, etc.

To multiply *a* by *b* we add the logarithms of *a* and *b*, written *log a* and *log b*, and find the number corresponding to the sum.

To divide *a* by *b*, we subtract *log b* from *log a* and find the number corresponding to the difference.

It may often occur in computations that the fifth place in a resulting logarithm, found by adding two or more logarithms, may be in error by one unit, thus making the quantity determined certain to one less significant figure than the number of places in the logarithm.

To find the product a^2b^3. Find the logarithm of *a* and multiply it by 2; find the logarithm of *b* and multiply it by 3; add the two results for the logarithm of the product and find the corresponding number. Thus:

What is the product $1.2^2 \times 4.3^3$?

page 512, line 21, col. 2, log 1.2 = 0.07918 × 2 = 0.15836
page 518, line 31, col. 2, log 4.3 = 0.63347 × 3 = 1.90041
page 512, line 15, col. 6 & 7, = 2.05877 = log 114.49. Ans.

This example checked by a 7-place table gives the same result to the fifth significant figure even though both logarithms were multiplied and then added.

The following example, worked by 5-place and 7-place logarithms, shows the uncertainty of the last figure in computations made by any set of tables, by showing the uncertainty of the fifth figure in the computations made with the 5-place tables. Required the product $1.65^2 \times 1.8^3$.

5-Place

$$\begin{aligned}\text{Log } 1.65 &= 0.21748 \times 2 = 0.43496\\ \text{Log } 1.8 &= 0.25527 \times 3 = \underline{0.76581}\\ &\qquad 1.20077 = \log 15.877. \quad \text{Ans.}\end{aligned}$$

7-Place

$$\begin{aligned}\text{Log } 1.65 &= 0.2174839 \times 2 = 0.4349678\\ \text{Log } 1.8 &= 0.2552725 \times 3 = \underline{0.7658175}\\ &\qquad 1.2007853 = \log 15.8776. \quad \text{Ans.}\end{aligned}$$

Thus 15.878 is nearer right than 15.877, though the error of the latter is only about 0.6 of one in the last place and the error of the former is about 0.4 of one in the last place. Greater differences can occur so that the fifth place is not certain within 1 unit.

Logarithms of fractions. The logarithm of $1 = \frac{1}{10} = 10^{1-1} = 10^0$ is 0.00000. Similarly the logarithm of $0.1 = \frac{1}{10} = 10^{0-1}$ is -1.00000; the logarithm of $\frac{1}{100}$ is -2; of $\frac{1}{1000}$ is -3; etc. Therefore, if the sequence is all fractional the characteristic is *minus* and with a numerical value expressed by the number of the place on the right of the decimal point in which the first significant figure appears. Thus the logarithm of 0.00126 is $-3 + 0.10037$, usually written $\bar{3}.10037$. The mantissa is not minus, only the characteristic is minus. This sometimes gives the beginner trouble, but need not if he remembers that the mantissa is always plus, while the characteristic is plus or minus according as the number corresponding is equal to or greater than 1 or is less than 1, and that in general the two must be treated separately. The following examples will make the use of the signs clear:

Required the product 43.0×0.43

$$\begin{aligned}\text{Log } 43 &= 1.63347 \qquad 1.63347\\ \text{Log } .43 &= \bar{1}.63347 \qquad \underline{.63347} - 1\\ \text{Log } 18.49 &= 1.26694 \qquad 2.26694 - 1 = 1.26694\end{aligned}$$

Required the product 43.0×0.43^2

$$\begin{aligned}\text{Log } 43.0 &= 1.63347 \qquad \text{Log } 0.43^2 = \bar{1}.63347 \times 2\\ \text{Log } 0.43^2 &= \bar{1}.26694 \qquad = \bar{2} + 1.26694 = \bar{1}.26694\\ \text{Log } 7.951 &= 0.90041\end{aligned}$$

Required the product $43.0 \times 0.43^{\frac{1}{2}}$

Log 43.0	= 1.63347
Log $0.43^{\frac{1}{2}}$	= $\bar{1}$.81674
Log 28.207	= 1.45021

$\log 0.43^{\frac{1}{2}} = \bar{1}.63347 \div 2 = \bar{1}.816735$ found thus: Add 10 to the characteristic and subtract 10 before dividing by 2, getting $\frac{9.63347 - 10}{2} = \bar{1}.816735 - 5$ or 1.816735.

Instead of adding and subtracting 10 in the preceding example, only enough may be added and subtracted to make the negative characteristic divisible by the divisor. Thus: In the foregoing example add and subtract 1, getting $\frac{\bar{1}.63347 - 1 + 1}{2} = \frac{\bar{2} + 1.63347}{2} = \bar{1}.816735$, as before, without the necessity of considering the 4 and -5. If the $\frac{1}{3}$ power had been required, the divisor would have been 3, and 2 should be added and subtracted getting $\frac{1.63347 - 2 + 2}{3} = \frac{\bar{3} + 2.63347}{3} = \bar{1}.877823 +$. This work need not be written out in full in an example, as the computer can see the figures mentally. Thus, he knows the characteristic will be $\bar{1}$ so he writes this down and then imagines a 2, or whatever number is to be added, at the left of the decimal and at once writes down the quotient after the $\bar{1}$. In handling very small numbers when the negative characteristic is large, it may be necessary to write out the operation to guard against error.

In making the tables, when the remainder after the 5th figure was more than 0.5, the fifth figure was increased by 1; when less than 0.5, the fifth figure was not changed.

When the computer is finding a logarithm, and the operation brings a 5 in the place following the last place of the table (as the sixth place when using the tables of this book) and the 5 is followed by ciphers, a good rule to adopt is to make the last place the nearest *even* number. Thus in getting $\log 0.43^{\frac{1}{2}}$ in the example above we add 5 in the sixth place. This rule will tend to balance inaccuracies in a long series of computations.

To divide 3.68×4.21 we find (page 517) $\log 3.68 = 0.56585$
(page 518) $\log 4.21 = 0.62428$

Now to subtract, we may add and subtract 10 from the log 3.68, getting

$$\begin{array}{r} 10.56585 - 10 \\ .62428 \\ \hline 9.94157 - 10 = \bar{1}.94157 \end{array}$$

or, we may add the arithmetical complement of log 4.21 instead of subtracting the log 4.21. That is, we may multiply by $\frac{1}{4.21}$ instead of dividing by 4.21. The logarithm of $\frac{1}{4.21}$ is $0.00000 - 0.62428$ or $\bar{1}.37572$ and is

called the arithmetical complement of the log 4.21. It is the logarithm of the reciprocal of 4.21. In a series of operations where several factors are to be multiplied and the product divided by the product of several factors, the arithmetical complements of the divisor factors are used to avoid the double operation of two additions and one subtraction, since the arithmetical complements may be written down from the logs almost as readily as the logs themselves. Each digit of the mantissa of the log is subtracted from 9 except the last which is subtracted from 10. The characteristic is negative, and one greater than the characteristic of the log when the log characteristic is positive; and is positive, and one less than the characteristic of the log if that characteristic is negative. Thus:

Log 8.364 is 0.92241	its complement is $\bar{1}$.07759
Log 83.64 is 1.92241	its complement is $\bar{2}$.07759
Log 836.4 is 2.92241	its complement is $\bar{3}$.07759
Log 0.8364 is $\bar{1}$.92241	its complement is 0.07759
Log 0.08364 is $\bar{2}$.92241	its complement is 1.07759

Instead of thinking of the characteristic by the rule above, the log may be subtracted from 10 instead of zero when a characteristic of 9 in the complement will correspond to $\bar{1}$; 8 to 2, etc. And the computer can as readily think of 9 as indicating that the first significant figure of the result is in the first or tenths' place of the decimal; 8, that it is in the second or hundredths' place, etc., and this will cause no confusion unless very large quantities are being dealt with, so that the 8 or 9 might sometimes be a positive characteristic not standing in the place of a $\bar{2}$ or $\bar{1}$; but this difficulty is not likely to arise in using 5- or 6-place tables, since such tables would not be used with such large numbers.

S′	T′	N	L 0	1	2	3	4	5	6	7	8	9
6.46												
366	385	**100**	00 000	043	087	130	173	217	260	303	346	389
366	385	101	432	475	518	561	604	647	689	732	775	817
366	385	102	860	903	945	988	*030	*072	*115	*157	*199	*242
366	386	103	01 284	326	368	410	452	494	536	578	620	662
366	386	104	703	745	787	828	870	912	953	995	*036	*078
366	386	105	02 119	160	202	243	284	325	366	407	449	490
366	386	106	531	572	612	653	694	735	776	816	857	898
366	387	107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
365	387	108	03 342	383	423	463	503	543	583	623	663	703
365	387	109	743	782	822	862	902	941	981	*021	*060	*100
365	387	**110**	04 139	179	218	258	297	336	376	415	454	493
365	388	111	532	571	610	650	689	727	766	805	844	883
365	388	112	922	961	999	*038	*077	*115	*154	*192	*231	*269
365	388	113	05 308	346	385	423	461	500	538	576	614	652
365	389	114	690	729	767	805	843	881	918	956	994	*032
365	389	115	06 070	108	145	183	221	258	296	333	371	408
364	389	116	446	483	521	558	595	633	670	707	744	781
364	389	117	819	856	893	930	967	*004	*041	*078	*115	*151
364	390	118	07 188	225	262	298	335	372	408	445	482	518
364	390	119	555	591	628	664	700	737	773	809	846	882
364	390	**120**	918	954	990	*027	*063	*099	*135	*171	*207	*243
364	391	121	08 279	314	350	386	422	458	493	529	565	600
363	391	122	636	672	707	743	778	814	849	884	920	955
363	391	123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
363	391	124	09 342	377	412	447	482	517	552	587	621	656
363	392	125	691	726	760	795	830	864	899	934	968	*003
363	392	126	10 037	072	106	140	175	209	243	278	312	346
363	392	127	380	415	449	483	517	551	585	619	653	687
363	393	128	721	755	789	823	857	890	924	958	992	*025
362	393	129	11 059	093	126	160	193	227	261	294	327	361
362	393	**130**	394	428	461	494	528	561	594	628	661	694
362	394	131	727	760	793	826	860	893	926	959	992	*024
362	394	132	12 057	090	123	156	189	222	254	287	320	352
362	394	133	385	418	450	483	516	548	581	613	646	678
362	395	134	710	743	775	808	840	872	905	937	969	*001
361	395	135	13 033	066	098	130	162	194	226	258	290	322
361	395	136	354	386	418	450	481	513	545	577	609	640
361	396	137	672	704	735	767	799	830	862	893	925	956
361	396	138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
361	396	139	14 301	333	364	395	426	457	489	520	551	582
361	397	**140**	613	644	675	706	737	768	799	829	860	891
360	397	141	922	953	983	*014	*045	*076	*106	*137	*168	*198
360	398	142	15 229	259	290	320	351	381	412	442	473	503
360	398	143	534	564	594	625	655	685	715	746	776	806
360	398	144	836	866	897	927	957	987	*017	*047	*077	*107
360	398	145	16 137	167	197	227	256	286	316	346	376	406
360	399	146	435	465	495	524	554	584	613	643	673	702
359	399	147	732	761	791	820	850	879	909	938	967	997
359	399	148	17 026	056	085	114	143	173	202	231	260	289
359	400	149	319	348	377	406	435	464	493	522	551	580
		N	L 0	1	2	3	4	5	6	7	8	9

P P		
	44	**43**	**42**
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8
	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1
	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4
	35	**34**	**33**
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7
	32	**31**	**30**
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

P P		
	S′	**T′**
1′	372	373
5	372	373

	S′	T′		S′	T′		S′	T′
1′	6.46 373	373	10′	373	373	14′	372	373
2	373	373	13	373	373	15	372	373

S′	T′ 6.46	N	L 0	1	2	3	4	5	6	7	8	9
359	400	**150**	17 609	638	667	696	725	754	782	811	840	869
359	401	151	898	926	955	984	*013	*041	*070	*099	*127	*156
358	401	152	18 184	213	241	270	298	327	355	384	412	441
358	401	153	469	498	526	554	583	611	639	667	696	724
358	402	154	752	780	808	837	865	893	921	949	977	*005
358	402	155	19 033	061	089	117	145	173	201	229	257	285
358	402	156	312	340	368	396	424	451	479	507	535	562
358	403	157	590	618	645	673	700	728	756	783	811	838
357	403	158	866	893	921	948	976	*003	*030	*058	*085	*112
357	404	159	20 140	167	194	222	249	276	303	330	358	385
357	404	**160**	412	439	466	493	520	548	575	602	629	656
357	404	161	683	710	737	763	790	817	844	871	898	925
357	405	162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
356	405	163	21 219	245	272	299	325	352	378	405	431	458
356	406	164	484	511	537	564	590	617	643	669	696	722
356	406	165	748	775	801	827	854	880	906	932	958	985
356	406	166	22 011	037	063	089	115	141	167	194	220	246
356	407	167	272	298	324	350	376	401	427	453	479	505
355	407	168	531	557	583	608	634	660	686	712	737	763
355	408	169	789	814	840	866	891	917	943	968	994	*019
355	408	**170**	23 045	070	096	121	147	172	198	223	249	274
355	408	171	300	325	350	376	401	426	452	477	502	528
354	409	172	553	578	603	629	654	679	704	729	754	779
354	409	173	805	830	855	880	905	930	955	980	*005	*030
354	410	174	24 055	080	105	130	155	180	204	229	254	279
354	410	175	304	329	353	378	403	428	452	477	502	527
354	411	176	551	576	601	625	650	674	699	724	748	773
353	411	177	797	822	846	871	895	920	944	969	993	*018
353	411	178	25 042	066	091	115	139	164	188	212	237	261
353	412	179	285	310	334	358	382	406	431	455	479	503
353	412	**180**	527	551	575	600	624	648	672	696	720	744
353	413	181	768	792	816	840	864	888	912	935	959	983
352	413	182	26 007	031	055	079	102	126	150	174	198	221
352	414	183	245	269	293	316	340	364	387	411	435	458
352	414	184	482	505	529	553	576	600	623	647	670	694
352	415	185	717	741	764	788	811	834	858	881	905	928
351	415	186	951	975	998	*021	*045	*068	*091	*114	*138	*161
351	415	187	27 184	207	231	254	277	300	323	346	370	393
351	416	188	416	439	462	485	508	531	554	577	600	623
351	416	189	646	669	692	715	738	761	784	807	830	852
350	417	**190**	875	898	921	944	967	989	*012	*035	*058	*081
350	417	191	28 103	126	149	171	194	217	240	262	285	307
350	418	192	330	353	375	398	421	443	466	488	511	533
350	418	193	556	578	601	623	646	668	691	713	735	758
350	419	194	780	803	825	847	870	892	914	937	959	981
349	419	195	29 003	026	048	070	092	115	137	159	181	203
349	420	196	226	248	270	292	314	336	358	380	403	425
349	420	197	447	469	491	513	535	557	579	601	623	645
349	421	198	667	688	710	732	754	776	798	820	842	863
348	421	199	885	907	929	951	973	994	*016	*038	*060	*081
		N	L 0	1	2	3	4	5	6	7	8	9

P P

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

P P

	S′	T′
1′	6.46 373	373
2	373	373

	S′	T′
15′	372	373
20	372	373

N	L	0	1	2	3	4	5	6	7	8	9
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
211		428	449	469	490	510	531	552	572	593	613
212		634	654	675	695	715	736	756	777	797	818
213		838	858	879	899	919	940	960	980	*001	*021
214	33	041	062	082	102	122	143	163	183	203	224
215		244	264	284	304	325	345	365	385	405	425
216		445	465	486	506	526	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	826
218		846	866	885	905	925	945	965	985	*005	*025
219	34	044	064	084	104	124	143	163	183	203	223
220		242	262	282	301	321	341	361	380	400	420
221		439	459	479	498	518	537	557	577	596	616
222		635	655	674	694	713	733	753	772	792	811
223		830	850	869	889	908	928	947	967	986	*005
224	35	025	044	064	083	102	122	141	160	180	199
225		218	238	257	276	295	315	334	353	372	392
226		411	430	449	468	488	507	526	545	564	583
227		603	622	641	660	679	698	717	736	755	774
228		793	813	832	851	870	889	908	927	946	965
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36	173	192	211	229	248	267	286	305	324	342
231		361	380	399	418	436	455	474	493	511	530
232		549	568	586	605	624	642	661	680	698	717
233		736	754	773	791	810	829	847	866	884	903
234		922	940	959	977	996	*014	*033	*051	*070	*088
235	37	107	125	144	162	181	199	218	236	254	273
236		291	310	328	346	365	383	401	420	438	457
237		475	493	511	530	548	566	585	603	621	639
238		658	676	694	712	731	749	767	785	803	822
239		840	858	876	894	912	931	949	967	985	*003
240	38	021	039	057	075	093	112	130	148	166	184
241		202	220	238	256	274	292	310	328	346	364
242		382	399	417	435	453	471	489	507	525	543
243		561	578	596	614	632	650	668	686	703	721
244		739	757	775	792	810	828	846	863	881	899
245		917	934	952	970	987	*005	*023	*041	*058	*076
246	39	094	111	129	146	164	182	199	217	235	252
247		270	287	305	322	340	358	375	393	410	428
248		445	463	480	498	515	533	550	568	585	602
249		620	637	655	672	690	707	724	742	759	777

N	L	0	1	2	3	4	5	6	7	8	9

P P		
	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

P P	
	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

P P	
	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

P P	
	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

P P	
	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

		S′	T′
2′	6.46	373	373
3		373	373

		S′	T′
20′		372	373
25		372	373

N	L 0	1	2	3	4	5	6	7	8	9	P P
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	18
252	40 140	157	175	192	209	226	243	261	278	295	1 1.8
253	312	329	346	364	381	398	415	432	449	466	2 3.6
254	483	500	518	535	552	569	586	603	620	637	3 5.4
255	654	671	688	705	722	739	756	773	790	807	4 7.2
256	824	841	858	875	892	909	926	943	960	976	5 9.0
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	6 10.8
258	41 162	179	196	212	229	246	263	280	296	313	7 12.6
259	330	347	363	380	397	414	430	447	464	481	8 14.4
260	497	514	531	547	564	581	597	614	631	647	9 16.2
261	664	681	697	714	731	747	764	780	797	814	
262	830	847	863	880	896	913	929	946	963	979	17
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	1 1.7
264	42 160	177	193	210	226	243	259	275	292	308	2 3.4
265	325	341	357	374	390	406	423	439	455	472	3 5.1
266	488	504	521	537	553	570	586	602	619	635	4 6.8
267	651	667	684	700	716	732	749	765	781	797	5 8.5
268	813	830	846	862	878	894	911	927	943	959	6 10.2
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	7 11.9
270	43 136	152	169	185	201	217	233	249	265	281	8 13.6
271	297	313	329	345	361	377	393	409	425	441	9 15.3
272	457	473	489	505	521	537	553	569	584	600	
273	616	632	648	664	680	696	712	727	743	759	16
274	775	791	807	823	838	854	870	886	902	917	1 1.6
275	933	949	965	981	996	*012	*028	*044	*059	*075	2 3.2
276	44 091	107	122	138	154	170	185	201	217	232	3 4.8
277	248	264	279	295	311	326	342	358	373	389	4 6.4
278	404	420	436	451	467	483	498	514	529	545	5 8.0
279	560	576	592	607	623	638	654	669	685	700	6 9.6
280	716	731	747	762	778	793	809	824	840	855	7 11.2
281	871	886	902	917	932	948	963	979	994	*010	8 12.8
282	45 025	040	056	071	086	102	117	133	148	163	9 14.4
283	179	194	209	225	240	255	271	286	301	317	
284	332	347	362	378	393	408	423	439	454	469	15
285	484	500	515	530	545	561	576	591	606	621	1 1.5
286	637	652	667	682	697	712	728	743	758	773	2 3.0
287	788	803	818	834	849	864	879	894	909	924	3 4.5
288	939	954	969	984	*000	*015	*030	*045	*060	*075	4 6.0
289	46 090	105	120	135	150	165	180	195	210	225	5 7.5
290	240	255	270	285	300	315	330	345	359	374	6 9.0
291	389	404	419	434	449	464	479	494	509	523	7 10.5
292	538	553	568	583	598	613	627	642	657	672	8 12.0
293	687	702	716	731	746	761	776	790	805	820	9 13.5
294	835	850	864	879	894	909	923	938	953	967	
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	14
296	47 129	144	159	173	188	202	217	232	246	261	1 1.4
297	276	290	305	319	334	349	363	378	392	407	2 2.8
298	422	436	451	465	480	494	509	524	538	553	3 4.2
299	567	582	596	611	625	640	654	669	683	698	4 5.6
N	L 0	1	2	3	4	5	6	7	8	9	5 7.0
											6 8.4
		S′	T′			S′	T′			S′	T′ 7 9.8
2′	6.46	373	373	25′		372	373	27′		372	374 8 11.2
3		373	373	26		372	373	30		372	374 9 12.6

N	L O	1	2	3	4	5	6	7	8	9	P P
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	**15**
304	287	302	316	330	344	359	373	387	401	416	1 | 1.5
305	430	444	458	473	487	501	515	530	544	558	2 | 3.0
306	572	586	601	615	629	643	657	671	686	700	3 | 4.5
307	714	728	742	756	770	785	799	813	827	841	4 | 6.0
308	855	869	883	897	911	926	940	954	968	982	5 | 7.5
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 | 9.0
310	49 136	150	164	178	192	206	220	234	248	262	7 | 10.5
311	276	290	304	318	332	346	360	374	388	402	8 | 12.0
312	415	429	443	457	471	485	499	513	527	541	9 | 13.5
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	**14**
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	1 | 1.4
317	50 106	120	133	147	161	174	188	202	215	229	2 | 2.8
318	243	256	270	284	297	311	325	338	352	365	3 | 4.2
319	379	393	406	420	433	447	461	474	488	501	4 | 5.6
320	515	529	542	556	569	583	596	610	623	637	5 | 7.0
321	651	664	678	691	705	718	732	745	759	772	6 | 8.4
322	786	799	813	826	840	853	866	880	893	907	7 | 9.8
323	920	934	947	961	974	987	*001	*014	*028	*041	8 | 11.2
324	51 055	068	081	095	108	121	135	148	162	175	9 | 12.6
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	**13**
328	587	601	614	627	640	654	667	680	693	706	1 | 1.3
329	720	733	746	759	772	786	799	812	825	838	2 | 2.6
330	851	865	878	891	904	917	930	943	957	970	3 | 3.9
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	4 | 5.2
332	52 114	127	140	153	166	179	192	205	218	231	5 | 6.5
333	244	257	270	284	297	310	323	336	349	362	6 | 7.8
334	375	388	401	414	427	440	453	466	479	492	7 | 9.1
335	504	517	530	543	556	569	582	595	608	621	8 | 10.4
336	634	647	660	673	686	699	711	724	737	750	9 | 11.7
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	**12**
339	53 020	033	046	058	071	084	097	110	122	135	1 | 1.2
340	148	161	173	186	199	212	224	237	250	263	2 | 2.4
341	275	288	301	314	326	339	352	364	377	390	3 | 3.6
342	403	415	428	441	453	466	479	491	504	517	4 | 4.8
343	529	542	555	567	580	593	605	618	631	643	5 | 6.0
344	656	668	681	694	706	719	732	744	757	769	6 | 7.2
345	782	794	807	820	832	845	857	870	882	895	7 | 8.4
346	908	920	933	945	958	970	983	995	*008	*020	8 | 9.6
347	54 033	045	058	070	083	095	108	120	133	145	9 | 10.8
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	

N	L O	1	2	3	4	5	6	7	8	9	P P

	S′	T′		S′	T′
3′	6.46 373	373	30′	372	374
4	373	373	35	372	374

N	L 0	1	2	3	4	5	6	7	8	9	PP
350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	**13**
354	900	913	925	937	949	962	974	986	998	*011	1 1.3
355	55 023	035	047	060	072	084	096	108	121	133	2 2.6
356	145	157	169	182	194	206	218	230	242	255	3 3.9
357	267	279	291	303	315	328	340	352	364	376	4 5.2
358	388	400	413	425	437	449	461	473	485	497	5 6.5
359	509	522	534	546	558	570	582	594	606	618	6 7.8
360	630	642	654	666	678	691	703	715	727	739	7 9.1
361	751	763	775	787	799	811	823	835	847	859	8 10.4
362	871	883	895	907	919	931	943	955	967	979	9 11.7
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	**12**
366	348	360	372	384	396	407	419	431	443	455	1 1.2
367	467	478	490	502	514	526	538	549	561	573	2 2.4
368	585	597	608	620	632	644	656	667	679	691	3 3.6
369	703	714	726	738	750	761	773	785	797	808	4 4.8
370	820	832	844	855	867	879	891	902	914	926	5 6.0
371	937	949	961	972	984	996	*008	*019	*031	*043	6 7.2
372	57 054	066	078	089	101	113	124	136	148	159	7 8.4
373	171	183	194	206	217	229	241	252	264	276	8 9.6
374	287	299	310	322	334	345	357	368	380	392	9 10.8
375	403	415	426	438	449	461	473	484	496	507	
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	**11**
378	749	761	772	784	795	807	818	830	841	852	1 1.1
379	864	875	887	898	910	921	933	944	955	967	2 2.2
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	3 3.3
381	58 092	104	115	127	138	149	161	172	184	195	4 4.4
382	206	218	229	240	252	263	274	286	297	309	5 5.5
383	320	331	343	354	365	377	388	399	410	422	6 6.6
384	433	444	456	467	478	490	501	512	524	535	7 7.7
385	546	557	569	580	591	602	614	625	636	647	8 8.8
386	659	670	681	692	704	715	726	737	749	760	9 9.9
387	771	782	794	805	816	827	838	850	861	872	
388	883	894	906	917	928	939	950	961	973	984	**10**
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	1 1.0
390	59 106	118	129	140	151	162	173	184	195	207	2 2.0
391	218	229	240	251	262	273	284	295	306	318	3 3.0
392	329	340	351	362	373	384	395	406	417	428	4 4.0
393	439	450	461	472	483	494	506	517	528	539	5 5.0
394	550	561	572	583	594	605	616	627	638	649	6 6.0
395	660	671	682	693	704	715	726	737	748	759	7 7.0
396	770	780	791	802	813	824	835	846	857	868	8 8.0
397	879	890	901	912	923	934	945	956	966	977	9 9.0
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	
399	60 097	108	119	130	141	152	163	173	184	195	

N	L 0	1	2	3	4	5	6	7	8	9	PP

		S′	T′			S′	T′			S′	T′
3′	6.46	373	373	35′		372	374	40′		372	375
4		373	373	39		372	374				

N	L O	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885	895	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312

PP	
11	
1	1.1
2	2.2
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9	9.9
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1	1.0
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3	3.0
4	4.0
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7	7.0
8	8.0
9	9.0
9	
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

N	L O	1	2	3	4	5	6	7	8	9

		S′	T′			S′	T′			S′	T′
4′	6.46	373	373	40′		372	375	43′		371	375
5		373	373	42		372	375	44		371	375
								45		371	375

N	L 0	1	2	3	4	5	6	7	8	9	P P
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	**10**
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	1 | 1.0
458	66 087	096	106	115	124	134	143	153	162	172	2 | 2.0
459	181	191	200	210	219	229	238	247	257	266	3 | 3.0
460	276	285	295	304	314	323	332	342	351	361	4 | 4.0
461	370	380	389	398	408	417	427	436	445	455	5 | 5.0
462	464	474	483	492	502	511	521	530	539	549	6 | 6.0
463	558	567	577	586	596	605	614	624	633	642	7 | 7.0
464	652	661	671	680	689	699	708	717	727	736	8 | 8.0
465	745	755	764	773	783	792	801	811	820	829	9 | 9.0
466	839	848	857	867	876	885	894	904	913	922	
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	**9**
471	302	311	321	330	339	348	357	367	376	385	1 | 0.9
472	394	403	413	422	431	440	449	459	468	477	2 | 1.8
473	486	495	504	514	523	532	541	550	560	569	3 | 2.7
474	578	587	596	605	614	624	633	642	651	660	4 | 3.6
475	669	679	688	697	706	715	724	733	742	752	5 | 4.5
476	761	770	779	788	797	806	815	825	834	843	6 | 5.4
477	852	861	870	879	888	897	906	916	925	934	7 | 6.3
478	943	952	961	970	979	988	997	*006	*015	*024	8 | 7.2
479	68 034	043	052	061	070	079	088	097	106	115	9 | 8.1
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	**8**
486	664	673	681	690	699	708	717	726	735	744	1 | 0.8
487	753	762	771	780	789	797	806	815	824	833	2 | 1.6
488	842	851	860	869	878	886	895	904	913	922	3 | 2.4
489	931	940	949	958	966	975	984	993	*002	*011	4 | 3.2
490	69 020	028	037	046	055	064	073	082	090	099	5 | 4.0
491	108	117	126	135	144	152	161	170	179	188	6 | 4.8
492	197	205	214	223	232	241	249	258	267	276	7 | 5.6
493	285	294	302	311	320	329	338	346	355	364	8 | 6.4
494	373	381	390	399	408	417	425	434	443	452	9 | 7.2
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	

N	L 0	1	2	3	4	5	6	7	8	9	P P

		S′	T′			S′	T′			S′	T′
4′	6.46	373	373	45′		371	375	49′		371	376
5		373	373	48		371	375	50		371	376

N	L 0	1	2	3	4	5	6	7	8	9	PP
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	9
508	586	595	603	612	621	629	638	646	655	663	1 | 0.9
509	672	680	689	697	706	714	723	731	740	749	2 | 1.8
510	757	766	774	783	791	800	808	817	825	834	3 | 2.7
511	842	851	859	868	876	885	893	902	910	919	4 | 3.6
512	927	935	944	952	961	969	978	986	995	*003	5 | 4.5
513	71 012	020	029	037	046	054	063	071	079	088	6 | 5.4
514	096	105	113	122	130	139	147	155	164	172	7 | 6.3
515	181	189	198	206	214	223	231	240	248	257	8 | 7.2
516	265	273	282	290	299	307	315	324	332	341	9 | 8.1
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	8
522	767	775	784	792	800	809	817	825	834	842	1 | 0.8
523	850	858	867	875	883	892	900	908	917	925	2 | 1.6
524	933	941	950	958	966	975	983	991	999	*008	3 | 2.4
525	72 016	024	032	041	049	057	066	074	082	090	4 | 3.2
526	099	107	115	123	132	140	148	156	165	173	5 | 4.0
527	181	189	198	206	214	222	230	239	247	255	6 | 4.8
528	263	272	280	288	296	304	313	321	329	337	7 | 5.6
529	346	354	362	370	378	387	395	403	411	419	8 | 6.4
530	428	436	444	452	460	469	477	485	493	501	9 | 7.2
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	7
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	1 | 0.7
538	73 078	086	094	102	111	119	127	135	143	151	2 | 1.4
539	159	167	175	183	191	199	207	215	223	231	3 | 2.1
540	239	247	255	263	272	280	288	296	304	312	4 | 2.8
541	320	328	336	344	352	360	368	376	384	392	5 | 3.5
542	400	408	416	424	432	440	448	456	464	472	6 | 4.2
543	480	488	496	504	512	520	528	536	544	552	7 | 4.9
544	560	568	576	584	592	600	608	616	624	632	8 | 5.6
545	640	648	656	664	672	679	687	695	703	711	9 | 6.3
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
N	L 0	1	2	3	4	5	6	7	8	9	PP

	S′	T′		S′	T′
5′	6.46 373	373	50′		371
6	373	373	55		371

N	L O	1	2	3	4	5	6	7	8	9	P P
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	1 0.8
564	128	136	143	151	159	166	174	182	189	197	2 1.6
565	205	213	220	228	236	243	251	259	266	274	3 2.4
566	282	289	297	305	312	320	328	335	343	351	4 3.2
567	358	366	374	381	389	397	404	412	420	427	5 4.0
568	435	442	450	458	465	473	481	488	496	504	6 4.8
569	511	519	526	534	542	549	557	565	572	580	7 5.6
570	587	595	603	610	618	626	633	641	648	656	8 6.4
571	664	671	679	686	694	702	709	717	724	732	9 7.2
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	7
581	418	425	433	440	448	455	462	470	477	485	1 0.7
582	492	500	507	515	522	530	537	545	552	559	2 1.4
583	567	574	582	589	597	604	612	619	626	634	3 2.1
584	641	649	656	664	671	678	686	693	701	708	4 2.8
585	716	723	730	738	745	753	760	768	775	782	5 3.5
586	790	797	805	812	819	827	834	842	849	856	6 4.2
587	864	871	879	886	893	901	908	916	923	930	7 4.9
588	938	945	953	960	967	975	982	989	997	*004	8 5.6
589	77 012	019	026	034	041	048	056	063	070	078	9 6.3
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	

N	L O	1	2	3	4	5	6	7	8	9	P P

	S′	T′		S′	T′		S′	T′
6′	6.46 373	373	56′	371	376	58′	371	377
55	371	376	57	371	377	59	370	377
						60	370	377

N	L O	1	2	3	4	5	6	7	8	9	P P
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	8
607	319	326	333	340	347	355	362	369	376	383	1 0.8
608	390	398	405	412	419	426	433	440	447	455	2 1.6
609	462	469	476	483	490	497	504	512	519	526	3 2.4
610	533	540	547	554	561	569	576	583	590	597	4 3.2
611	604	611	618	625	633	640	647	654	661	668	5 4.0
612	675	682	689	696	704	711	718	725	732	739	6 4.8
613	746	753	760	767	774	781	789	796	803	810	7 5.6
614	817	824	831	838	845	852	859	866	873	880	8 6.4
615	888	895	902	909	916	923	930	937	944	951	9 7.2
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	7
622	379	386	393	400	407	414	421	428	435	442	1 0.7
623	449	456	463	470	477	484	491	498	505	511	2 1.4
624	518	525	532	539	546	553	560	567	574	581	3 2.1
625	588	595	602	609	616	623	630	637	644	650	4 2.8
626	657	664	671	678	685	692	699	706	713	720	5 3.5
627	727	734	741	748	754	761	768	775	782	789	6 4.2
628	796	803	810	817	824	831	837	844	851	858	7 4.9
629	865	872	879	886	893	900	906	913	920	927	8 5.6
630	934	941	948	955	962	969	975	982	989	996	9 6.3
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	6
637	414	421	428	434	441	448	455	462	468	475	1 0.6
638	482	489	496	502	509	516	523	530	536	543	2 1.2
639	550	557	564	570	577	584	591	598	604	611	3 1.8
640	618	625	632	638	645	652	659	665	672	679	4 2.4
641	686	693	699	706	713	720	726	733	740	747	5 3.0
642	754	760	767	774	781	787	794	801	808	814	6 3.6
643	821	828	835	841	848	855	862	868	875	882	7 4.2
644	889	895	902	909	916	922	929	936	943	949	8 4.8
645	956	963	969	976	983	990	996	*003	*010	*017	9 5.4
646	81 023	030	037	043	050	057	064	070	077	084	
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
N	L O	1	2	3	4	5	6	7	8	9	P P

	S′	T′		S′	T′		S′	T′
6′	6.46 373	373	60′	370	377	64′	370	378
7	373	373	63	370	377	65	370	378

N	L O	1	2	3	4	5	6	7	8	9	P P
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	7
662	086	092	099	105	112	119	125	132	138	145	1 | 0.7
663	151	158	164	171	178	184	191	197	204	210	2 | 1.4
664	217	223	230	236	243	249	256	263	269	276	3 | 2.1
665	282	289	295	302	308	315	321	328	334	341	4 | 2.8
666	347	354	360	367	373	380	387	393	400	406	5 | 3.5
667	413	419	426	432	439	445	452	458	465	471	6 | 4.2
668	478	484	491	497	504	510	517	523	530	536	7 | 4.9
669	543	549	556	562	569	575	582	588	595	601	8 | 5.6
670	607	614	620	627	633	640	646	653	659	666	9 | 6.3
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	6
681	315	321	327	334	340	347	353	359	366	372	1 | 0.6
682	378	385	391	398	404	410	417	423	429	436	2 | 1.2
683	442	448	455	461	467	474	480	487	493	499	3 | 1.8
684	506	512	518	525	531	537	544	550	556	563	4 | 2.4
685	569	575	582	588	594	601	607	613	620	626	5 | 3.0
686	632	639	645	651	658	664	670	677	683	689	6 | 3.6
687	696	702	708	715	721	727	734	740	746	753	7 | 4.2
688	759	765	771	778	784	790	797	803	809	816	8 | 4.8
689	822	828	835	841	847	853	860	866	872	879	9 | 5.4
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	

N	L O	1	2	3	4	5	6	7	8	9	P P

		S′	T′			S′	T′			S′	T′
6′	6.46	373	373	65′		370	378	70′		370	379
7		373	373	69		370	378				

N	L 0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	535	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	850	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	285	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	425
715	431	437	443	449	455	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	625	631	637	643	649	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	064	070	076	082	088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	629	635	641	646	652	658	664	670	676	682
736	688	694	700	705	711	717	723	729	735	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*035
742	87 040	046	052	058	064	070	075	081	087	093
743	099	105	111	116	122	128	134	140	146	151
744	157	163	169	175	181	186	192	198	204	210
745	216	221	227	233	239	245	251	256	262	268
746	274	280	286	291	297	303	309	315	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	425	431	437	442
749	448	454	460	466	471	477	483	489	495	500
N	L 0	1	2	3	4	5	6	7	8	9

PP	
7	
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3
6	
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4
5	
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

PP		
	S′	T′
7′	6.46 373	373
8	373	373
70′	370	379
71	370	379
72′	369	379
74	369	379
75	369	380

N	L 0	1	2	3	4	5	6	7	8	9	P P
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	6
762	195	201	207	213	218	224	230	235	241	247	1 0.6
763	252	258	264	270	275	281	287	292	298	304	2 1.2
764	309	315	321	326	332	338	343	349	355	360	3 1.8
765	366	372	377	383	389	395	400	406	412	417	4 2.4
766	423	429	434	440	446	451	457	463	468	474	5 3.0
767	480	485	491	497	502	508	513	519	525	530	6 3.6
768	536	542	547	553	559	564	570	576	581	587	7 4.2
769	593	598	604	610	615	621	627	632	638	643	8 4.8
770	649	655	660	666	672	677	683	689	694	700	9 5.4
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	5
782	321	326	332	337	343	348	354	360	365	371	1 0.5
783	376	382	387	393	398	404	409	415	421	426	2 1.0
784	432	437	443	448	454	459	465	470	476	481	3 1.5
785	487	492	498	504	509	515	520	526	531	537	4 2.0
786	542	548	553	559	564	570	575	581	586	592	5 2.5
787	597	603	609	614	620	625	631	636	642	647	6 3.0
788	653	658	664	669	675	680	686	691	697	702	7 3.5
789	708	713	719	724	730	735	741	746	752	757	8 4.0
790	763	768	774	779	785	790	796	801	807	812	9 4.5
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	

N	L 0	1	2	3	4	5	6	7	8	9	P P

		S′	T′			S′	T′
7′	6.46	373	373	75′		369	380
8		373	373	80		369	380

N	L 0	1	2	3	4	5	6	7	8	9	PP
800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
809	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
811	902	907	913	918	924	929	934	940	945	950	6
812	956	961	966	972	977	982	988	993	998	*004	1 0.6
813	91 009	014	020	025	030	036	041	046	052	057	2 1.2
814	062	068	073	078	084	089	094	100	105	110	3 1.8
815	116	121	126	132	137	142	148	153	158	164	4 2.4
816	169	174	180	185	190	196	201	206	212	217	5 3.0
817	222	228	233	238	243	249	254	259	265	270	6 3.6
818	275	281	286	291	297	302	307	312	318	323	7 4.2
819	328	334	339	344	350	355	360	365	371	376	8 4.8
820	381	387	392	397	403	408	413	418	424	429	9 5.4
821	434	440	445	450	455	461	466	471	477	482	
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	5
832	92 012	018	023	028	033	038	044	049	054	059	1 0.5
833	065	070	075	080	085	091	096	101	106	111	2 1.0
834	117	122	127	132	137	143	148	153	158	163	3 1.5
835	169	174	179	184	189	195	200	205	210	215	4 2.0
836	221	226	231	236	241	247	252	257	262	267	5 2.5
837	273	278	283	288	293	298	304	309	314	319	6 3.0
838	324	330	335	340	345	350	355	361	366	371	7 3.5
839	376	381	387	392	397	402	407	412	418	423	8 4.0
840	428	433	438	443	449	454	459	464	469	474	9 4.5
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	

N	L 0	1	2	3	4	5	6	7	8	9	PP

	S′	T′		S′	T′		S′	T′
8′	6.46 373	373	80′	369	380	82′	368	381
9	373	373	81	369	381	85	368	381

N	L 0	1	2	3	4	5	6	7	8	9	P P
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	6
858	349	354	359	364	369	374	379	384	389	394	1 0.6
859	399	404	409	414	420	425	430	435	440	445	2 1.2
860	450	455	460	465	470	475	480	485	490	495	3 1.8
861	500	505	510	515	520	526	531	536	541	546	4 2.4
862	551	556	561	566	571	576	581	586	591	596	5 3.0
863	601	606	611	616	621	626	631	636	641	646	6 3.6
864	651	656	661	666	671	676	682	687	692	697	7 4.2
865	702	707	712	717	722	727	732	737	742	747	8 4.8
866	752	757	762	767	772	777	782	787	792	797	9 5.4
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	5
872	052	057	062	067	072	077	082	086	091	096	1 0.5
873	101	106	111	116	121	126	131	136	141	146	2 1.0
874	151	156	161	166	171	176	181	186	191	196	3 1.5
875	201	206	211	216	221	226	231	236	240	245	4 2.0
876	250	255	260	265	270	275	280	285	290	295	5 2.5
877	300	305	310	315	320	325	330	335	340	345	6 3.0
878	349	354	359	364	369	374	379	384	389	394	7 3.5
879	399	404	409	414	419	424	429	433	438	443	8 4.0
880	448	453	458	463	468	473	478	483	488	493	9 4.5
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	4
887	792	797	802	807	812	817	822	827	832	836	1 0.4
888	841	846	851	856	861	866	871	876	880	885	2 0.8
889	890	895	900	905	910	915	919	924	929	934	3 1.2
890	939	944	949	954	959	963	968	973	978	983	4 1.6
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	5 2.0
892	95 036	041	046	051	056	061	066	071	075	080	6 2.4
893	085	090	095	100	105	109	114	119	124	129	7 2.8
894	134	139	143	148	153	158	163	168	173	177	8 3.2
895	182	187	192	197	202	207	211	216	221	226	9 3.6
896	231	236	240	245	250	255	260	265	270	274	
897	279	284	289	294	299	303	308	313	318	323	
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	

N	L 0	1	2	3	4	5	6	7	8	9	P P

		S′	T′			S′	T′			S′	T′
8′	6.46	373	373	85′		368	381	89′		368	382
9		373	373	86		368	382	90		368	383

N	L O	1	2	3	4	5	6	7	8	9
900	95 424	429	434	439	444	448	453	458	463	468
901	472	477	482	487	492	497	501	506	511	516
902	521	525	530	535	540	545	550	554	559	564
903	569	574	578	583	588	593	598	602	607	612
904	617	622	626	631	636	641	646	650	655	660
905	665	670	674	679	684	689	694	698	703	708
906	713	718	722	727	732	737	742	746	751	756
907	761	766	770	775	780	785	789	794	799	804
908	809	813	818	823	828	832	837	842	847	852
909	856	861	866	871	875	880	885	890	895	899
910	904	909	914	918	923	928	933	938	942	947
911	952	957	961	966	971	976	980	985	990	995
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042
913	96 047	052	057	061	066	071	076	080	085	090
914	095	099	104	109	114	118	123	128	133	137
915	142	147	152	156	161	166	171	175	180	185
916	190	194	199	204	209	213	218	223	227	232
917	237	242	246	251	256	261	265	270	275	280
918	284	289	294	298	303	308	313	317	322	327
919	332	336	341	346	350	355	360	365	369	374
920	379	384	388	393	398	402	407	412	417	421
921	426	431	435	440	445	450	454	459	464	468
922	473	478	483	487	492	497	501	506	511	515
923	520	525	530	534	539	544	548	553	558	562
924	567	572	577	581	586	591	595	600	605	609
925	614	619	624	628	633	638	642	647	652	656
926	661	666	670	675	680	685	689	694	699	703
927	708	713	717	722	727	731	736	741	745	750
928	755	759	764	769	774	778	783	788	792	797
929	802	806	811	816	820	825	830	834	839	844
930	848	853	858	862	867	872	876	881	886	890
931	895	900	904	909	914	918	923	928	932	937
932	942	946	951	956	960	965	970	974	979	984
933	988	993	997	*002	*007	*011	*016	*021	*025	*030
934	97 035	039	044	049	053	058	063	067	072	077
935	081	086	090	095	100	104	109	114	118	123
936	128	132	137	142	146	151	155	160	165	169
937	174	179	183	188	192	197	202	206	211	216
938	220	225	230	234	239	243	248	253	257	262
939	267	271	276	280	285	290	294	299	304	308
940	313	317	322	327	331	336	340	345	350	354
941	359	364	368	373	377	382	387	391	396	400
942	405	410	414	419	424	428	433	437	442	447
943	451	456	460	465	470	474	479	483	488	493
944	497	502	506	511	516	520	525	529	534	539
945	543	548	552	557	562	566	571	575	580	585
946	589	594	598	603	607	612	617	621	626	630
947	635	640	644	649	653	658	663	667	672	676
948	681	685	690	695	699	704	708	713	717	722
949	727	731	736	740	745	749	754	759	763	768

N	L O	1	2	3	4	5	6	7	8	9

	S′	T′
9′ 6.46	373	373
10	373	373

	S′	T′
90′	368	383
91	368	383

	S′	T′
92′	367	383
94	367	383
95	367	384

PP	
5	
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5
4	
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

PP	

N	L	0	1	2	3	4	5	6	7	8	9	P P
950	97	772	777	782	786	791	795	800	804	809	813	
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	
961		272	277	281	286	290	295	299	304	308	313	5
962		318	322	327	331	336	340	345	349	354	358	1 | 0.5
963		363	367	372	376	381	385	390	394	399	403	2 | 1.0
964		408	412	417	421	426	430	435	439	444	448	3 | 1.5
965		453	457	462	466	471	475	480	484	489	493	4 | 2.0
966		498	502	507	511	516	520	525	529	534	538	5 | 2.5
967		543	547	552	556	561	565	570	574	579	583	6 | 3.0
968		588	592	597	601	605	610	614	619	623	628	7 | 3.5
969		632	637	641	646	650	655	659	664	668	673	8 | 4.0
970		677	682	686	691	695	700	704	709	713	717	9 | 4.5
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	4
982		211	216	220	224	229	233	238	242	247	251	1 | 0.4
983		255	260	264	269	273	277	282	286	291	295	2 | 0.8
984		300	304	308	313	317	322	326	330	335	339	3 | 1.2
985		344	348	352	357	361	366	370	374	379	383	4 | 1.6
986		388	392	396	401	405	410	414	419	423	427	5 | 2.0
987		432	436	441	445	449	454	458	463	467	471	6 | 2.4
988		476	480	484	489	493	498	502	506	511	515	7 | 2.8
989		520	524	528	533	537	542	546	550	555	559	8 | 3.2
990		564	568	572	577	581	585	590	594	599	603	9 | 3.6
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	

N	L	0	1	2	3	4	5	6	7	8	9	P P

	S′	T′		S′	T′		S′	T′
9′	6.46 373	373	95′	367	384	99′	367	385
10	373	373	98	367	384	100	366	385

N	L	0	1	2	3	4	5	6	7	8	9
1000	000	0000	0434	0869	1303	1737	2171	2605	3039	3473	3907
1001		4341	4775	5208	5642	6076	6510	6943	7377	7810	8244
1002		8677	9111	9544	9977	*0411	*0844	*1277	*1710	*2143	*2576
1003	001	3009	3442	3875	4308	4741	5174	5607	6039	6472	6905
1004		7337	7770	8202	8635	9067	9499	9932	*0364	*0796	*1228
1005	002	1661	2093	2525	2957	3389	3821	4253	4685	5116	5548
1006		5980	6411	6843	7275	7706	8138	8569	9001	9432	9863
1007	003	0295	0726	1157	1588	2019	2451	2882	3313	3744	4174
1008		4605	5036	5467	5898	6328	6759	7190	7620	8051	8481
1009		8912	9342	9772	*0203	*0633	*1063	*1493	*1924	*2354	*2784
1010	004	3214	3644	4074	4504	4933	5363	5793	6223	6652	7082
1011		7512	7941	8371	8800	9229	9659	*0088	*0517	*0947	*1376
1012	005	1805	2234	2663	3092	3521	3950	4379	4808	5237	5666
1013		6094	6523	6952	7380	7809	8238	8666	9094	9523	9951
1014	006	0380	0808	1236	1664	2092	2521	2949	3377	3805	4233
1015		4660	5088	5516	5944	6372	6799	7227	7655	8082	8510
1016		8937	9365	9792	*0219	*0647	*1074	*1501	*1928	*2355	*2782
1017	007	3210	3637	4064	4490	4917	5344	5771	6198	6624	7051
1018		7478	7904	8331	8757	9184	9610	*0037	*0463	*0889	*1316
1019	008	1742	2168	2594	3020	3446	3872	4298	4724	5150	5576
1020		6002	6427	6853	7279	7704	8130	8556	8981	9407	9832
1021	009	0257	0683	1108	1533	1959	2384	2809	3234	3659	4084
1022		4509	4934	5359	5784	6208	6633	7058	7483	7907	8332
1023		8756	9181	9605	*0030	*0454	*0878	*1303	*1727	*2151	*2575
1024	010	3000	3424	3848	4272	4696	5120	5544	5967	6391	6815
1025		7239	7662	8086	8510	8933	9357	9780	*0204	*0627	*1050
1026	011	1474	1897	2320	2743	3166	3590	4013	4436	4859	5282
1027		5704	6127	6550	6973	7396	7818	8241	8664	9086	9509
1028		9931	*0354	*0776	*1198	*1621	*2043	*2465	*2887	*3310	*3732
1029	012	4154	4576	4998	5420	5842	6264	6685	7107	7529	7951
1030		8372	8794	9215	9637	*0059	*0480	*0901	*1323	*1744	*2165
1031	013	2587	3008	3429	3850	4271	4692	5113	5534	5955	6376
1032		6797	7218	7639	8059	8480	8901	9321	9742	*0162	*0583
1033	014	1003	1424	1844	2264	2685	3105	3525	3945	4365	4785
1034		5205	5625	6045	6465	6885	7305	7725	8144	8564	8984
1035		9403	9823	*0243	*0662	*1082	*1501	*1920	*2340	*2759	*3178
1036	015	3598	4017	4436	4855	5274	5693	6112	6531	6950	7369
1037		7788	8206	8625	9044	9462	9881	*0300	*0718	*1137	*1555
1038	016	1974	2392	2810	3229	3647	4065	4483	4901	5319	5737
1039		6155	6573	6991	7409	7827	8245	8663	9080	9498	9916
1040	017	0333	0751	1168	1586	2003	2421	2838	3256	3673	4090
1041		4507	4924	5342	5759	6176	6593	7010	7427	7844	8260
1042		8677	9094	9511	9927	*0344	*0761	*1177	*1594	*2010	*2427
1043	018	2843	3259	3676	4092	4508	4925	5341	5757	6173	6589
1044		7005	7421	7837	8253	8669	9084	9500	9916	*0332	*0747
1045	019	1163	1578	1994	2410	2825	3240	3656	4071	4486	4902
1046		5317	5732	6147	6562	6977	7392	7807	8222	8637	9052
1047		9467	9882	*0296	*0711	*1126	*1540	*1955	*2369	*2784	*3198
1048	020	3613	4027	4442	4856	5270	5684	6099	6513	6927	7341
1049		7755	8169	8583	8997	9411	9824	*0238	*0652	*1066	*1479
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614

N	L	0	1	2	3	4	5	6	7	8	9

N	L	0	1	2	3	4	5	6	7	8	9
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614
1051		6027	6440	6854	7267	7680	8093	8506	8919	9332	9745
1052	022	0157	0570	0983	1396	1808	2221	2634	3046	3459	3871
1053		4284	4696	5109	5521	5933	6345	6758	7170	7582	7994
1054		8406	8818	9230	9642	*0054	*0466	*0878	*1289	*1701	*2113
1055	023	2525	2936	3348	3759	4171	4582	4994	5405	5817	6228
1056		6639	7050	7462	7873	8284	8695	9106	9517	9928	*0339
1057	024	0750	1161	1572	1982	2393	2804	3214	3625	4036	4446
1058		4857	5267	5678	6088	6498	6909	7319	7729	8139	8549
1059		8960	9370	9780	*0190	*0600	*1010	*1419	*1829	*2239	*2649
1060	025	3059	3468	3878	4288	4697	5107	5516	5926	6335	6744
1061		7154	7563	7972	8382	8791	9200	9609	*0018	*0427	*0836
1062	026	1245	1654	2063	2472	2881	3289	3698	4107	4515	4924
1063		5333	5741	6150	6558	6967	7375	7783	8192	8600	9008
1064		9416	9824	*0233	*0641	*1049	*1457	*1865	*2273	*2680	*3088
1065	027	3496	3904	4312	4719	5127	5535	5942	6350	6757	7165
1066		7572	7979	8387	8794	9201	9609	*0016	*0423	*0830	*1237
1067	028	1644	2051	2458	2865	3272	3679	4086	4492	4899	5306
1068		5713	6119	6526	6932	7339	7745	8152	8558	8964	9371
1069		9777	*0183	*0590	*0996	*1402	*1808	*2214	*2620	*3026	*3432
1070	029	3838	4244	4649	5055	5461	5867	6272	6678	7084	7489
1071		7895	8300	8706	9111	9516	9922	*0327	*0732	*1138	*1543
1072	030	1948	2353	2758	3163	3568	3973	4378	4783	5188	5592
1073		5997	6402	6807	7211	7616	8020	8425	8830	9234	9638
1074	031	0043	0447	0851	1256	1660	2064	2468	2872	3277	3681
1075		4085	4489	4893	5296	5700	6104	6508	6912	7315	7719
1076		8123	8526	8930	9333	9737	*0140	*0544	*0947	*1350	*1754
1077	032	2157	2560	2963	3367	3770	4173	4576	4979	5382	5785
1078		6188	6590	6993	7396	7799	8201	8604	9007	9409	9812
1079	033	0214	0617	1019	1422	1824	2226	2629	3031	3433	3835
1080		4238	4640	5042	5444	5846	6248	6650	7052	7453	7855
1081		8257	8659	9060	9462	9864	*0265	*0667	*1068	*1470	*1871
1082	034	2273	2674	3075	3477	3878	4279	4680	5081	5482	5884
1083		6285	6686	7087	7487	7888	8289	8690	9091	9491	9892
1084	035	0293	0693	1094	1495	1895	2296	2696	3096	3497	3897
1085		4297	4698	5098	5498	5898	6298	6698	7098	7498	7898
1086		8298	8698	9098	9498	9898	*0297	*0697	*1097	*1496	*1896
1087	036	2295	2695	3094	3494	3893	4293	4692	5091	5491	5890
1088		6289	6688	7087	7486	7885	8284	8683	9082	9481	9880
1089	037	0279	0678	1076	1475	1874	2272	2671	3070	3468	3867
1090		4265	4663	5062	5460	5858	6257	6655	7053	7451	7849
1091		8248	8646	9044	9442	9839	*0237	*0635	*1033	*1431	*1829
1092	038	2226	2624	3022	3419	3817	4214	4612	5009	5407	5804
1093		6202	6599	6996	7393	7791	8188	8585	8982	9379	9776
1094	039	0173	0570	0967	1364	1761	2158	2554	2951	3348	3745
1095		4141	4538	4934	5331	5727	6124	6520	6917	7313	7709
1096		8106	8502	8898	9294	9690	*0086	*0482	*0878	*1274	*1670
1097	040	2066	2462	2858	3254	3650	4045	4441	4837	5232	5628
1098		6023	6419	6814	7210	7605	8001	8396	8791	9187	9582
1099		9977	*0372	*0767	*1162	*1557	*1952	*2347	*2742	*3137	*3532
1100	041	3927	4322	4716	5111	5506	5900	6295	6690	7084	7479
N	L	0	1	2	3	4	5	6	7	8	9

TABLE XIX

The Logarithms of the Trigonometric Functions for Each Minute of the Quadrant

USE OF TABLE XIX[1]

In this table the logarithms of the sines, cosines, tangents, and cotangents of all angles through 90° measured in whole minutes are given, together with headings that enable the computer to read the functions directly for all angles through 180°. Whenever the trigonometric function is fractional, as it always is for sines and cosines, except when it is 1, the logarithms have been increased by 10 to avoid the negative characteristic. But where the function is greater than 1, as are tangents and cotangents for certain parts of the circle, the logarithm is given without the ten. Thus, in the columns headed *cot* at the *top* of the page the logarithms are unaugmented; in all other columns they are augmented by the addition of 10. This must be remembered in working with them.

To find the square root of the sine of 26° 16′ we find on page 561, line 17, the logarithmic sine to be 9.64596 which must be divided by 2 to give the logarithm of the square root and this operation is performed thus:

$$\begin{aligned} 2)\underline{9.64596 - 10} \\ 4.82298 - 5 &= \bar{1}.82298 \end{aligned}$$

whence the $\sqrt{\sin 26° 16'}$ = the number whose log is $\bar{1}.82298$ or 0.66524+.

But to find the square root of the cotangent of 24° 16′ (page 559, line 17) we have 0.34600 ÷ 2 = 0.17300 the number corresponding to which is 1.4894−.

The numbering of the pages. On each page four degree numbers will be found which are to be used as follows: The upper left-hand figure is used with the headings at the tops of the columns and the minutes running down the left side of the page. The lower right-hand figure is used with the headings at the bottom of the columns, and the minutes running up the right side of the page. The upper right-hand figure is used with the headings at the tops of the columns, but with the minutes running up the right side of the page. The lower left-hand figure is used with the headings at the bottom of the page and the minutes running down the left side of the page. Generally stated the figures are used with the headings and minute columns nearest them.

Example

See page 567. The log sine of 32° 20′ is found in the 2d column 21st line to be 9.72823. The log sine of 122° 20′ is found in the 7th column 21st line

[1] The methods of interpolating when the angle is near 0° or 90° are explained at page 533.

to be 9.92683. The log sine of 147° 20′ is found in the second column 21st line from the bottom to be 9.73219. The log sine 57° 20′ is found in the 7th column 21st line from the bottom to be 9.92522.

When the function or angle sought is not in the table as when the log sine of 14° 26.42′ is wanted, or the angle whose log sine is 9.42715, we must interpolate between the quantities of the table. Thus:

a. Log sine 14° 26′, page 549, line 27, is 9.39664 and the difference or change for 1 minute is 49 as found in the column headed *d*, then the change for 0.42 of a minute will be $\frac{42}{100} \times 49$ or 20.58 flushed up to 21 and added to 9.39664 giving 9.39685. This computation is facilitated by the table of proportional parts at the right of the page. Inspection will show that for a tabular difference of 49, 0.4′ gives 19.6 and, moving the decimal point, 0.02′ gives 0.98, which together are 20.58 as before. This computation may thus be done mentally.

b. The angle whose sine is 9.42715 is between 15° 30′ and 15° 31′ determined by looking for the tabular sine next smaller than 9.42715 and finding it on page 550, line 31, to be 9.42690. The difference is $9.42715 - 9.42690 = .00025$ and the tabular difference for 1 minute is .00045. Therefore, the angle sought is larger than 15° 30′ by $\frac{25}{45}$ of 1′ or 0.555+ minute and the angle is 15° 30.56′. Again the table of proportional parts may be used. For a tabular difference of 45, the difference next less than 25 is 22.5 corresponding to 0.5 minutes, and leaving 2.5 difference to be accounted for which, moving the decimal point one digit to the left, corresponds to .05 minutes leaving 0.25 difference unaccounted for which in turn corresponds to .005, etc.

Fractional angles near 0° or 90°. Near 0°, the tabular differences of the logarithms of sines and tangents are changing too rapidly to consider the change between two consecutive minutes as proportional to the fraction of a minute involved. For angles near 90° the same thing is true of the cosines and cotangents. To aid in making correct interpolations, certain quantities S and T are introduced in the tables for angles under 3° 21′. They are given to 5 decimal places, and are found in the common logarithm tables on the same pages as the numbers corresponding to the minutes in the angle. Thus if the angle is 1° 42.6′, the values of S and T are found on the page of the table of logarithms where the logarithm of 102.6 is found, page 512. If the angle is 0° 56.7′ the values of S and T are found on the page of the log tables bearing the number 56.7, page 521.

These quantities S and T are such that if a is an angle expressed in degrees and minutes as usual, and m is the number of minutes in the angle, then

$$S = \log \sin a - \log m$$
$$T = \log \tan a - \log m$$

So that
$$\log \sin a = S + \log m$$
$$\log \tan a = T + \log m$$

Therefore, to find the log sin 1° 42.6′ we reduce to minutes, getting 102.6, turn to the log tables for the number 102.6 and find in the first column the value of S

$= 6.46366$

and the log of m (102.6) $= 2.01115$

whence, log sine 1° 42.6′ $= 8.47481$

To find the log sin 0° 56.7′, we turn to the log tables for the number 56.7, page 521, and at the bottom of the page find S for 56.7 minutes to be

6.46371

and the logarithm of 56.7 1.75358

whence log sin 0° 56.7′ is 8.21729

To find the log tan 0° 03.4′, we turn to the log tables for the number 34, page 516, and find at the bottom of the page T for 3.4′ $= 6.46373$

and log m = log 3.4 $= 0.53148$

whence log tan 0° 03.4′ $= 6.99521$

To find the log cos 88° 03.6′, we find the sine of the complement thus:
Complement of 88° 03.6′ = 1° 56.4′ = 116.4′.

On page 512, where the number 1164 is found, we find $S = 6.46364$

log 116.4 $= 2.06595$

Whence log sin 1° 56.4′ = log cos 88° 03.6′ $= 8.52959$

To find the log cot a, the log tan of the complementary angle may be found, or, more simply, since $\cot = \frac{1}{\tan}$, the log tangent may be found and subtracted from 10. This is called the *complement of the tangent.*

To find an angle corresponding to a given logarithm the same quantities are used. The formulas are reversed and the angle found in minutes. Thus:

$$S = \log \sin a - \log m$$
$$T = \log \tan a - \log m$$

or,

$$\log m = \log \sin a - S$$
$$\log m = \log \tan a - T$$

As it is easier to add than to subtract, the complements of S and T up to 3° are given in the table following. These may be added to log sin a and log tan a respectively for log m.

Having log sin $a = 8.47481$, to find a, we turn to page 536 and find that the angle is between 1° 42′ and 1° 43′ and that *Cpl* S is 3.53634. Therefore, log m = log sin a + *Cpl* S = 8.47481 + 3.53634 or 2.01115 = log 102.6′ or 1° 42.6′

Given log tan $a = 6.99521$, we turn to page 535 and find the angle between 03′ and 04′ and *Cpl* $T = 3.53627$. Therefore, log m = log tan a + *Cpl* T = 6.99521 + 3.53627 or 0.53148 = log 3.4 or 0° 03.4′.

′	′	L Sin	d	Cpl S′	Cpl T′	L Tan	c d	L Cot	L Cos	
0	0	———		———	———	———		———	0.00 000	60
1	1	6.46 373	30103	3.53 627	3.53 627	6.46 373	30103	3.53 627	0.00 000	59
2	2	6.76 476	17609	3.53 627	3.53 627	6.76 476	17609	3.23 524	0.00 000	58
3	3	6.94 085	12494	3.53 627	3.53 627	6.94 085	12494	3.05 915	0.00 000	57
4	4	7.06 579	9691	3.53 627	3.53 627	7.06 579	9691	2.93 421	0.00 000	56
5	5	7.16 270	7918	3.53 627	3.53 627	7.16 270	7918	2.83 730	0.00 000	55
6	6	7.24 188	6694	3.53 627	3.53 627	7.24 188	6694	2.75 812	0.00 000	54
7	7	7.30 882	5800	3.53 627	3.53 627	7.30 882	5800	2.69 118	0.00 000	53
8	8	7.36 682	5115	3.53 627	3.53 627	7.36 682	5115	2.63 318	0.00 000	52
9	9	7.41 797	4576	3.53 627	3.53 627	7.41 797	4576	2.58 203	0.00 000	51
10	10	7.46 373	4139	3.53 627	3.53 627	7.46 373	4139	2.53 627	0.00 000	50
11	11	7.50 512	3779	3.53 627	3.53 627	7.50 512	3779	2.49 488	0.00 000	49
12	12	7.54 291	3476	3.53 627	3.53 627	7.54 291	3476	2.45 709	0.00 000	48
13	13	7.57 767	3218	3.53 627	3.53 627	7.57 767	3219	2.42 233	0.00 000	47
14	14	7.60 985	2997	3.53 628	3.53 627	7.60 986	2996	2.39 014	0.00 000	46
15	15	7.63 982	2802	3.53 628	3.53 627	7.63 982	2803	2.36 018	0.00 000	45
16	16	7.66 784	2633	3.53 628	3.53 627	7.66 785	2633	2.33 215	0.00 000	44
17	17	7.69 417	2483	3.53 628	3.53 627	7.69 418	2482	2.30 582	9.99 999	43
18	18	7.71 900	2348	3.53 628	3.53 627	7.71 900	2348	2.28 100	9.99 999	42
19	19	7.74 248	2227	3.53 628	3.53 627	7.74 248	2228	2.25 752	9.99 999	41
20	20	7.76 475	2119	3.53 628	3.53 627	7.76 476	2119	2.23 524	9.99 999	40
21	21	7.78 594	2021	3.53 628	3.53 627	7.78 595	2020	2.21 405	9.99 999	39
22	22	7.80 615	1930	3.53 628	3.53 627	7.80 615	1931	2.19 385	9.99 999	38
23	23	7.82 545	1848	3.53 628	3.53 627	7.82 546	1848	2.17 454	9.99 999	37
24	24	7.84 393	1773	3.53 628	3.53 627	7.84 394	1773	2.15 606	9.99 999	36
25	25	7.86 166	1704	3.53 628	3.53 627	7.86 167	1704	2.13 833	9.99 999	35
26	26	7.87 870	1639	3.53 628	3.53 627	7.87 871	1639	2.12 129	9.99 999	34
27	27	7.89 509	1579	3.53 628	3.53 626	7.89 510	1579	2.10 490	9.99 999	33
28	28	7.91 088	1524	3.53 628	3.53 626	7.91 089	1524	2.08 911	9.99 999	32
29	29	7.92 612	1472	3.53 628	3.53 626	7.92 613	1473	2.07 387	9.99 998	31
30	30	7.94 084	1424	3.53 628	3.53 626	7.94 086	1424	2.05 914	9.99 998	30
31	31	7.95 508	1379	3.53 628	3.53 626	7.95 510	1379	2.04 490	9.99 998	29
32	32	7.96 887	1336	3.53 628	3.53 626	7.96 889	1336	2.03 111	9.99 998	28
33	33	7.98 223	1297	3.53 628	3.53 626	7.98 225	1297	2.01 775	9.99 998	27
34	34	7.99 520	1259	3.53 628	3.53 626	7.99 522	1259	2.00 478	9.99 998	26
35	35	8.00 779	1223	3.53 628	3.53 626	8.00 781	1223	1.99 219	9.99 998	25
36	36	8.02 002	1190	3.53 628	3.53 626	8.02 004	1190	1.97 996	9.99 998	24
37	37	8.03 192	1158	3.53 628	3.53 626	8.03 194	1159	1.96 806	9.99 997	23
38	38	8.04 350	1128	3.53 628	3.53 626	8.04 353	1128	1.95 647	9.99 997	22
39	39	8.05 478	1100	3.53 628	3.53 626	8.05 481	1100	1.94 519	9.99 997	21
40	40	8.06 578	1072	3.53 628	3.53 625	8.06 581	1072	1.93 419	9.99 997	20
41	41	8.07 650	1046	3.53 628	3.53 625	8.07 653	1047	1.92 347	9.99 997	19
42	42	8.08 696	1022	3.53 628	3.53 625	8.08 700	1022	1.91 300	9.99 997	18
43	43	8.09 718	999	3.53 629	3.53 625	8.09 722	998	1.90 278	9.99 997	17
44	44	8.10 717	976	3.53 629	3.53 625	8.10 720	976	1.89 280	9.99 996	16
45	45	8.11 693	954	3.53 629	3.53 625	8.11 696	955	1.88 304	9.99 996	15
46	46	8.12 647	934	3.53 629	3.53 625	8.12 651	934	1.87 349	9.99 996	14
47	47	8.13 581	914	3.53 629	3.53 625	8.13 585	915	1.86 415	9.99 996	13
48	48	8.14 495	896	3.53 629	3.53 625	8.14 500	895	1.85 500	9.99 996	12
49	49	8.15 391	877	3.53 629	3.53 624	8.15 395	878	1.84 605	9.99 996	11
50	50	8.16 268	860	3.53 629	3.53 624	8.16 273	860	1.83 727	9.99 995	10
51	51	8.17 128	843	3.53 629	3.53 624	8.17 133	843	1.82 867	9.99 995	9
52	52	8.17 971	827	3.53 629	3.53 624	8.17 976	828	1.82 024	9.99 995	8
53	53	8.18 798	812	3.53 629	3.53 624	8.18 804	812	1.81 196	9.99 995	7
54	54	8.19 610	797	3.53 629	3.53 624	8.19 616	797	1.80 384	9.99 995	6
55	55	8.20 407	782	3.53 629	3.53 624	8.20 413	782	1.79 587	9.99 994	5
56	56	8.21 189	769	3.53 629	3.53 624	8.21 195	769	1.78 805	9.99 994	4
57	57	8.21 958	755	3.53 629	3.53 623	8.21 964	756	1.78 036	9.99 994	3
58	58	8.22 713	743	3.53 629	3.53 623	8.22 720	742	1.77 280	9.99 994	2
59	59	8.23 456	730	3.53 630	3.53 623	8.23 462	730	1.76 538	9.99 994	1
60	60	8.24 186		3.53 630	3.53 623	8.24 192		1.75 808	9.99 993	0
		L Cos	d			L Cot	c d	L Tan	L Sin	′

′	′	L Sin	d	Cpl S′	Cpl T′	L Tan	c d	L Cot	L Cos	
60	0	8.24 186	717	3.53 630	3.53 623	8.24 192	718	1.75 808	9.99 993	60
61	1	8.24 903	706	3.53 630	3.53 623	8.24 910	706	1.75 090	9.99 993	59
62	2	8.25 609	695	3.53 630	3.53 623	8.25 616	696	1.74 384	9.99 993	58
63	3	8.26 304	684	3.53 630	3.53 622	8.26 312	684	1.73 688	9.99 993	57
64	4	8.26 988	673	3.53 630	3.53 622	8.26 996	673	1.73 004	9.99 992	56
65	5	8.27 661	663	3.53 630	3.53 622	8.27 669	663	1.72 331	9.99 992	55
66	6	8.28 324	653	3.53 630	3.53 622	8.28 332	654	1.71 668	9.99 992	54
67	7	8.28 977	644	3.53 630	3.53 622	8.28 986	643	1.71 014	9.99 992	53
68	8	8.29 621	634	3.53 630	3.53 622	8.29 629	634	1.70 371	9.99 992	52
69	9	8.30 255	624	3.53 630	3.53 622	8.30 263	625	1.69 737	9.99 991	51
70	10	8.30 879	616	3.53 630	3.53 621	8.30 888	617	1.69 112	9.99 991	50
71	11	8.31 495	608	3.53 630	3.53 621	8.31 505	607	1.68 495	9.99 991	49
72	12	8.32 103	599	3.53 631	3.53 621	8.32 112	599	1.67 888	9.99 990	48
73	13	8.32 702	590	3.53 631	3.53 621	8.32 711	591	1.67 289	9.99 990	47
74	14	8.33 292	583	3.53 631	3.53 621	8.33 302	584	1.66 698	9.99 990	46
75	15	8.33 875	575	3.53 631	3.53 620	8.33 886	575	1.66 114	9.99 990	45
76	16	8.34 450	568	3.53 631	3.53 620	8.34 461	568	1.65 539	9.99 989	44
77	17	8.35 018	560	3.53 631	3.53 620	8.35 029	561	1.64 971	9.99 989	43
78	18	8.35 578	553	3.53 631	3.53 620	8.35 590	553	1.64 410	9.99 989	42
79	19	8.36 131	547	3.53 631	3.53 620	8.36 143	546	1.63 857	9.99 989	41
80	20	8.36 678	539	3.53 631	3.53 620	8.36 689	540	1.63 311	9.99 988	40
81	21	8.37 217	533	3.53 631	3.53 619	8.37 229	533	1.62 771	9.99 988	39
82	22	8.37 750	526	3.53 632	3.53 619	8.37 762	527	1.62 238	9.99 988	38
83	23	8.38 276	520	3.53 632	3.53 619	8.38 289	520	1.61 711	9.99 987	37
84	24	8.38 796	514	3.53 632	3.53 619	8.38 809	514	1.61 191	9.99 987	36
85	25	8.39 310	508	3.53 632	3.53 619	8.39 323	509	1.60 677	9.99 987	35
86	26	8.39 818	502	3.53 632	3.53 618	8.39 832	502	1.60 168	9.99 986	34
87	27	8.40 320	496	3.53 632	3.53 618	8.40 334	496	1.59 666	9.99 986	33
88	28	8.40 816	491	3.53 632	3.53 618	8.40 830	491	1.59 170	9.99 986	32
89	29	8.41 307	485	3.53 632	3.53 618	8.41 321	486	1.58 679	9.99 985	31
90	30	8.41 792	480	3.53 632	3.53 617	8.41 807	480	1.58 193	9.99 985	30
91	31	8.42 272	474	3.53 632	3.53 617	8.42 287	475	1.57 713	9.99 985	29
92	32	8.42 746	470	3.53 633	3 53 617	8.42 762	470	1.57 238	9.99 984	28
93	33	8.43 216	464	3.53 633	3.53 617	8.43 232	464	1.56 768	9.99 984	27
94	34	8.43 680	459	3.53 633	3.53 617	8.43 696	460	1.56 304	9.99 984	26
95	35	8.44 139	455	3.53 633	3.53 616	8.44 156	455	1.55 844	9.99 983	25
96	36	8.44 594	450	3.53 633	3.53 616	8.44 611	450	1.55 389	9.99 983	24
97	37	8.45 044	445	3.53 633	3.53 616	8.45 061	446	1.54 939	9.99 983	23
98	38	8.45 489	441	3.53 633	3.53 616	8.45 507	441	1.54 493	9.99 982	22
99	39	8.45 930	436	3.53 633	3.53 615	8.45 948	437	1.54 052	9.99 982	21
100	40	8.46 366	433	3.53 634	3.53 615	8.46 385	432	1.53 615	9.99 982	20
101	41	8.46 799	427	3.53 634	3.53 615	8.46 817	428	1.53 183	9.99 981	19
102	42	8.47 226	424	3.53 634	3.53 615	8.47 245	424	1.52 755	9.99 981	18
103	43	8.47 650	419	3.53 634	3.53 614	8.47 669	420	1.52 331	9.99 981	17
104	44	8.48 069	416	3.53 634	3.53 614	8.48 089	416	1.51 911	9.99 980	16
105	45	8.48 485	411	3.53 634	3.53 614	8.48 505	412	1.51 495	9.99 980	15
106	46	8.48 896	408	3.53 634	3.53 614	8.48 917	408	1.51 083	9.99 979	14
107	47	8.49 304	404	3.53 634	3.53 613	8.49 325	404	1.50 675	9.99 979	13
108	48	8.49 708	400	3.53 635	3.53 613	8.49 729	401	1.50 271	9.99 979	12
109	49	8.50 108	396	3.53 635	3.53 613	8.50 130	397	1.49 870	9.99 978	11
110	50	8.50 504	393	3.53 635	3.53 613	8.50 527	393	1.49 473	9.99 978	10
111	51	8.50 897	390	3.53 635	3.53 612	8.50 920	390	1.49 080	9.99 977	9
112	52	8.51 287	386	3.53 635	3.53 612	8.51 310	386	1.48 690	9.99 977	8
113	53	8.51 673	382	3.53 635	3.53 612	8.51 696	383	1.48 304	9.99 977	7
114	54	8.52 055	379	3.53 635	3.53 611	8.52 079	380	1.47 921	9.99 976	6
115	55	8.52 434	376	3.53 635	3.53 611	8.52 459	376	1.47 541	9.99 976	5
116	56	8.52 810	373	3.53 636	3.53 611	8.52 835	373	1.47 165	9.99 975	4
117	57	8.53 183	369	3.53 636	3.53 611	8.53 208	370	1.46 792	9.99 975	3
118	58	8.53 552	367	3.53 636	3.53 610	8.53 578	367	1.46 422	9.99 974	2
119	59	8.53 919	363	3.53 636	3.53 610	8.53 945	363	1.46 055	9.99 974	1
120	60	8.54 282		3.53 636	3.53 610	8.54 308		1.45 692	9.99 974	0
		L Cos	d			L Cot	c d	L Tan	L Sin	′

′	′	L Sin	d	Cpl S′	Cpl T′	L Tan	c d	L Cot	L Cos	
120	0	8.54 282	360	3.53 636	3.53 610	8.54 308	361	1.45 692	9.99 974	60
121	1	8.54 642	357	3.53 636	3.53 609	8.54 669	358	1.45 331	9.99 973	59
122	2	8.54 999	355	3.53 637	3.53 609	8.55 027	355	1.44 973	9.99 973	58
123	3	8.55 354	351	3.53 637	3.53 609	8.55 382	352	1.44 618	9.99 972	57
124	4	8.55 705	349	3.53 637	3.53 609	8.55 734	349	1.44 266	9.99 972	56
125	5	8.56 054	346	3.53 637	3.53 608	8.56 083	346	1.43 917	9.99 971	55
126	6	8.56 400	343	3.53 637	3.53 608	8.56 429	344	1.43 571	9.99 971	54
127	7	8.56 743	341	3.53 637	3.53 608	8.56 773	341	1.43 227	9.99 970	53
128	8	8.57 084	337	3.53 637	3.53 607	8.57 114	338	1.42 886	9.99 970	52
129	9	8.57 421	336	3.53 638	3.53 607	8.57 452	336	1.42 548	9.99 969	51
130	10	8.57 757	332	3.53 638	3.53 607	8.57 788	333	1.42 212	9.99 969	50
131	11	8.58 089	330	3.53 638	3.53 606	8.58 121	330	1.41 879	9.99 968	49
132	12	8.58 419	328	3.53 638	3.53 606	8.58 451	328	1.41 549	9.99 968	48
133	13	8.58 747	325	3.53 638	3.53 606	8.58 779	326	1.41 221	9.99 967	47
134	14	8.59 072	323	3.53 638	3.53 605	8.59 105	323	1.40 895	9.99 967	46
135	15	8.59 395	320	3.53 639	3.53 605	8.59 428	321	1.40 572	9.99 967	45
136	16	8.59 715	318	3.53 639	3.53 605	8.59 749	319	1.40 251	9.99 966	44
137	17	8.60 033	316	3.53 639	3.53 604	8.60 068	316	1.39 932	9.99 966	43
138	18	8.60 349	313	3.53 639	3.53 604	8.60 384	314	1.39 616	9.99 965	42
139	19	8.60 662	311	3.53 639	3.53 604	8.60 698	311	1.39 302	9.99 964	41
140	20	8.60 973	309	3.53 639	3.53 603	8.61 009	310	1.38 991	9.99 964	40
141	21	8.61 282	307	3.53 640	3.53 603	8.61 319	307	1.38 681	9.99 963	39
142	22	8.61 589	305	3.53 640	3.53 603	8.61 626	305	1.38 374	9.99 963	38
143	23	8.61 894	302	3.53 640	3.53 602	8.61 931	303	1.38 069	9.99 962	37
144	24	8.62 196	301	3.53 640	3.53 602	8.62 234	301	1.37 766	9.99 962	36
145	25	8.62 497	298	3.53 640	3.53 602	8.62 535	299	1.37 465	9.99 961	35
146	26	8.62 795	296	3.53 640	3.53 601	8.62 834	297	1.37 166	9.99 961	34
147	27	8.63 091	294	3.53 641	3.53 601	8.63 131	295	1.36 869	9.99 960	33
148	28	8.63 385	293	3.53 641	3.53 601	8.63 426	292	1.36 574	9.99 960	32
149	29	8.63 678	290	3.53 641	3.53 600	8.63 718	291	1.36 282	9.99 959	31
150	30	8.63 968	288	3.53 641	3.53 600	8.64 009	289	1.35 991	9.99 959	30
151	31	8.64 256	287	3.53 641	3.53 599	8.64 298	287	1.35 702	9.99 958	29
152	32	8.64 543	284	3.53 642	3.53 599	8.64 585	285	1.35 415	9.99 958	28
153	33	8.64 827	283	3.53 642	3.53 599	8.64 870	284	1.35 130	9.99 957	27
154	34	8.65 110	281	3.53 642	3.53 598	8.65 154	281	1.34 846	9.99 956	26
155	35	8.65 391	279	3.53 642	3.53 598	8.65 435	280	1.34 565	9.99 956	25
156	36	8.65 670	277	3.53 642	3.53 598	8.65 715	278	1.34 285	9.99 955	24
157	37	8.65 947	276	3.53 642	3.53 597	8.65 993	276	1.34 007	9.99 955	23
158	38	8.66 223	274	3.53 643	3.53 597	8.66 269	274	1.33 731	9.99 954	22
159	39	8.66 497	272	3.53 643	3.53 596	8.66 543	273	1.33 457	9.99 954	21
160	40	8.66 769	270	3.53 643	3.53 596	8.66 816	271	1.33 184	9.99 953	20
161	41	8.67 039	269	3.53 643	3.53 596	8.67 087	269	1.32 913	9.99 952	19
162	42	8.67 308	267	3.53 643	3.53 595	8.67 356	268	1.32 644	9.99 952	18
163	43	8.67 575	266	3.53 644	3.53 595	8.67 624	266	1.32 376	9.99 951	17
164	44	8.67 841	263	3.53 644	3.53 594	8.67 890	264	1.32 110	9.99 951	16
165	45	8.68 104	263	3.53 644	3.53 594	8.68 154	263	1.31 846	9.99 950	15
166	46	8.68 367	260	3.53 644	3.53 594	8.68 417	261	1.31 583	9.99 949	14
167	47	8.68 627	259	3.53 644	3.53 593	8.68 678	260	1.31 322	9.99 949	13
168	48	8.68 886	258	3.53 645	3.53 593	8.68 938	258	1.31 062	9.99 948	12
169	49	8.69 144	256	3.53 645	3.53 592	8.69 196	257	1.30 804	9.99 948	11
170	50	8.69 400	254	3.53 645	3.53 592	8.69 453	255	1.30 547	9.99 947	10
171	51	8.69 654	253	3.53 645	3.53 592	8.69 708	254	1.30 292	9.99 946	9
172	52	8.69 907	252	3.53 646	3.53 591	8.69 962	252	1.30 038	9.99 946	8
173	53	8.70 159	250	3.53 646	3.53 591	8.70 214	251	1.29 786	9.99 945	7
174	54	8.70 409	249	3.53 646	3.53 590	8.70 465	249	1.29 535	9.99 944	6
175	55	8.70 658	247	3.53 646	3.53 590	8.70 714	248	1.29 286	9.99 944	5
176	56	8.70 905	246	3.53 646	3.53 589	8.70 962	246	1.29 038	9.99 943	4
177	57	8.71 151	244	3.53 647	3.53 589	8.71 208	245	1.28 792	9.99 942	3
178	58	8.71 395	243	3.53 647	3.53 589	8.71 453	244	1.28 547	9.99 942	2
179	59	8.71 538	242	3.53 647	3.53 588	8.71 697	243	1.28 303	9.99 941	1
180	60	8.71 880		3.53 647	3.53 588	8.71 940		1.28 060	9.99 940	0
		L Cos	d			L Cot	c d	L Tan	L Sin	′

′	L Sin	d	L Tan	c d	L Cot	L Cos		′	P P				
									241	239	237	236	234
0	8.71 880		8.71 940		1.28 060	9.99 940	60	1	24.1	23.9	23.7	23.6	23.4
		240		241				2	48.2	47.8	47.4	47.2	46.8
1	8.72 120		8 72 181		1.27 819	9.99 940	59	3	72.3	71.7	71.1	70.8	70.2
2	8.72 359	239	8.72 420	239	1.27 580	9.99 939	58	4	96.4	95.6	94.8	94.4	93.6
3	8.72 597	238	8.72 659	239	1.27 341	9.99 938	57	5	120.5	119.5	118.5	118.0	117.0
		237		237				6	144.6	143.4	142.2	141.6	140.4
4	8.72 834		8.72 896		1.27 104	9.99 938	56	7	168.7	167.3	165.9	165.2	163.8
5	8.73 069	235	8.73 132	236	1.26 868	9.99 937	55	8	192.8	191.2	189.6	188.8	187.2
6	8.73 303	234	8.73 366	234	1.26 634	9.99 936	54	9	216.9	215.1	213.3	212.4	210.6
		232		234									
7	8.73 535		8.73 600		1.26 400	9.99 936	53		232	231	229	227	226
8	8.73 767	232	8.73 832	232	1.26 168	9.99 935	52	1	23.2	23.1	22.9	22.7	22.6
9	8.73 997	230	8.74 063	231	1 25 937	9.99 934	51	2	46.4	46.2	45.8	45.4	45.2
		229		229				3	69.6	69.3	68.7	68.1	67.8
10	8.74 226		8.74 292		1.25 708	9.99 934	50	4	92.8	92.4	91.6	90.8	90.4
		228		229				5	116.0	115.5	114.5	113.5	113.0
11	8.74 454		8.74 521		1.25 479	9.99 933	49	6	139.2	138.6	137.4	136.2	135.6
12	8.74 680	226	8 74 748	227	1.25 252	9.99 932	48	7	162.4	161.7	160.3	158.9	158.2
13	8.74 906	226	8.74 974	226	1.25 026	9.99 932	47	8	185.6	184.8	183.2	181.6	180.8
		224		225				9	208.8	207.9	206.1	204.3	203.4
14	8 75 130		8.75 199		1.24 801	9.99 931	46						
15	8.75 353	223	8.75 423	224	1.24 577	9.99 930	45		224	222	220	219	217
16	8.75 575	222	8.75 645	222	1.24 355	9.99 929	44	1	22.4	22.2	22.0	21.9	21.7
		220		222				2	44.8	44.4	44.0	43.8	43.4
17	8.75 795		8.75 867		1.24 133	9.99 929	43	3	67.2	66.6	66.0	65.7	65.1
18	8.76 015	220	8.76 087	220	1.23 913	9.99 928	42	4	89.6	88.8	88.0	87.6	86.8
19	8.76 234	219	8.76 306	219	1.23 694	9.99 927	41	5	112.0	111.0	110.0	109.5	108.5
		217		219				6	134.4	133.2	132.0	131.4	130.2
20	8.76 451		8.76 525		1.23 475	9.99 926	40	7	156.8	155.4	154.0	153.3	151.9
		216		217				8	179.2	177.6	176.0	175.2	173.6
21	8.76 667		8.76 742		1.23 258	9.99 926	39	9	201.6	199.8	198.0	197 1	195.3
22	8.76 883	216	8.76 958	216	1.23 042	9.99 925	38						
23	8.77 097	214	8.77 173	215	1.22 827	9.99 924	37		216	214	213	211	209
		213		214				1	21.6	21.4	21.3	21.1	20.9
24	8.77 310		8.77 387		1 22 613	9.99 923	36	2	43.2	42.8	42.6	42.2	41.8
25	8.77 522	212	8.77 600	213	1.22 400	9.99 923	35	3	64.8	64.2	63.9	63.3	62.7
26	8.77 733	211	8.77 811	211	1.22 189	9.99 922	34	4	86.4	85.6	85.2	84.4	83.6
		210		211				5	108.0	107.0	106.5	105.5	104.5
27	8.77 943		8.78 022		1.21 978	9.99 921	33	6	129.6	128.4	127.8	126.6	125.4
28	8.78 152	209	8.78 232	210	1.21 768	9.99 920	32	7	151.2	149.8	149.1	147.7	146.3
29	8.78 360	208	8.78 441	209	1.21 559	9.99 920	31	8	172.8	171.2	170.4	168.8	167.2
		208		208				9	194.4	192.6	191.7	189.9	188.1
30	8.78 568		8.78 649		1.21 351	9.99 919	30						
		206		206					208	206	203	201	199
31	8.78 774		8.78 855		1.21 145	9.99 918	29	1	20.8	20.6	20.3	20.1	19.9
32	8.78 979	205	8.79 061	206	1.20 939	9.99 917	28	2	41.6	41.2	40.6	40.2	39.8
33	8.79 183	204	8.79 266	205	1.20 734	9.99 917	27	3	62.4	61.8	60.9	60.3	59.7
		203		204				4	83.2	82.4	81.2	80.4	79.6
34	8.79 386		8.79 470		1.20 530	9.99 916	26	5	104.0	103.0	101.5	100.5	99.5
35	8.79 588	202	8.79 673	203	1.20 327	9.99 915	25	6	124.8	123.6	121.8	120.6	119.4
36	8.79 789	201	8.79 875	202	1.20 125	9.99 914	24	7	145.6	144.2	142.1	140.7	139.3
		201		201				8	166.4	164.8	162.4	160.8	159.2
37	8.79 990		8.80 076		1.19 924	9.99 913	23	9	187.2	185.4	182.7	180.9	179.1
38	8.80 189	199	8.80 277	201	1.19 723	9.99 913	22						
39	8.80 388	199	8.80 476	199	1.19 524	9.99 912	21		198	196	194	192	190
		197		198				1	19.8	19.6	19.4	19.2	19.0
40	8.80 585		8.80 674		1.19 326	9.99 911	20	2	39.6	39.2	38.8	38.4	38.0
		197		198				3	59.4	58.8	58.2	57.6	57.0
41	8.80 782		8.80 872		1.19 128	9.99 910	19	4	79.2	78.4	77.6	76.8	76.0
42	8.80 978	196	8.81 068	196	1.18 932	9.99 909	18	5	99.0	98.0	97.0	96.0	95.0
43	8.81 173	195	8.81 264	196	1.18 736	9.99 909	17	6	118.8	117.6	116.4	115.2	114.0
		194		195				7	138.6	137.2	135.8	134.4	133.0
44	8.81 367		8.81 459		1.18 541	9.99 908	16	8	158.4	156.8	155.2	153.6	152.0
45	8.81 560	193	8.81 653	194	1.18 347	9.99 907	15	9	178.2	176.4	174.6	172.8	171.0
46	8.81 752	192	8.81 846	193	1.18 154	9.99 906	14						
		192		192					188	186	184	182	181
47	8.81 944		8.82 038		1.17 962	9.99 905	13	1	18.8	18.6	18.4	18.2	18.1
48	8.82 134	190	8.82 230	192	1.17 770	9.99 904	12	2	37.6	37.2	36.8	36.4	36.2
49	8.82 324	190	8.82 420	190	1.17 580	9.99 904	11	3	56.4	55.8	55.2	54.6	54.3
		189		190				4	75.2	74.4	73.6	72.8	72.4
50	8.82 513		8.82 610		1.17 390	9.99 903	10	5	94.0	93.0	92.0	91.0	90.5
		188		189				6	112.8	111.6	110.4	109.2	108.6
51	8.82 701		8.82 799		1.17 201	9.99 902	9	7	131.6	130.2	128.8	127.4	126.7
52	8.82 888	187	8.82 987	188	1 17 013	9.99 901	8	8	150.4	148.8	147.2	145.6	144.8
53	8.83 075	187	8.83 175	188	1.16 825	9.99 900	7	9	169.2	167.4	165.6	163.8	162.9
		186		186									
54	8.83 261		8.83 361		1.16 639	9.99 899	6						
55	8.83 446	185	8.83 547	186	1.16 453	9.99 898	5						
56	8.83 630	184	8.83 732	185	1.16 268	9.99 898	4						
		183		184									
57	8.83 813		8.83 916		1.16 084	9.99 897	3						
58	8.83 996	183	8.84 100	184	1.15 900	9.99 896	2						
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1						
		181		182									
60	8.84 358		8.84 464		1.15 536	9.99 894	0						
	L Cos	d	L Cot	c d	L Tan	L Sin	′		P P				

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.84 358		8.84 464		1.15 536	9.99 894	60
		181		182			
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	1 14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	1 14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	55
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	1 14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	1 14 107	9.99 887	52
9	8.85 955		8.86 069		1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	50
		173		174			
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	1.12 894	9.99 880	45
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	1.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 878	42
19	8.87 661	168	8.87 785	169	1.12 215	9.99 877	41
20	8.87 829	166	8.87 953	168	1.12 047	9.99 876	40
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39
22	8.88 161	166	8.88 287	167	1.11 713	9.99 874	38
23	8.88 326	165	8.88 453	166	1.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36
25	8.88 654	164	8.88 783	165	1.11 217	9.99 871	35
26	8.88 817	163	8.88 948	165	1.11 052	9.99 870	34
27	8.88 980	163	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32
29	8.89 304	162	8.89 437	163	1.10 563	9.99 867	31
30	8.89 464	160	8.89 598	161	1.10 402	9.99 866	30
31	8.89 625	161	8.89 760	162	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27
34	8.90 102	159	8.90 240	160	1.09 760	9.99 862	26
35	8.90 260	158	8.90 399	159	1.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24
37	8.90 574	157	8.90 715	158	1.09 285	9.99 859	23
38	8.90 730	156	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 029	157	1.08 971	9.99 857	21
40	8.91 040	155	8.91 185	156	1.08 815	9.99 856	20
		155		155			
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	1.08 505	9.99 854	18
43	8.91 502	153	8.91 650	155	1.08 350	9.99 853	17
44	8.91 655	153	8.91 803	153	1.08 197	9.99 852	16
45	8.91 807	152	8.91 957	154	1.08 043	9.99 851	15
46	8.91 959	152	8.92 110	153	1.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13
48	8.92 261	151	8.92 414	152	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11
50	8.92 561	150	8.92 716	151	1.07 284	9.99 845	10
		149		150			
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 165	149	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	148	1.06 687	9.99 841	6
55	8.93 301	147	8.93 462	149	1.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	1.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3
58	8.93 740	145	8.93 903	147	1.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1
60	8.94 030		8.94 195		1.05 805	9.99 834	0
	L Cos	d	L Cot	c d	L Tan	L Sin	′

P P

	182	181	180	179	178
1	18.2	18.1	18.0	17.9	17.8
2	36.4	36.2	36.0	35.8	35.6
3	54.6	54.3	54.0	53.7	53.4
4	72.8	72.4	72.0	71.6	71.2
5	91.0	90.5	90.0	89.5	89.0
6	109.2	108.6	108.0	107.4	106.8
7	127.4	126.7	126.0	125.3	124.6
8	145.6	144.8	144.0	143.2	142.4
9	163.8	162.9	162.0	161.1	160.2

	177	176	175	174	173
1	17.7	17.6	17.5	17.4	17.3
2	35.4	35.2	35.0	34.8	34.6
3	53.1	52.8	52.5	52.2	51.9
4	70.8	70.4	70.0	69.6	69.2
5	88.5	88.0	87.5	87.0	86.5
6	106.2	105.6	105.0	104.4	103.8
7	123.9	123.2	122.5	121.8	121.1
8	141.6	140.8	140.0	139.2	138.4
9	159.3	158.4	157.5	156.6	155.7

	172	171	170	169	168
1	17.2	17.1	17.0	16.9	16.8
2	34.4	34.2	34.0	33.8	33.6
3	51.6	51.3	51.0	50.7	50.4
4	68.8	68.4	68.0	67.6	67.2
5	86.0	85.5	85.0	84.5	84.0
6	103.2	102.6	102.0	101.4	100.8
7	120.4	119.7	119.0	118.3	117.6
8	137.6	136.8	136.0	135.2	134.4
9	154.8	153.9	153.0	152.1	151.2

	167	166	165	164	163
1	16.7	16.6	16.5	16.4	16.3
2	33.4	33.2	33.0	32.8	32.6
3	50.1	49.8	49.5	49.2	48.9
4	66.8	66.4	66.0	65.6	65.2
5	83.5	83.0	82.5	82.0	81.5
6	100.2	99.6	99.0	98.4	97.8
7	116.9	116.2	115.5	114.8	114.1
8	133.6	132.8	132.0	131.2	130.4
9	150.3	149.4	148.5	147.6	146.7

	162	161	160	159	158
1	16.2	16.1	16.0	15.9	15.8
2	32.4	32.2	32.0	31.8	31.6
3	48.6	48.3	48.0	47.7	47.4
4	64.8	64.4	64.0	63.6	63.2
5	81.0	80.5	80.0	79.5	79.0
6	97.2	96.6	96.0	95.4	94.8
7	113.4	112.7	112.0	111.3	110.6
8	129.6	128.8	128.0	127.2	126.4
9	145.8	144.9	144.0	143.1	142.2

	157	156	155	154	153
1	15.7	15.6	15.5	15.4	15.3
2	31.4	31.2	31.0	30.8	30.6
3	47.1	46.8	46.5	46.2	45.9
4	62.8	62.4	62.0	61.6	61.2
5	78.5	78.0	77.5	77.0	76.5
6	94.2	93.6	93.0	92.4	91.8
7	109.9	109.2	108.5	107.8	107.1
8	125.6	124.8	124.0	123.2	122.4
9	141.3	140.4	139.5	138.6	137.7

	152	151	150	149	148
1	15.2	15.1	15.0	14.9	14.8
2	30.4	30.2	30.0	29.8	29.6
3	45.6	45.3	45.0	44.7	44.4
4	60.8	60.4	60.0	59.6	59.2
5	76.0	75.5	75.0	74.5	74.0
6	91.2	90.6	90.0	89.4	88.8
7	106.4	105.7	105.0	104.3	103.6
8	121.6	120.8	120.0	119.2	118.4
9	136.8	135.9	135.0	134.1	133.2

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.94 030		8.94 195		1.05 805	9.99 834	**60**
		144		145			
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52
9	8.95 310		8.95 486		1.04 514	9.99 824	51
10	8.95 450	140	8.95 627	141	1.04 373	9.99 823	**50**
		139		140			
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	**40**
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	**30**
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	**20**
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	**10**
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

P P

	147	146	145	144
1	14.7	14.6	14.5	14.4
2	29.4	29.2	29.0	28.8
3	44.1	43.8	43.5	43.2
4	58.8	58.4	58.0	57.6
5	73.5	73.0	72.5	72.0
6	88.2	87.6	87.0	86.4
7	102.9	102.2	101.5	100.8
8	117.6	116.8	116.0	115.2
9	132.3	131.4	130.5	129.6

	143	142	141	140
1	14.3	14.2	14.1	14.0
2	28.6	28.4	28.2	28.0
3	42.9	42.6	42.3	42.0
4	57.2	56.8	56.4	56.0
5	71.5	71.0	70.5	70.0
6	85.8	85.2	84.6	84.0
7	100.1	99.4	98.7	98.0
8	114.4	113.6	112.8	112.0
9	128.7	127.8	126.9	126.0

	139	138	137	136
1	13.9	13.8	13.7	13.6
2	27.8	27.6	27.4	27.2
3	41.7	41.4	41.1	40.8
4	55.6	55.2	54.8	54.4
5	69.5	69.0	68.5	68.0
6	83.4	82.8	82.2	81.6
7	97.3	96.6	95.9	95.2
8	111.2	110.4	109.6	108.8
9	125.1	124.2	123.3	122.4

	135	134	133	132
1	13.5	13.4	13.3	13.2
2	27.0	26.8	26.6	26.4
3	40.5	40.2	39.9	39.6
4	54.0	53.6	53.2	52.8
5	67.5	67.0	66.5	66.0
6	81.0	80.4	79.8	79.2
7	94.5	93.8	93.1	92.4
8	108.0	107.2	106.4	105.6
9	121.5	120.6	119.7	118.8

	131	130	129	128
1	13.1	13.0	12.9	12.8
2	26.2	26.0	25.8	25.6
3	39.3	39.0	38.7	38.4
4	52.4	52.0	51.6	51.2
5	65.5	65.0	64.5	64.0
6	78.6	78.0	77.4	76.8
7	91.7	91.0	90.3	89.6
8	104.8	104.0	103.2	102.4
9	117.9	117.0	116.1	115.2

	127	126	125	124
1	12.7	12.6	12.5	12.4
2	25.4	25.2	25.0	24.8
3	38.1	37.8	37.5	37.2
4	50.8	50.4	50.0	49.6
5	63.5	63.0	62.5	62.0
6	76.2	75.6	75.0	74.4
7	88.9	88.2	87.5	86.8
8	101.6	100.8	100.0	99.2
9	114.3	113.4	112.5	111.6

	123	122	121	120
1	12.3	12.2	12.1	12.0
2	24.6	24.4	24.2	24.0
3	36.9	36.6	36.3	36.0
4	49.2	48.8	48.4	48.0
5	61.5	61.0	60.5	60.0
6	73.8	73.2	72.6	72.0
7	86.1	85.4	84.7	84.0
8	98.4	97.6	96.8	96.0
9	110.7	109.8	108.9	108.0

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	0.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46
15	9.03 690	116	9.03 948	117	0.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44
17	9.03 920	115	9.04 181	116	0.95 819	9.99 738	43
18	9.04 034	114	9.04 297	116	0.95 703	9.99 737	42
19	9.04 149	115	9.04 413	115	0.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	40
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	30
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23
38	9.06 264	109	9.06 556	110	0.93 444	9.99 708	22
39	9.06 372	108	9.06 666	109	0.93 334	9.99 707	21
40	9.06 481	109	9.06 775	110	0.93 225	9.99 705	20
41	9.06 589	108	9.06 885	109	0.93 115	9.99 704	19
42	9.06 696	107	9.06 994	109	0.93 006	9.99 702	18
43	9.06 804	108	9.07 103	108	0.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	0.92 789	9.99 699	16
45	9.07 018	107	9.07 320	108	0.92 680	9.99 698	15
46	9.07 124	106	9.07 428	108	0.92 572	9.99 696	14
47	9.07 231	107	9.07 536	107	0.92 464	9.99 695	13
48	9.07 337	106	9.07 643	108	0.92 357	9.99 693	12
49	9.07 442	105	9.07 751	107	0.92 249	9.99 692	11
50	9.07 548	106	9.07 858	106	0.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	0.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7
54	9.07 968	105	9.08 283	106	0.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4
57	9.08 280	104	9.08 600	105	0.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	105	0.91 190	9.99 677	1
60	9.08 589	103	9.08 914	104	0.91 086	9.99 675	0
	L Cos	d	L Cot	c d	L Tan	L Sin	′

P P

	121	120	119	118
1	12.1	12.0	11.9	11.8
2	24.2	24.0	23.8	23.6
3	36.3	36.0	35.7	35.4
4	48.4	48.0	47.6	47.2
5	60.5	60.0	59.5	59.0
6	72.6	72.0	71.4	70.8
7	84.7	84.0	83.3	82.6
8	96.8	96.0	95.2	94.4
9	108.9	108.0	107.1	106.2

	117	116	115	114
1	11.7	11.6	11.5	11.4
2	23.4	23.2	23.0	22.8
3	35.1	34.8	34.5	34.2
4	46.8	46.4	46.0	45.6
5	58.5	58.0	57.5	57.0
6	70.2	69.6	69.0	68.4
7	81.9	81.2	80.5	79.8
8	93.6	92.8	92.0	91.2
9	105.3	104.4	103.5	102.6

	113	112	111	110
1	11.3	11.2	11.1	11.0
2	22.6	22.4	22.2	22.0
3	33.9	33.6	33.3	33.0
4	45.2	44.8	44.4	44.0
5	56.5	56.0	55.5	55.0
6	67.8	67.2	66.6	66.0
7	79.1	78.4	77.7	77.0
8	90.4	89.6	88.8	88.0
9	101.7	100.8	99.9	99.0

	109	108	107	106
1	10.9	10.8	10.7	10.6
2	21.8	21.6	21.4	21.2
3	32.7	32.4	32.1	31.8
4	43.6	43.2	42.8	42.4
5	54.5	54.0	53.5	53.0
6	65.4	64.8	64.2	63.6
7	76.3	75.6	74.9	74.2
8	87.2	86.4	85.6	84.8
9	98.1	97.2	96.3	95.4

	105	104	103
1	10.5	10.4	10.3
2	21.0	20.8	20.6
3	31.5	31.2	30.9
4	42.0	41.6	41.2
5	52.5	52.0	51.5
6	63.0	62.4	61.8
7	73.5	72.8	72.1
8	84.0	83.2	82.4
9	94.5	93.6	92.7

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.08 589		9.08 914		0.91 086	9.99 675	**60**
		103		105			
1	9.08 692		9.09 019		0.90 981	9.99 674	59
		103		104			
2	9.08 795		9.09 123		0.90 877	9.99 672	58
		102		104			
3	9.08 897		9.09 227		0.90 773	9.99 670	57
		102		103			
4	9.08 999		9.09 330		0.90 670	9.99 669	56
		102		104			
5	9.09 101		9.09 434		0.90 566	9.99 667	55
		101		103			
6	9.09 202		9.09 537		0.90 463	9.99 666	54
		102		103			
7	9.09 304		9.09 640		0.90 360	9.99 664	53
		101		102			
8	9.09 405		9.09 742		0.90 258	9.99 663	52
		101		103			
9	9.09 506		9.09 845		0.90 155	9.99 661	51
		100		102			
10	9.09 606		9.09 947		0.90 053	9.99 659	**50**
		101		102			
11	9.09 707		9.10 049		0.89 951	9.99 658	49
		100		101			
12	9.09 807		9.10 150		0.89 850	9.99 656	48
		100		102			
13	9.09 907		9.10 252		0.89 748	9.99 655	47
		99		101			
14	9.10 006		9.10 353		0.89 647	9.99 653	46
		100		101			
15	9.10 106		9.10 454		0.89 546	9.99 651	45
		99		101			
16	9.10 205		9.10 555		0.89 445	9.99 650	44
		99		101			
17	9.10 304		9.10 656		0.89 344	9.99 648	43
		98		100			
18	9.10 402		9.10 756		0.89 244	9.99 647	42
		98		100			
19	9.10 501		9.10 856		0.89 144	9.99 645	41
		98		100			
20	9.10 599		9.10 956		0.89 044	9.99 643	**40**
		98		100			
21	9.10 697		9.11 056		0.88 944	9.99 642	39
		98		99			
22	9.10 795		9.11 155		0.88 845	9.99 640	38
		98		99			
23	9.10 893		9.11 254		0.88 746	9.99 638	37
		97		99			
24	9.10 990		9.11 353		0.88 647	9.99 637	36
		97		99			
25	9.11 087		9.11 452		0.88 548	9.99 635	35
		97		99			
26	9.11 184		9.11 551		0.88 449	9.99 633	34
		97		98			
27	9.11 281		9.11 649		0.88 351	9.99 632	33
		96		98			
28	9.11 377		9.11 747		0.88 253	9.99 630	32
		97		98			
29	9.11 474		9.11 845		0.88 155	9.99 629	31
		96		98			
30	9.11 570		9.11 943		0.88 057	9.99 627	**30**
		96		97			
31	9.11 666		9.12 040		0.87 960	9.99 625	29
		95		98			
32	9.11 761		9.12 138		0.87 862	9.99 624	28
		96		97			
33	9.11 857		9.12 235		0.87 765	9.99 622	27
		95		97			
34	9.11 952		9.12 332		0.87 668	9.99 620	26
		95		96			
35	9.12 047		9.12 428		0.87 572	9.99 618	25
		95		97			
36	9.12 142		9.12 525		0.87 475	9.99 617	24
		94		96			
37	9.12 236		9.12 621		0.87 379	9.99 615	23
		95		96			
38	9.12 331		9.12 717		0.87 283	9.99 613	22
		94		96			
39	9.12 425		9.12 813		0.87 187	9.99 612	21
		94		96			
40	9.12 519		9.12 909		0.87 091	9.99 610	**20**
		93		95			
41	9.12 612		9.13 004		0.86 996	9.99 608	19
		94		95			
42	9.12 706		9.13 099		0.86 901	9.99 607	18
		93		95			
43	9.12 799		9.13 194		0.86 806	9.99 605	17
		93		95			
44	9.12 892		9.13 289		0.86 711	9.99 603	16
		93		95			
45	9.12 985		9.13 384		0.86 616	9.99 601	15
		93		94			
46	9.13 078		9.13 478		0.86 522	9.99 600	14
		93		95			
47	9.13 171		9.13 573		0.86 427	9.99 598	13
		92		94			
48	9.13 263		9.13 667		0.86 333	9.99 596	12
		92		94			
49	9.13 355		9.13 761		0.86 239	9.99 595	11
		92		93			
50	9.13 447		9.13 854		0.86 146	9.99 593	**10**
		92		94			
51	9.13 539		9.13 948		0.86 052	9.99 591	9
		91		93			
52	9.13 630		9.14 041		0.85 959	9.99 589	8
		92		93			
53	9.13 722		9.14 134		0.85 866	9.99 588	7
		91		93			
54	9.13 813		9.14 227		0.85 773	9.99 586	6
		91		93			
55	9.13 904		9.14 320		0.85 680	9.99 584	5
		90		92			
56	9.13 994		9.14 412		0.85 588	9.99 582	4
		91		92			
57	9.14 085		9.14 504		0.85 496	9.99 581	3
		90		93			
58	9.14 175		9.14 597		0.85 403	9.99 579	2
		91		91			
59	9.14 266		9.14 688		0.85 312	9.99 577	1
		90		92			
60	9.14 356		9.14 780		0.85 220	9.99 575	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

P P			
	105	**104**	**103**
1	10.5	10.4	10.3
2	21.0	20.8	20.6
3	31.5	31.2	30.9
4	42.0	41.6	41.2
5	52.5	52.0	51.5
6	63.0	62.4	61.8
7	73.5	72.8	72.1
8	84.0	83.2	82.4
9	94.5	93.6	92.7
	102	**101**	**99**
1	10.2	10.1	9.9
2	20.4	20.2	19.8
3	30.6	30.3	29.7
4	40.8	40.4	39.6
5	51.0	50.5	49.5
6	61.2	60.6	59.4
7	71.4	70.7	69.3
8	81.6	80.8	79.2
9	91.8	90.9	89.1
	98	**97**	**96**
1	9.8	9.7	9.6
2	19.6	19.4	19.2
3	29.4	29.1	28.8
4	39.2	38.8	38.4
5	49.0	48.5	48.0
6	58.8	58.2	57.6
7	68.6	67.9	67.2
8	78.4	77.6	76.8
9	88.2	87.3	86.4
	95	**94**	**93**
1	9.5	9.4	9.3
2	19.0	18.8	18.6
3	28.5	28.2	27.9
4	38.0	37.6	37.2
5	47.5	47.0	46.5
6	57.0	56.4	55.8
7	66.5	65.8	65.1
8	76.0	75.2	74.4
9	85.5	84.6	83.7
	92	**91**	**90**
1	9.2	9.1	9.0
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	**60**
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	**50**
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	**40**
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	0.82 550	9.99 520	**30**
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	**20**
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17
44	9.18 137	82	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	15
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	82	9.18 896	84	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	83	0.81 021	9.99 486	12
49	9.18 547	81	9.19 063	84	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	**10**
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433		9.19 971		0.80 029	9.99 462	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

P P

	92	91	90
1	9.2	9.1	9.0
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

	89	88
1	8.9	8.8
2	17.8	17.6
3	26.7	26.4
4	35.6	35.2
5	44.5	44.0
6	53.4	52.8
7	62.3	61.6
8	71.2	70.4
9	80.1	79.2

	87	86	85
1	8.7	8.6	8.5
2	17.4	17.2	17.0
3	26.1	25.8	25.5
4	34.8	34.4	34.0
5	43.5	43.0	42.5
6	52.2	51.6	51.0
7	60.9	60.2	59.5
8	69.6	68.8	68.0
9	78.3	77.4	76.5

	84	83
1	8.4	8.3
2	16.8	16.6
3	25.2	24.9
4	33.6	33.2
5	42.0	41.5
6	50.4	49.8
7	58.8	58.1
8	67.2	66.4
9	75.6	74.7

	82	81	80
1	8.2	8.1	8.0
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.6	72.0

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	**60**
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	79	9.20 621	80	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51
10	9.20 223	78	9.20 782	80	0.79 218	9.99 442	**50**
11	9.20 302	79	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	78	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	77	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	80	0.78 818	9.99 432	45
16	9.20 691	78	9.21 261	79	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	80	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	77	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	76	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	77	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	76	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	75	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761	76	9.22 361	77	0.77 639	9.99 400	**30**
31	9.21 836	75	9.22 438	78	0.77 562	9.99 398	29
32	9.21 912	76	9.22 516	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	75	9.22 747	77	0.77 253	9.99 390	25
36	9.22 211	74	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	77	0.77 099	9.99 385	23
38	9.22 361	75	9.22 977	76	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	77	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	76	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	77	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	74	9.23 435	76	0.76 565	9.99 370	16
45	9.22 878	73	9.23 510	75	0.76 490	9.99 368	15
46	9.22 952	74	9.23 586	76	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	75	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	76	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	73	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	72	9.24 112	75	0.75 888	9.99 351	7
54	9.23 535	73	9.24 186	74	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	75	0.75 739	9.99 346	5
56	9.23 679	72	9.24 335	74	0.75 665	9.99 344	4
57	9.23 752	73	9.24 410	75	0.75 590	9.99 342	3
58	9.23 823	71	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967	72	9.24 632	74	0.75 368	9.99 335	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

P P			
	82	**81**	**80**
1	8.2	8.1	8.0
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0
	79	**78**	**77**
1	7.9	7.8	7.7
2	15.8	15.6	15.4
3	23.7	23.4	23.1
4	31.6	31.2	30.8
5	39.5	39.0	38.5
6	47.4	46.8	46.2
7	55.3	54.6	53.9
8	63.2	62.4	61.6
9	71.1	70.2	69.3
	76	**75**	**74**
1	7.6	7.5	7.4
2	15.2	15.0	14.8
3	22.8	22.5	22.2
4	30.4	30.0	29.6
5	38.0	37.5	37.0
6	45.6	45.0	44.4
7	53.2	52.5	51.8
8	60.8	60.0	59.2
9	68.4	67.5	66.6
	73	**72**	**71**
1	7.3	7.2	7.1
2	14.6	14.4	14.2
3	21.9	21.6	21.3
4	29.2	28.8	28.4
5	36.5	36.0	35.5
6	43.8	43.2	42.6
7	51.1	50.4	49.7
8	58.4	57.6	56.8
9	65.7	64.8	63.9

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	2	**60**
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	2	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	3	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	2	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	3	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	2	**50**
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	3	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	2	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	3	45
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	2	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	3	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	**40**
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	2	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	3	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	2	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	3	35
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	**30**
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	3	25
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	70	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	2	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	3	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	2	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	3	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	2	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	3	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	15
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	2	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	3	13
48	9.27 273	66	9.28 049	69	0.71 951	9.99 224	2	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	3	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	2	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	3	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	2	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	3	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	2	5
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	3	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	2	2
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	3	1
60	9.28 060		9 28 865		0.71 135	9.99 195	2	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	74	73	72
1	7.4	7.3	7.2
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	70	69
1	7.1	7.0	6.9
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	67	66
1	6.8	6.7	6.6
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	8
1	6.5	0.3
2	13.0	0.6
3	19.5	0.9
4	26.0	1.2
5	32.5	1.5
6	39.0	1.8
7	45.5	2.1
8	52.0	2.4
9	58.5	2.7

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.28 060		9.28 865		0.71 135	9.99 195		**60**
		65		68			3	
1	9.28 125		9.28 933		0.71 067	9.99 192		59
		65		67			2	
2	9.28 190		9.29 000		0.71 000	9.99 190		58
		64		67			3	
3	9.28 254		9.29 067		0.70 933	9.99 187		57
		65		67			2	
4	9.28 319		9.29 134		0.70 866	9.99 185		56
		65		67			3	
5	9.28 384		9.29 201		0.70 799	9.99 182		55
		64		67			2	
6	9.28 448		9.29 268		0.70 732	9.99 180		54
		64		67			3	
7	9.28 512		9.29 335		0.70 665	9.99 177		53
		65		66			2	
8	9.28 577		9.29 402		0.70 598	9.99 175		52
		64		66			3	
9	9.28 641		9.29 468		0.70 532	9.99 172		51
		64		67			2	
10	9.28 705		9.29 535		0.70 465	9.99 170		**50**
		64		66			3	
11	9.28 769		9.29 601		0.70 399	9.99 167		49
		64		67			2	
12	9.28 833		9.29 668		0.70 332	9.99 165		48
		63		66			3	
13	9.28 896		9.29 734		0.70 266	9.99 162		47
		64		66			2	
14	9.28 960		9.29 800		0.70 200	9.99 160		46
		64		66			3	
15	9.29 024		9.29 866		0.70 134	9.99 157		45
		63		66			2	
16	9.29 087		9.29 932		0.70 068	9.99 155		44
		63		66			3	
17	9.29 150		9.29 998		0.70 002	9.99 152		43
		64		66			2	
18	9.29 214		9.30 064		0.69 936	9.99 150		42
		63		66			3	
19	9.29 277		9.30 130		0.69 870	9.99 147		41
		63		65			2	
20	9.29 340		9.30 195		0.69 805	9.99 145		**40**
		63		66			3	
21	9.29 403		9.30 261		0.69 739	9.99 142		39
		63		65			2	
22	9.29 466		9.30 326		0.69 674	9.99 140		38
		63		65			3	
23	9.29 529		9.30 391		0.69 609	9.99 137		37
		62		66			2	
24	9.29 591		9.30 457		0.69 543	9.99 135		36
		63		65			3	
25	9.29 654		9.30 522		0.69 478	9.99 132		35
		62		65			2	
26	9.29 716		9.30 587		0.69 413	9.99 130		34
		63		65			3	
27	9.29 779		9.30 652		0.69 348	9.99 127		33
		62		65			3	
28	9.29 841		9.30 717		0.69 283	9.99 124		32
		62		65			2	
29	9.29 903		9.30 782		0.69 218	9.99 122		31
		63		64			3	
30	9.29 966		9.30 846		0.69 154	9.99 119		**30**
		62		65			2	
31	9.30 028		9.30 911		0.69 089	9.99 117		29
		62		64			3	
32	9.30 090		9.30 975		0.69 025	9.99 114		28
		61		65			2	
33	9.30 151		9.31 040		0.68 960	9.99 112		27
		62		64			3	
34	9.30 213		9.31 104		0.68 896	9.99 109		26
		62		64			3	
35	9.30 275		9.31 168		0.68 832	9.99 106		25
		61		65			2	
36	9.30 336		9.31 233		0.68 767	9.99 104		24
		62		64			3	
37	9.30 398		9.31 297		0.68 703	9.99 101		23
		61		64			2	
38	9.30 459		9.31 361		0.68 639	9.99 099		22
		62		64			3	
39	9.30 521		9.31 425		0.68 575	9.99 096		21
		61		64			3	
40	9.30 582		9.31 489		0.68 511	9.99 093		**20**
		61		63			2	
41	9.30 643		9.31 552		0.68 448	9.99 091		19
		61		64			3	
42	9.30 704		9.31 616		0.68 384	9.99 088		18
		61		63			2	
43	9.30 765		9.31 679		0.68 321	9.99 086		17
		61		64			3	
44	9.30 826		9.31 743		0.68 257	9.99 083		16
		61		63			3	
45	9.30 887		9.31 806		0.68 194	9.99 080		15
		60		64			2	
46	9.30 947		9.31 870		0.68 130	9.99 078		14
		61		63			3	
47	9.31 008		9.31 933		0.68 067	9.99 075		13
		60		63			3	
48	9.31 068		9.31 996		0.68 004	9.99 072		12
		61		63			2	
49	9.31 129		9.32 059		0.67 941	9.99 070		11
		60		63			3	
50	9.31 189		9.32 122		0.67 878	9.99 067		**10**
		61		63			3	
51	9.31 250		9.32 185		0.67 815	9.99 064		9
		60		63			2	
52	9.31 310		9.32 248		0.67 752	9.99 062		8
		60		63			3	
53	9.31 370		9.32 311		0.67 689	9.99 059		7
		60		62			3	
54	9.31 430		9.32 373		0.67 627	9.99 056		6
		60		63			2	
55	9.31 490		9.32 436		0.67 564	9.99 054		5
		59		62			3	
56	9.31 549		9.32 498		0.67 502	9.99 051		4
		60		63			3	
57	9.31 609		9.32 561		0.67 439	9.99 048		3
		60		62			2	
58	9.31 669		9.32 623		0.67 377	9.99 046		2
		59		62			3	
59	9.31 728		9.32 685		0.67 315	9.99 043		1
		60		62			3	
60	9.31 788		9.32 747		0.67 253	9.99 040		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	68	67	66
1	6.8	6.7	6.6
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	64	63
1	6.5	6.4	6.3
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7

	62	61	60
1	6.2	6.1	6.0
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0

	59	3
1	5.9	0.3
2	11.8	0.6
3	17.7	0.9
4	23.6	1.2
5	29.5	1.5
6	35.4	1.8
7	41.3	2.1
8	47.2	2.4
9	53.1	2.7

P P

′	L Sin	d	L Tan	cd	L Cot	L Cos	d	′
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	2	60
1	9.31 847	59	9.32 810	62	0.67 190	9.99 038	2	59
2	9.31 907	60	9.32 872	61	0.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	3	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	2	56
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	3	55
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	3	54
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	2	53
8	9.32 261	59	9.33 242	61	0.66 758	9.99 019	3	52
9	9.32 319	58	9.33 303	62	0.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	3	50
11	9.32 437	59	9.33 426	61	0.66 574	9.99 011	2	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48
13	9.32 553	58	9.33 548	61	0.66 452	9.99 005	3	47
14	9.32 612	59	9.33 609	61	0.66 391	9.99 002	3	46
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	2	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	3	42
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	2	41
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	40
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	3	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	2	37
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	3	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	3	34
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	2	33
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	30
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	3	29
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	2	28
33	9.33 704	57	9.34 755	60	0.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	59	0.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	60	0.65 126	9.98 944	3	25
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	3	23
38	9.33 987	56	9.35 051	59	0.64 949	9.98 936	2	22
39	9.34 043	57	9.35 111	60	0.64 889	9.98 933	3	21
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	3	20
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	3	17
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	2	16
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	15
46	9.34 436	56	9.35 523	58	0.64 477	9.98 913	3	14
47	9.34 491	55	9.35 581	59	0.64 419	9.98 910	3	13
48	9.34 547	56	9.35 640	58	0.64 360	9.98 907	3	12
49	9.34 602	55	9.35 698	59	0.64 302	9.98 904	3	11
50	9.34 658	56	9.35 757	58	0.64 243	9.98 901	3	10
51	9.34 713	55	9.35 815	58	0.64 185	9.98 898	3	9
52	9.34 769	56	9.35 873	58	0.64 127	9.98 896	2	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	5
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	58	0.63 721	9.98 875	3	1
60	9.35 209	55	9.36 336	57	0.63 664	9.98 872	3	0
	L Cos	d	L Cot	cd	L Tan	L Sin	d	′

P P

	63	62	61
1	6.3	6.2	6.1
2	12.6	12.4	12.2
3	18.9	18.6	18.3
4	25.2	24.8	24.4
5	31.5	31.0	30.5
6	37.8	37.2	36.6
7	44.1	43.4	42.7
8	50.4	49.6	48.8
9	56.7	55.8	54.9

	60	59
1	6.0	5.9
2	12.0	11.8
3	18.0	17.7
4	24.0	23.6
5	30.0	29.5
6	36.0	35.4
7	42.0	41.3
8	48.0	47.2
9	54.0	53.1

	58	57
1	5.8	5.7
2	11.6	11.4
3	17.4	17.1
4	23.2	22.8
5	29.0	28.5
6	34.8	34.2
7	40.6	39.9
8	46.4	45.6
9	52.2	51.3

	56	55	3
1	5.6	5.5	0.3
2	11.2	11.0	0.6
3	16.8	16.5	0.9
4	22.4	22.0	1.2
5	28.0	27.5	1.5
6	33.6	33.0	1.8
7	39.2	38.5	2.1
8	44.8	44.0	2.4
9	50.4	49.5	2.7

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	3	60
1	9.35 263	54	9.36 394	58	0.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	3	58
3	9.35 373	55	9.36 509	57	0.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	3	56
5	9.35 481	54	9.36 624	57	0.63 376	9.98 858	3	55
6	9.35 536	55	9.36 681	57	0.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	50
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46
15	9.36 022	54	9.37 193	57	0.62 807	9.98 828	3	45
16	9.36 075	53	9.37 250	56	0.62 750	9.98 825	3	44
17	9.36 129	54	9.37 306	57	0.62 694	9.98 822	3	43
18	9.36 182	53	9.37 363	56	0.62 637	9.98 819	3	42
19	9.36 236	54	9.37 419	57	0.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	40
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39
22	9.36 395	53	9.37 588	56	0.62 412	9.98 807	3	38
23	9.36 449	54	9.37 644	56	0.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	35
26	9.36 608	53	9.37 812	56	0.62 188	9.98 795	3	34
27	9.36 660	52	9.37 868	56	0.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	56	0.62 020	9.98 786	3	31
30	9.36 819	53	9.38 035	55	0.61 965	9.98 783	3	30
31	9.36 871	52	9.38 091	56	0.61 909	9.98 780	3	29
32	9.36 924	53	9.38 147	56	0.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27
34	9.37 028	52	9.38 257	55	0.61 743	9.98 771	3	26
35	9.37 081	53	9.38 313	56	0.61 687	9.98 768	3	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	55	0.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	56	0.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	20
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	3	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	4	18
43	9.37 497	52	9.38 754	55	0.61 246	9.98 743	3	17
44	9.37 549	52	9.38 808	54	0.61 192	9.98 740	3	16
45	9.37 600	51	9.38 863	55	0.61 137	9.98 737	3	15
46	9.37 652	52	9.38 918	55	0.61 082	9.98 734	3	14
47	9.37 703	51	9.38 972	54	0.61 028	9.98 731	3	13
48	9.37 755	52	9.39 027	55	0.60 973	9.98 728	3	12
49	9.37 806	51	9.39 082	55	0.60 918	9.98 725	3	11
50	9.37 858	52	9.39 136	54	0.60 864	9.98 722	3	10
51	9.37 909	51	9.39 190	54	0.60 810	9.98 719	3	9
52	9.37 960	51	9.39 245	55	0.60 755	9.98 715	4	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	5
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	3	1
60	9.38 368	51	9.39 677	54	0.60 323	9.98 690	4	0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	58	57	56
1	5.8	5.7	5.6
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	54	53
1	5.5	5.4	5.3
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
1	5.2	5.1
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25 5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		P P
0	9.38 368		9.39 677		0.60 323	9.98 690		60	
		50		54			3		
1	9.38 418		9.39 731		0.60 269	9.98 687		59	
		51		54			3		
2	9.38 469		9.39 785		0.60 215	9.98 684		58	
		50		53			3		
3	9.38 519		9.39 838		0.60 162	9.98 681		57	
		51		54			3		
4	9.38 570		9.39 892		0.60 108	9.98 678		56	
		50		53			3		
5	9.38 620		9.39 945		0.60 055	9.98 675		55	
		50		54			4		
6	9.38 670		9.39 999		0.60 001	9.98 671		54	
		51		53			4		
7	9.38 721		9.40 052		0.59 948	9.98 668		53	
		50		54			3		
8	9.38 771		9.40 106		0.59 894	9.98 665		52	
		50		53			3		
9	9.38 821		9.40 159		0.59 841	9.98 662		51	
		50		53			3		
10	9.38 871		9.40 212		0.59 788	9.98 659		50	
		50		54			3		
11	9.38 921		9.40 266		0.59 734	9.98 656		49	
		50		53			4		
12	9.38 971		9.40 319		0.59 681	9.98 652		48	
		50		53			3		
13	9.39 021		9.40 372		0.59 628	9.98 649		47	
		50		53			3		
14	9.39 071		9.40 425		0.59 575	9.98 646		46	
		50		53			3		
15	9.39 121		9.40 478		0.59 522	9.98 643		45	
		49		53			3		
16	9.39 170		9.40 531		0.59 469	9.98 640		44	
		50		53			4		
17	9.39 220		9.40 584		0.59 416	9.98 636		43	
		50		52			3		
18	9.39 270		9.40 636		0.59 364	9.98 633		42	
		49		53			3		
19	9.39 319		9.40 689		0.59 311	9.98 630		41	
		50		53			3		
20	9.39 369		9.40 742		0.59 258	9.98 627		40	
		49		53			4		
21	9.39 418		9.40 795		0.59 205	9.98 623		39	
		49		52			3		
22	9.39 467		9.40 847		0.59 153	9.98 620		38	
		49		53			3		
23	9.39 517		9.40 900		0.59 100	9.98 617		37	
		49		52			3		
24	9.39 566		9.40 952		0.59 048	9.98 614		36	
		49		53			4		
25	9.39 615		9.41 005		0.58 995	9.98 610		35	
		49		52			3		
26	9.39 664		9.41 057		0.58 943	9.98 607		34	
		49		52			3		
27	9.39 713		9.41 109		0.58 891	9.98 604		33	
		49		52			3		
28	9.39 762		9.41 161		0.58 839	9.98 601		32	
		49		53			4		
29	9.39 811		9.41 214		0.58 786	9.98 597		31	
		49		52			3		
30	9.39 860		9.41 266		0.58 734	9.98 594		30	
		49		52			3		
31	9.39 909		9.41 318		0.58 682	9.98 591		29	
		49		52			3		
32	9.39 958		9.41 370		0.58 630	9.98 588		28	
		48		52			4		
33	9.40 006		9.41 422		0.58 578	9.98 584		27	
		49		52			3		
34	9.40 055		9.41 474		0.58 526	9.98 581		26	
		48		52			3		
35	9.40 103		9.41 526		0.58 474	9.98 578		25	
		49		52			4		
36	9.40 152		9.41 578		0.58 422	9.98 574		24	
		48		51			3		
37	9.40 200		9.41 629		0.58 371	9.98 571		23	
		49		52			3		
38	9.40 249		9.41 681		0.58 319	9.98 568		22	
		48		52			3		
39	9.40 297		9.41 733		0.58 267	9.98 565		21	
		49		51			4		
40	9.40 346		9.41 784		0.58 216	9.98 561		20	
		48		52			3		
41	9.40 394		9.41 836		0.58 164	9.98 558		19	
		48		51			3		
42	9.40 442		9.41 887		0.58 113	9.98 555		18	
		48		52			4		
43	9.40 490		9.41 939		0.58 061	9.98 551		17	
		48		51			3		
44	9.40 538		9.41 990		0.58 010	9.98 548		16	
		48		51			3		
45	9.40 586		9.42 041		0.57 959	9.98 545		15	
		48		52			4		
46	9.40 634		9.42 093		0.57 907	9.98 541		14	
		48		51			3		
47	9.40 682		9.42 144		0.57 856	9.98 538		13	
		48		51			3		
48	9.40 730		9.42 195		0.57 805	9.98 535		12	
		48		51			4		
49	9.40 778		9.42 246		0.57 754	9.98 531		11	
		47		51			3		
50	9.40 825		9.42 297		0.57 703	9.98 528		10	
		48		51			3		
51	9.40 873		9.42 348		0.57 652	9.98 525		9	
		48		51			4		
52	9.40 921		9.42 399		0.57 601	9.98 521		8	
		47		51			3		
53	9.40 968		9.42 450		0.57 550	9.98 518		7	
		48		51			3		
54	9.41 016		9.42 501		0.57 499	9.98 515		6	
		47		51			4		
55	9.41 063		9.42 552		0.57 448	9.98 511		5	
		48		51			3		
56	9.41 111		9.42 603		0.57 397	9.98 508		4	
		47		50			3		
57	9.41 158		9.42 653		0.57 347	9.98 505		3	
		47		51			4		
58	9.41 205		9.42 704		0.57 296	9.98 501		2	
		47		51			4		
59	9.41 252		9.42 755		0.57 245	9.98 498		1	
		48		50			4		
60	9.41 300		9.42 805		0.57 195	9.98 494		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	P P

	54	53
1	5.4	5.3
2	10.8	10.6
3	16.2	15.9
4	21.6	21.2
5	27.0	26.5
6	32.4	31.8
7	37.8	37.1
8	43.2	42.4
9	48.6	47.7

	52	51	50
1	5.2	5.1	5.0
2	10.4	10.2	10.0
3	15.6	15.3	15.0
4	20.8	20.4	20.0
5	26.0	25.5	25.0
6	31.2	30.6	30.0
7	36.4	35.7	35.0
8	41.6	40.8	40.0
9	46.8	45.9	45.0

	49	48	47
1	4.9	4.8	4.7
2	9.8	9.6	9.4
3	14.7	14.4	14.1
4	19.6	19.2	18.8
5	24.5	24.0	23.5
6	29.4	28.8	28.2
7	34.3	33.6	32.9
8	39.2	38.4	37.6
9	44.1	43.2	42.3

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

′	L Sin	d	L Tan	cd	L Cot	L Cos	d	′
0	9.41 300		9.42 805		0.57 195	9.98 494		60
		47		51			3	
1	9.41 347		9.42 856		0.57 144	9.98 491		59
		47		50			3	
2	9.41 394		9.42 906		0.57 094	9.98 488		58
		47		51			4	
3	9.41 441		9.42 957		0.57 043	9.98 484		57
		47		50			3	
4	9.41 488		9.43 007		0.56 993	9.98 481		56
		47		50			4	
5	9.41 535		9.43 057		0.56 943	9.98 477		55
		47		51			3	
6	9.41 582		9.43 108		0.56 892	9.98 474		54
		46		50			3	
7	9.41 628		9.43 158		0.56 842	9.98 471		53
		47		50			4	
8	9.41 675		9.43 208		0.56 792	9.98 467		52
		47		50			3	
9	9.41 722		9.43 258		0.56 742	9.98 464		51
		46		50			4	
10	9.41 768		9.43 308		0.56 692	9.98 460		50
		47		50			3	
11	9.41 815		9.43 358		0.56 642	9.98 457		49
		46		50			4	
12	9.41 861		9.43 408		0.56 592	9.98 453		48
		47		50			4	
13	9.41 908		9.43 458		0.56 542	9.98 450		47
		46		50			3	
14	9.41 954		9.43 508		0.56 492	9.98 447		46
		47		50			4	
15	9.42 001		9.43 558		0.56 442	9.98 443		45
		46		49			4	
16	9.42 047		9.43 607		0.56 393	9.98 440		44
		46		50			4	
17	9.42 093		9.43 657		0.56 343	9.98 436		43
		47		50			4	
18	9.42 140		9.43 707		0.56 293	9.98 433		42
		46		49			4	
19	9.42 186		9.43 756		0.56 244	9.98 429		41
		46		50			3	
20	9.42 232		9.43 806		0.56 194	9.98 426		40
		46		49			4	
21	9.42 278		9.43 855		0.56 145	9.98 422		39
		46		50			3	
22	9.42 324		9.43 905		0.56 095	9.98 419		38
		46		49			4	
23	9.42 370		9.43 954		0.56 046	9.98 415		37
		46		50			3	
24	9.42 416		9.44 004		0.55 996	9.98 412		36
		45		49			3	
25	9.42 461		9.44 053		0.55 947	9.98 409		35
		46		49			4	
26	9.42 507		9.44 102		0.55 898	9.98 405		34
		46		49			3	
27	9.42 553		9.44 151		0.55 849	9.98 402		33
		46		50			4	
28	9.42 599		9.44 201		0.55 799	9.98 398		32
		45		49			4	
29	9.42 644		9.44 250		0.55 750	9.98 395		31
		46		49			4	
30	9.42 690		9.44 299		0.55 701	9.98 391		30
		45		49			3	
31	9.42 735		9.44 348		0.55 652	9.98 388		29
		46		49			4	
32	9.42 781		9.44 397		0.55 603	9.98 384		28
		46		49			4	
33	9.42 826		9.44 446		0.55 554	9.98 381		27
		46		49			4	
34	9.42 872		9.44 495		0.55 505	9.98 377		26
		45		49			4	
35	9.42 917		9.44 544		0.55 456	9.98 373		25
		45		48			3	
36	9.42 962		9.44 592		0.55 408	9.98 370		24
		46		49			4	
37	9.43 008		9.44 641		0.55 359	9.98 366		23
		46		49			3	
38	9.43 053		9.44 690		0.55 310	9.98 363		22
		45		48			3	
39	9.43 098		9.44 738		0.55 262	9.98 359		21
		45		49			4	
40	9.43 143		9.44 787		0.55 213	9.98 356		20
		45		49			4	
41	9.43 188		9.44 836		0.55 164	9.98 352		19
		45		48			3	
42	9.43 233		9.44 884		0.55 116	9.98 349		18
		45		49			4	
43	9.43 278		9.44 933		0.55 067	9.98 345		17
		45		48			3	
44	9.43 323		9.44 981		0.55 019	9.98 342		16
		44		48			4	
45	9.43 367		9.45 029		0.54 971	9.98 338		15
		45		49			4	
46	9.43 412		9.45 078		0.54 922	9.98 334		14
		45		48			3	
47	9.43 457		9.45 126		0.54 874	9.98 331		13
		45		48			4	
48	9.43 502		9.45 174		0.54 826	9.98 327		12
		44		48			3	
49	9.43 546		9.45 222		0.54 778	9.98 324		11
		45		49			4	
50	9.43 591		9.45 271		0.54 729	9.98 320		10
		44		48			3	
51	9.43 635		9.45 319		0.54 681	9.98 317		9
		45		48			4	
52	9.43 680		9.45 367		0.54 633	9.98 313		8
		45		48			4	
53	9.43 724		9.45 415		0.54 585	9.98 309		7
		45		48			4	
54	9.43 769		9.45 463		0.54 537	9.98 306		6
		44		48			4	
55	9.43 813		9.45 511		0.54 489	9.98 302		5
		44		48			3	
56	9.43 857		9.45 559		0.54 441	9.98 299		4
		44		47			4	
57	9.43 901		9.45 606		0.54 394	9.98 295		3
		45		48			4	
58	9.43 946		9.45 654		0.54 346	9.98 291		2
		44		48			3	
59	9.43 990		9.45 702		0.54 298	9.98 288		1
		44		48			4	
60	9.44 034		9.45 750		0.54 250	9.98 284		0
	L Cos	d	L Cot	cd	L Tan	L Sin	d	′

P P

	51	50	49
1	5.1	5.0	4.9
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47	46
1	4.8	4.7	4.6
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44
1	4.5	4.4
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9 44 034		9.45 750		0.54 205	9.98 284		60
		44		47			3	
1	9.44 078		9.45 797		0.54 203	9.98 281		59
2	9.44 122	44	9.45 845	48	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	47	0.54 108	9.98 273	4	57
		44		48			3	
4	9.44 210		9.45 940		0.54 060	9.98 270		56
5	9.44 253	43	9.45 987	47	0.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	48	0.53 965	9.98 262	4	54
		44		47			3	
7	9.44 341		9.46 082		0.53 918	9.98 259		53
8	9.44 385	44	9.46 130	48	0.53 870	9.98 255	4	52
9	9.44 428	43	9.46 177	47	0.53 823	9.98 251	4	51
		44		47			3	
10	9.44 472		9.46 224		0.53 776	9.98 248		50
		44		47			4	
11	9.44 516		9.46 271		0.53 729	9.98 244		49
12	9.44 559	43	9.46 319	48	0.53 681	9.98 240	4	48
13	9.44 602	43	9.46 366	47	0.53 634	9.98 237	3	47
		44		47			4	
14	9.44 646		9.46 413		0.53 587	9.98 233		46
15	9.44 689	43	9.46 460	47	0.53 540	9.98 229	4	45
16	9.44 733	44	9.46 507	47	0.53 493	9.98 226	3	44
		43		47			4	
17	9.44 776		9.46 554		0.53 446	9.98 222		43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	4	42
19	9.44 862	43	9.46 648	47	0.53 352	9.98 215	3	41
		43		46			4	
20	9.44 905		9.46 694		0.53 306	9.98 211		40
		43		47			4	
21	9.44 948		9.46 741		0.53 259	9.98 207		39
22	9.44 992	44	9.46 788	47	0.53 212	9.98 204	3	38
23	9.45 035	43	9.46 835	47	0.53 165	9.98 200	4	37
		42		46			4	
24	9.45 077		9.46 881		0.53 119	9.98 196		36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	4	35
26	9.45 163	43	9.46 975	47	0.53 025	9.98 189	3	34
		43		46			4	
27	9.45 206		9.47 021		0.52 979	9.98 185		33
28	9.45 249	43	9.47 068	47	0.52 932	9.98 181	4	32
29	9.45 292	43	9.47 114	46	0.52 886	9.98 177	4	31
		42		46			3	
30	9.45 334		9.47 160		0.52 840	9.98 174		30
		43		47			4	
31	9.45 377		9.47 207		0.52 793	9.98 170		29
32	9.45 419	42	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	43	9.47 299	46	0.52 701	9.98 162	4	27
		42		47			4	
34	9.45 504		9.47 346		0.52 654	9.98 159		26
35	9.45 547	43	9.47 392	46	0.52 608	9.98 155	3	25
36	9.45 589	42	9.47 438	46	0.52 562	9.98 151	4	24
		43		46			4	
37	9.45 632		9.47 484		0.52 516	9.98 147		23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	3	21
		42		46			4	
40	9.45 758		9.47 622		0.52 378	9.98 136		20
		43		46			4	
41	9 45 801		9.47 668		0.52 332	9.98 132		19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	3	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
		42		46			4	
44	9.45 927		9.47 806		0.52 194	9.98 121		16
45	9.45 969	42	9.47 852	46	0.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	45	0.52 103	9.98 113	4	14
		42		46			4	
47	9.46 053		9.47 943		0.52 057	9.98 110		13
48	9.46 095	42	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	41	9.48 035	46	0.51 965	9.98 102	4	11
		42		45			4	
50	9.46 178		9.48 080		0.51 920	9.98 098		10
		42		46			4	
51	9.46 220		9.48 126		0.51 874	9.98 094		9
52	9.46 262	42	9.48 171	45	0.51 829	9.98 090	4	8
53	9.46 303	41	9.48 217	46	0.51 783	9.98 087	3	7
		42		45			4	
54	9.46 345		9.48 262		0.51 738	9.98 083		6
55	9.46 386	41	9.48 307	45	0.51 693	9.98 079	4	5
56	9.46 428	42	9.48 353	46	0.51 647	9.98 075	4	4
		41		45			4	
57	9.46 469		9.48 398		0.51 602	9.98 071		3
58	9.46 511	42	9.48 443	45	0.51 557	9.98 067	4	2
59	9.46 552	41	9.48 489	46	0.51 511	9.98 063	4	1
		42		45			3	
60	9.46 594		9.48 534		0.51 466	9.98 060		0
′	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	48	47	46
1	4.8	4.7	4.6
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44	43
1	4.5	4.4	4.3
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
1	4.2	4.1
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	**60**
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	41	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	42	9.48 759	45	0.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	4	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	3	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	**50**
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	45	0.50 837	9.98 005	4	46
15	9.47 209	41	9.49 207	44	0.50 793	9.98 001	4	45
16	9.47 249	40	9.49 252	45	0.50 748	9.97 997	4	44
17	9.47 290	41	9.49 296	44	0.50 704	9.97 993	4	43
18	9.47 330	40	9.49 341	45	0.50 659	9.97 989	4	42
19	9.47 371	41	9.49 385	44	0.50 615	9.97 986	3	41
20	9.47 411	40	9.49 430	45	0.50 570	9.97 982	4	**40**
21	9.47 452	41	9.49 474	44	0.50 526	9.97 978	4	39
22	9.47 492	40	9.49 519	45	0.50 481	9.97 974	4	38
23	9.47 533	41	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	44	0.50 393	9.97 966	4	36
25	9.47 613	40	9.49 652	45	0.50 348	9.97 962	4	35
26	9.47 654	41	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	**30**
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	40	9.50 180	44	0.49 820	9.97 914	4	23
38	9.48 133	39	9.50 223	43	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	40	9.50 311	44	0.49 689	9.97 902	4	**20**
41	9.48 252	39	9.50 355	44	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	43	0.49 602	9.97 894	4	18
43	9.48 332	40	9.50 442	44	0.49 558	9.97 890	4	17
44	9.48 371	39	9.50 485	43	0.49 515	9.97 886	4	16
45	9.48 411	40	9.50 529	44	0.49 471	9.97 882	4	15
46	9.48 450	39	9.50 572	43	0.49 428	9.97 878	4	14
47	9.48 490	40	9.50 616	44	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	43	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	44	0.49 297	9.97 866	4	11
50	9.48 607	39	9.50 746	43	0.49 254	9.97 861	5	**10**
51	9.48 647	40	9.50 789	43	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	44	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	43	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	44	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	45	44	43
1	4.5	4.4	4.3
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
1	4.2	4.1	4.0
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5	4	3
1	3.9	0.5	0.4	0.3
2	7.8	1.0	0.8	0.6
3	11.7	1.5	1.2	0.9
4	15.6	2.0	1.6	1.2
5	19.5	2.5	2.0	1.5
6	23.4	3.0	2.4	1.8
7	27.3	3.5	2.8	2.1
8	31.2	4.0	3.2	2.4
9	35.1	4.5	3.6	2.7

P P

′	L Sin	d	L Tan	cd	L Cot	L Cos	d		P P
0	9.48 998		9.51 178		0.48 822	9.97 821		**60**	
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	4	59	
2	9.49 076	39	9.51 264	43	0.48 736	9.97 812	5	58	
3	9.49 115	39	9.51 306	42	0.48 694	9.97 808	4	57	
4	9.49 153	38	9.51 349	43	0.48 651	9.97 804	4	56	
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55	
6	9.49 231	39	9.51 435	43	0.48 565	9.97 796	4	54	
7	9.49 269	38	9.51 478	43	0.48 522	9.97 792	4	53	
8	9.49 308	39	9.51 520	42	0.48 480	9.97 788	4	52	
9	9.49 347	39	9.51 563	43	0.48 437	9.97 784	4	51	
10	9.49 385	38	9.51 606	43	0.48 394	9.97 779	5	**50**	
11	9.49 424	39	9.51 648	42	0.48 352	9.97 775	4	49	
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48	
13	9.49 500	38	9.51 734	43	0.48 266	9.97 767	4	47	
14	9.49 539	39	9.51 776	42	0.48 224	9.97 763	4	46	
15	9.49 577	38	9.51 819	43	0.48 181	9.97 759	4	45	
16	9.49 615	38	9.51 861	42	0.48 139	9.97 754	5	44	
17	9.49 654	39	9.51 903	42	0.48 097	9.97 750	4	43	
18	9.49 692	38	9.51 946	43	0.48 054	9.97 746	4	42	
19	9.49 730	38	9.51 988	42	0.48 012	9.97 742	4	41	
20	9.49 768	38	9.52 031	43	0.47 969	9.97 738	4	**40**	
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	4	39	
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	5	38	
23	9.49 882	38	9.52 157	42	0.47 843	9.97 725	4	37	
24	9.49 920	38	9.52 200	43	0.47 800	9.97 721	4	36	
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35	
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	4	34	
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	5	33	
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32	
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31	
30	9.50 148	38	9.52 452	42	0.47 548	9.97 696	4	**30**	
31	9.50 185	37	9.52 494	42	0.47 506	9.97 691	5	29	
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28	
33	9.50 261	38	9.52 578	42	0.47 422	9.97 683	4	27	
34	9.50 298	37	9.52 620	42	0.47 380	9.97 679	4	26	
35	9.50 336	38	9.52 661	41	0.47 339	9.97 674	5	25	
36	9.50 374	38	9.52 703	42	0.47 297	9.97 670	4	24	
37	9.50 411	37	9.52 745	42	0.47 255	9.97 666	4	23	
38	9.50 449	38	9.52 787	42	0.47 213	9.97 662	4	22	
39	9.50 486	37	9.52 829	42	0.47 171	9.97 657	5	21	
40	9.50 523	37	9.52 870	41	0.47 130	9.97 653	4	**20**	
41	9.50 561	38	9.52 912	42	0.47 088	9.97 649	4	19	
42	9.50 598	37	9.52 953	41	0.47 047	9.97 645	4	18	
43	9.50 635	37	9.52 995	42	0.47 005	9.97 640	5	17	
44	9.50 673	38	9.53 037	42	0.46 963	9.97 636	4	16	
45	9.50 710	37	9.53 078	41	0.46 922	9.97 632	4	15	
46	9.50 747	37	9.53 120	42	0.46 880	9.97 628	4	14	
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	5	13	
48	9.50 821	37	9.53 202	41	0.46 798	9.97 619	4	12	
49	9.50 858	37	9.53 244	42	0.46 756	9.97 615	4	11	
50	9.50 896	38	9.53 285	41	0.46 715	9.97 610	5	**10**	
51	9.50 933	37	9.53 327	42	0.46 673	9.97 606	4	9	
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	4	8	
53	9.51 007	37	9.53 409	41	0.46 591	9.97 597	5	7	
54	9.51 043	36	9.53 450	41	0.46 550	9.97 593	4	6	
55	9.51 080	37	9.53 492	42	0.46 508	9.97 589	4	5	
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	5	4	
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3	
58	9.51 191	37	9.53 615	41	0.46 385	9.97 576	4	2	
59	9.51 227	36	9.53 656	41	0.46 344	9.97 571	5	1	
60	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	**0**	
′	L Cos	d	L Cot	cd	L Tan	L Sin	d	′	P P

	43	42	41
1	4.3	4.2	4.1
2	8.6	8.4	8.2
3	12.9	12.6	12.3
4	17.2	16.8	16.4
5	21.5	21.0	20.5
6	25.8	25.2	24.6
7	30.1	29.4	28.7
8	34.4	33.6	32.8
9	38.7	37.8	36.9

	39	38	37
1	3.9	3.8	3.7
2	7.8	7.6	7.4
3	11.7	11.4	11.1
4	15.6	15.2	14.8
5	19.5	19.0	18.5
6	23.4	22.8	22.2
7	27.3	26.6	25.9
8	31.2	30.4	29.6
9	35.1	34.2	33.3

	36	5	4
1	3.6	0.5	0.4
2	7.2	1.0	0.8
3	10.8	1.5	1.2
4	14.4	2.0	1.6
5	18.0	2.5	2.0
6	21.6	3.0	2.4
7	25.2	3.5	2.8
8	28.8	4.0	3.2
9	32.4	4.5	3.6

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.51 264		9.53 697		0.46 303	9.97 567		**60**
		37		41			4	
1	9.51 301		9.53 738		0.46 262	9.97 563		59
		37		41			5	
2	9.51 338		9.53 779		0.46 221	9.97 558		58
		36		41			4	
3	9.51 374		9.53 820		0.46 180	9.97 554		57
		37		41			4	
4	9.51 411		9.53 861		0.46 139	9.97 550		56
		36		41			5	
5	9.51 447		9.53 902		0.46 098	9.97 545		55
		37		41			5	
6	9.51 484		9.53 943		0.46 057	9.97 541		54
		36		41			5	
7	9.51 520		9.53 984		0.46 016	9.97 536		53
		37		41			4	
8	9.51 557		9.54 025		0.45 975	9.97 532		52
		36		40			4	
9	9.51 593		9.54 065		0.45 935	9.97 528		51
		36		41			5	
10	9.51 629		9.54 106		0.45 894	9.97 523		**50**
		37		41			4	
11	9.51 666		9.54 147		0.45 853	9.97 519		49
		36		40			4	
12	9.51 702		9.54 187		0.45 813	9.97 515		48
		36		41			5	
13	9.51 738		9.54 228		0.45 772	9.97 510		47
		36		41			4	
14	9.51 774		9.54 269		0.45 731	9.97 506		46
		37		40			5	
15	9.51 811		9.54 309		0.45 691	9.97 501		45
		36		41			5	
16	9.51 847		9.54 350		0.45 650	9.97 497		44
		36		40			4	
17	9.51 883		9.54 390		0.45 610	9.97 492		43
		36		41			4	
18	9.51 919		9.54 431		0.45 569	9.97 488		42
		36		40			4	
19	9.51 955		9.54 471		0.45 529	9.97 484		41
		36		41			5	
20	9.51 991		9.54 512		0.45 488	9.97 479		**40**
		36		40			4	
21	9.52 027		9.54 552		0.45 448	9.97 475		39
		36		41			5	
22	9.52 063		9.54 593		0.45 407	9.97 470		38
		36		40			4	
23	9.52 099		9.54 633		0.45 367	9.97 466		37
		36		40			5	
24	9.52 135		9.54 673		0.45 327	9.97 461		36
		36		41			4	
25	9.52 171		9.54 714		0.45 286	9.97 457		35
		36		40			4	
26	9.52 207		9.54 754		0.45 246	9.97 453		34
		35		40			5	
27	9.52 242		9.54 794		0.45 206	9.97 448		33
		36		41			4	
28	9.52 278		9.54 835		0.45 165	9.97 444		32
		36		40			5	
29	9.52 314		9.54 875		0.45 125	9.97 439		31
		36		40			4	
30	9.52 350		9.54 915		0.45 085	9.97 435		**30**
		35		40			5	
31	9.52 385		9.54 955		0.45 045	9.97 430		29
		36		40			4	
32	9.52 421		9.54 995		0.45 005	9.97 426		28
		35		40			5	
33	9.52 456		9.55 035		0.44 965	9.97 421		27
		36		40			4	
34	9.52 492		9.55 075		0.44 925	9.97 417		26
		35		40			5	
35	9.52 527		9.55 115		0.44 885	9.97 412		25
		36		40			4	
36	9.52 563		9.55 155		0.44 845	9.97 408		24
		35		40			5	
37	9.52 598		9.55 195		0.44 805	9.97 403		23
		36		40			4	
38	9.52 634		9.55 235		0.44 765	9.97 399		22
		35		40			5	
39	9.52 669		9.55 275		0.44 725	9.97 394		21
		36		40			4	
40	9.52 705		9.55 315		0.44 685	9.97 390		**20**
		35		40			5	
41	9.52 740		9.55 355		0.44 645	9.97 385		19
		35		40			4	
42	9.52 775		9.55 395		0.44 605	9.97 381		18
		36		39			5	
43	9.52 811		9.55 434		0.44 566	9.97 376		17
		35		40			4	
44	9.52 846		9.55 474		0.44 526	9.97 372		16
		35		40			5	
45	9.52 881		9.55 514		0.44 486	9.97 367		15
		35		40			4	
46	9.52 916		9.55 554		0.44 446	9.97 363		14
		35		39			5	
47	9.52 951		9.55 593		0.44 407	9.97 358		13
		35		40			5	
48	9.52 986		9.55 633		0.44 367	9.97 353		12
		35		40			4	
49	9.53 021		9.55 673		0.44 327	9.97 349		11
		35		39			5	
50	9.53 056		9.55 712		0.44 288	9.97 344		**10**
		36		40			4	
51	9.53 092		9.55 752		0.44 248	9.97 340		9
		34		39			5	
52	9.53 126		9.55 791		0.44 209	9.97 335		8
		35		40			4	
53	9.53 161		9.55 831		0.44 169	9.97 331		7
		35		39			5	
54	9.53 196		9.55 870		0.44 130	9.97 326		6
		35		40			4	
55	9.53 231		9.55 910		0.44 090	9.97 322		5
		35		39			5	
56	9.53 266		9.55 949		0.44 051	9.97 317		4
		35		40			5	
57	9.53 301		9.55 989		0.44 011	9.97 312		3
		35		39			4	
58	9.53 336		9.56 028		0.43 972	9.97 308		2
		34		39			5	
59	9.53 370		9.56 067		0.43 933	9.97 303		1
		35		40			4	
60	9.53 405		9.56 107		0.43 893	9.97 299		**0**
′	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P		
	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

P P		
	37	**36**	**35**
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

P P		
	34	**5**	**4**
1	3.4	0.5	0.4
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	**60**
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	35	9.56 185	39	0.43 815	9.97 289	5	58
3	9.53 509	34	9.56 224	39	0.43 776	9.97 285	4	57
4	9.53 544	35	9.56 264	40	0.43 736	9.97 280	5	56
5	9.53 578	34	9.56 303	39	0.43 697	9.97 276	4	55
6	9.53 613	35	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53
8	9.53 682	35	9.56 420	39	0.43 580	9.97 262	4	52
9	9.53 716	34	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	35	9.56 498	39	0.43 502	9.97 252	5	**50**
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	4	49
12	9.53 819	34	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	35	9.56 615	39	0.43 385	9.97 238	5	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	4	46
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	5	45
16	9.53 957	35	9.56 732	39	0.43 268	9.97 224	5	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	4	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	39	0.43 151	9.97 210	5	41
20	9.54 093	34	9.56 887	38	0.43 113	9.97 206	4	**40**
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	5	38
23	9.54 195	34	9.57 004	39	0.42 996	9.97 192	4	37
24	9.54 229	34	9.57 042	38	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	5	35
26	9.54 297	34	9.57 120	39	0.42 880	9.97 178	4	34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	39	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	38	0.42 765	9.97 163	5	31
30	9.54 433	33	9.57 274	39	0.42 726	9.97 159	4	**30**
31	9.54 466	34	9.57 312	38	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	39	0.42 649	9.97 149	5	28
33	9.54 534	33	9.57 389	38	0.42 611	9.97 145	4	27
34	9.54 567	34	9.57 428	39	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	25
36	9.54 635	33	9.57 504	38	0.42 496	9.97 130	5	24
37	9.54 668	34	9.57 543	39	0.42 457	9.97 126	4	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	38	0.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	39	0.42 342	9.97 111	5	**20**
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	4	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	38	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	39	0.42 151	9.97 087	5	15
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	4	14
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	**10**
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	5	5
56	9.55 301	33	9.58 267	38	0.41 733	9.97 035	4	4
57	9.55 334	33	9.58 304	37	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433	33	9.58 418	38	0.41 582	9.97 015	5	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P			
	40	**39**	**38**
1	4.0	3.9	3.8
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

P P			
	37	**35**	**34**
1	3.7	3.5	3.4
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

P P			
	33	**5**	**4**
1	3.3	0.5	0.4
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.55 433		9.58 418		0.41 582	9.97 015		**60**
		33		37			5	
1	9.55 466		9.58 455		0.41 545	9.97 010		59
		33		38			5	
2	9.55 499		9.58 493		0.41 507	9.97 005		58
		33		38			4	
3	9.55 532		9.58 531		0.41 469	9.97 001		57
		32		38			5	
4	9.55 564		9.58 569		0.41 431	9.96 996		56
		33		37			5	
5	9.55 597		9.58 606		0.41 394	9.96 991		55
		33		38			5	
6	9.55 630		9.58 644		0.41 356	9.96 986		54
		33		37			5	
7	9.55 663		9.58 681		0.41 319	9.96 981		53
		32		38			5	
8	9.55 695		9.58 719		0.41 281	9.96 976		52
		33		38			5	
9	9.55 728		9.58 757		0.41 243	9.96 971		51
		33		37			5	
10	9.55 761		9.58 794		0.41 206	9.96 966		**50**
		32		38			4	
11	9.55 793		9.58 832		0.41 168	9.96 962		49
		33		37			5	
12	9.55 826		9.58 869		0.41 131	9.96 957		48
		32		38			5	
13	9.55 858		9.58 907		0.41 093	9.96 952		47
		33		37			5	
14	9.55 891		9.58 944		0.41 056	9.96 947		46
		32		37			5	
15	9.55 923		9.58 981		0.41 019	9.96 942		45
		33		38			5	
16	9.55 956		9.59 019		0.40 981	9.96 937		44
		32		37			5	
17	9.55 988		9.59 056		0.40 944	9.96 932		43
		33		38			5	
18	9.56 021		9.59 094		0.40 906	9.96 927		42
		32		37			5	
19	9.56 053		9.59 131		0.40 869	9.96 922		41
		32		37			5	
20	9.56 085		9.59 168		0.40 832	9.96 917		**40**
		33		37			5	
21	9.56 118		9.59 205		0.40 795	9.96 912		39
		32		38			5	
22	9.56 150		9.59 243		0.40 757	9.96 907		38
		32		37			4	
23	9.56 182		9.59 280		0.40 720	9.96 903		37
		33		37			5	
24	9.56 215		9.59 317		0.40 683	9.96 898		36
		32		37			5	
25	9.56 247		9.59 354		0.40 646	9.96 893		35
		32		37			5	
26	9.56 279		9.59 391		0.40 609	9.96 888		34
		32		38			5	
27	9.56 311		9.59 429		0.40 571	9.96 883		33
		32		37			5	
28	9.56 343		9.59 466		0.40 534	9.96 878		32
		32		37			5	
29	9.56 375		9.59 503		0.40 497	9.96 873		31
		33		37			5	
30	9.56 408		9.59 540		0.40 460	9.96 868		**30**
		32		37			5	
31	9.56 440		9.59 577		0.40 423	9.96 863		29
		32		37			5	
32	9.56 472		9.59 614		0.40 386	9.96 858		28
		32		37			5	
33	9.56 504		9.59 651		0.40 349	9.96 853		27
		32		37			5	
34	9.56 536		9.59 688		0.40 312	9.96 848		26
		32		37			5	
35	9.56 568		9.59 725		0.40 275	9.96 843		25
		31		37			5	
36	9.56 599		9.59 762		0.40 238	9.96 838		24
		32		37			5	
37	9.56 631		9.59 799		0.40 201	9.96 833		23
		32		36			5	
38	9.56 663		9.59 835		0.40 165	9.96 828		22
		32		37			5	
39	9.56 695		9.59 872		0.40 128	9.96 823		21
		32		37			5	
40	9.56 727		9.59 909		0.40 091	9.96 818		**20**
		32		37			5	
41	9.56 759		9.59 946		0.40 054	9.96 813		19
		31		37			5	
42	9.56 790		9.59 983		0.40 017	9.96 808		18
		32		36			5	
43	9.56 822		9.60 019		0.39 981	9.96 803		17
		32		37			5	
44	9.56 854		9.60 056		0.39 944	9.96 798		16
		32		37			5	
45	9.56 886		9.60 093		0.39 907	9.96 793		15
		31		37			5	
46	9.56 917		9.60 130		0.39 870	9.96 788		14
		32		36			5	
47	9.56 949		9.60 166		0.39 834	9.96 783		13
		31		37			5	
48	9.56 980		9.60 203		0.39 797	9.96 778		12
		32		37			6	
49	9.57 012		9.60 240		0.39 760	9.96 772		11
		31		36			5	
50	9.57 044		9.60 276		0.39 724	9.96 767		**10**
		31		37			5	
51	9.57 075		9.60 313		0.39 687	9.96 762		9
		32		36			5	
52	9.57 107		9.60 349		0.39 651	9.96 757		8
		31		37			5	
53	9.57 138		9.60 386		0.39 614	9.96 752		7
		31		36			5	
54	9.57 169		9.60 422		0.39 578	9.96 747		6
		32		37			5	
55	9.57 201		9.60 459		0.39 541	9.96 742		5
		31		36			5	
56	9.57 232		9.60 495		0.39 505	9.96 737		4
		32		37			5	
57	9.57 264		9.60 532		0.39 468	9.96 732		3
		31		36			5	
58	9.57 295		9.60 568		0.39 432	9.96 727		2
		31		37			5	
59	9.57 326		9.60 605		0.39 395	9.96 722		1
		32		36			5	
60	9.57 358		9.60 641		0.39 359	9.96 717		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	33	**32**	**31**
1	3.3	3.2	3.1
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	6	**5**	**4**
1	0.6	0.5	0.4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	**60**
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	31	9.60 786	36	0.39 214	9.96 696	5	56
5	9.57 514	32	9.60 823	37	0.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	5	52
9	9.57 638	31	9.60 967	36	0.39 033	9.96 670	6	51
10	9.57 669	31	9.61 004	37	0.38 996	9.96 665	5	**50**
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	5	45
16	9.57 855	31	9.61 220	36	0.38 780	9.96 634	6	44
17	9.57 885	30	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	31	9.61 364	36	0.38 636	9.96 614	5	**40**
21	9.58 008	30	9.61 400	36	0.38 600	9.96 608	6	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	31	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	30	9.61 544	36	0.38 456	9.96 588	5	35
26	9.58 162	31	9.61 579	35	0.38 421	9.96 582	6	34
27	9.58 192	30	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	31	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	30	9.61 687	36	0.38 313	9.96 567	5	31
30	9.58 284	31	9.61 722	35	0.38 278	9.96 562	6	**30**
31	9.58 314	30	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	31	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	30	9.61 830	36	0.38 170	9.96 546	5	27
34	9.58 406	31	9.61 865	35	0.38 135	9.96 541	5	26
35	9.58 436	30	9.61 901	36	0.38 099	9.96 535	6	25
36	9.58 467	31	9.61 936	35	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	36	0.37 992	9.96 520	5	22
39	9.58 557	30	9.62 043	35	0.37 957	9.96 514	6	21
40	9.58 588	31	9.62 079	36	0.37 921	9.96 509	5	**20**
41	9.58 618	30	9.62 114	35	0.37 886	9.96 504	5	19
42	9.58 648	30	9.62 150	36	0.37 850	9.96 498	6	18
43	9.58 678	30	9.62 185	35	0.37 815	9.96 493	5	17
44	9.58 709	31	9.62 221	36	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	35	0.37 744	9.96 483	5	15
46	9.58 769	30	9.62 292	36	0.37 708	9.96 477	6	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	35	0.37 638	9.96 467	5	12
49	9.58 859	30	9.62 398	36	0.37 602	9.96 461	6	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	**10**
51	9.58 919	30	9.62 468	35	0.37 532	9.96 451	5	9
52	9.58 949	30	9.62 504	36	0.37 496	9.96 445	6	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	5	6
55	9.59 039	30	9.62 609	35	0.37 391	9.96 429	6	5
56	9.59 069	30	9.62 645	36	0.37 355	9.96 424	5	4
57	9.59 098	29	9.62 680	35	0.37 320	9.96 419	5	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	6	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188	30	9.62 785	35	0.37 215	9.96 403	5	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	37	36	35
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
1	2.9	0.6	0.5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.59 188		9.62 785		0.37 215	9.96 403	6	**60**
1	9.59 218	30	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	29	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	35	0.37 110	9.96 387	6	57
4	9.59 307	30	9.62 926	36	0.37 074	9.96 381		56
5	9.59 336	29	9.62 961	35	0.37 039	9.96 376	5	55
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	6	54
7	9.59 396	30	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	29	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 455	30	9.63 101	35	0.36 899	9.96 354		51
10	9.59 484	29	9.63 135	34	0.36 865	9.96 349	5	**50**
11	9.59 514	30	9.63 170	35	0.36 830	9.96 343	6	49
12	9.59 543	29	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	30	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327		46
15	9.59 632	30	9.63 310	35	0.36 690	9.96 322	5	45
16	9.59 661	29	9.63 345	35	0.36 655	9.96 316	6	44
17	9.59 690	29	9.63 379	34	0.36 621	9.96 311	5	43
18	9.59 720	30	9.63 414	35	0.36 586	9.96 305	6	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	5	41
20	9.59 778	29	9.63 484	35	0.36 516	9.96 294	6	**40**
21	9.59 808	30	9.63 519	35	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	34	0.36 447	9.96 284	5	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	6	37
24	9.59 895	29	9.63 623	35	0.36 377	9.96 273	5	36
25	9.59 924	29	9.63 657	34	0.36 343	9.96 267	6	35
26	9.59 954	30	9.63 692	35	0.36 308	9.96 262	5	34
27	9.59 983	29	9.63 726	34	0.36 274	9.96 256	6	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	5	32
29	9.60 041	29	9.63 796	35	0.36 204	9.96 245	6	31
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	5	**30**
31	9.60 099	29	9.63 865	35	0.36 135	9.96 234	6	29
32	9.60 128	29	9.63 899	34	0.36 101	9.96 229	5	28
33	9.60 157	29	9.63 934	35	0.36 066	9.96 223	6	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26
35	9.60 215	29	9.64 003	35	0.35 997	9.96 212	6	25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 207	5	24
37	9.60 273	29	9.64 072	35	0.35 928	9.96 201	6	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	5	22
39	9.60 331	29	9.64 140	34	0.35 860	9.96 190	6	21
40	9.60 359	28	9.64 175	35	0.35 825	9.96 185	5	**20**
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	6	19
42	9.60 417	29	9.64 243	34	0.35 757	9.96 174	5	18
43	9.60 446	29	9.64 278	35	0.35 722	9.96 168	6	17
44	9.60 474	28	9.64 312	34	0.35 688	9.96 162		16
45	9.60 503	29	9.64 346	34	0.35 654	9.96 157	5	15
46	9.60 532	29	9.64 381	35	0.35 619	9.96 151	6	14
47	9.60 561	29	9.64 415	34	0.35 585	9.96 146	5	13
48	9.60 589	28	9.64 449	34	0.35 551	9.96 140	6	12
49	9.60 618	29	9.64 483	34	0.35 517	9.96 135	5	11
50	9.60 646	28	9.64 517	34	0.35 483	9.96 129	6	**10**
51	9.60 675	29	9.64 552	35	0.35 448	9.96 123		9
52	9.60 704	29	9.64 586	34	0.35 414	9.96 118	5	8
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	6	7
54	9.60 761	29	9.64 654	34	0.35 346	9.96 107	5	6
55	9.60 789	28	9.64 688	34	0.35 312	9.96 101	6	5
56	9.60 818	29	9.64 722	34	0.35 278	9.96 095	6	4
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	5	3
58	9.60 875	29	9.64 790	34	0.35 210	9.96 084	6	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	5	1
60	9.60 931	28	9.64 858	34	0.35 142	9.96 073	6	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	36	35	34
1	3.6	3.5	3.4
2	7.2	7.0	6.8
3	10.8	10.5	10.2
4	14.4	14.0	13.6
5	18.0	17.5	17.0
6	21.6	21.0	20.4
7	25.2	24.5	23.8
8	28.8	28.0	27.2
9	32.4	31.5	30.6

	30	29	28
1	3.0	2.9	2.8
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.60 931		9.64 858		0.35 142	9.96 073		**60**
		29		34			6	
1	9.60 960		9.64 892		0.35 108	9.96 067		59
		28		34			5	
2	9.60 988		9.64 926		0.35 074	9.96 062		58
		28		34			6	
3	9.61 016		9.64 960		0.35 040	9.96 056		57
		29		34			6	
4	9.61 045		9.64 994		0.35 006	9.96 050		56
		28		34			5	
5	9.61 073		9.65 028		0.34 972	9.96 045		55
		28		34			6	
6	9.61 101		9.65 062		0.34 938	9.96 039		54
		28		34			5	
7	9.61 129		9.65 096		0.34 904	9.96 034		53
		29		34			6	
8	9.61 158		9.65 130		0.34 870	9.96 028		52
		28		34			6	
9	9.61 186		9.65 164		0.34 836	9.96 022		51
		28		33			5	
10	9.61 214		9.65 197		0.34 803	9.96 017		**50**
		28		34			6	
11	9.61 242		9.65 231		0.34 769	9.96 011		49
		28		34			6	
12	9.61 270		9.65 265		0.34 735	9.96 005		48
		28		34			5	
13	9.61 298		9.65 299		0.34 701	9.96 000		47
		28		34			6	
14	9.61 326		9.65 333		0.34 667	9.95 994		46
		28		33			6	
15	9.61 354		9.65 366		0.34 634	9.95 988		45
		28		34			6	
16	9.61 382		9.65 400		0.34 600	9.95 982		44
		29		34			5	
17	9.61 411		9.65 434		0.34 566	9.95 977		43
		27		33			6	
18	9.61 438		9.65 467		0.34 533	9.95 971		42
		28		34			6	
19	9.61 466		9.65 501		0.34 499	9.95 965		41
		28		34			6	
20	9.61 494		9.65 535		0.34 465	9.95 960		**40**
		28		33			6	
21	9.61 522		9.65 568		0.34 432	9.95 954		39
		28		34			6	
22	9.61 550		9.65 602		0.34 398	9.95 948		38
		28		34			6	
23	9.61 578		9.65 636		0.34 364	9.95 942		37
		28		33			6	
24	9.61 606		9.65 669		0.34 331	9.95 937		36
		28		34			6	
25	9.61 634		9.65 703		0.34 297	9.95 931		35
		28		33			6	
26	9.61 662		9.65 736		0.34 264	9.95 925		34
		27		34			5	
27	9.61 689		9.65 770		0.34 230	9.95 920		33
		28		33			6	
28	9.61 717		9.65 803		0.34 197	9.95 914		32
		28		34			6	
29	9.61 745		9.65 837		0.34 163	9.95 908		31
		28		33			6	
30	9.61 773		9.65 870		0.34 130	9.95 902		**30**
		27		34			5	
31	9.61 800		9.65 904		0.34 096	9.95 897		29
		28		33			6	
32	9.61 828		9.65 937		0.34 063	9.95 891		28
		28		34			6	
33	9.61 856		9.65 971		0.34 029	9.95 885		27
		27		33			6	
34	9.61 883		9.66 004		0.33 996	9.95 879		26
		28		34			6	
35	9.61 911		9.66 038		0.33 962	9.95 873		25
		28		33			6	
36	9.61 939		9.66 071		0.33 929	9.95 868		24
		27		33			5	
37	9.61 966		9.66 104		0.33 896	9.95 862		23
		28		34			6	
38	9.61 994		9.66 138		0.33 862	9.95 856		22
		27		33			6	
39	9.62 021		9.66 171		0.33 829	9.95 850		21
		28		33			6	
40	9.62 049		9.66 204		0.33 796	9.95 844		**20**
		27		34			5	
41	9.62 076		9.66 238		0.33 762	9.95 839		19
		28		33			6	
42	9.62 104		9.66 271		0.33 729	9.95 833		18
		27		33			6	
43	9.62 131		9.66 304		0.33 696	9.95 827		17
		28		33			6	
44	9.62 159		9.66 337		0.33 663	9.95 821		16
		27		34			6	
45	9.62 186		9.66 371		0.33 629	9.95 815		15
		28		33			5	
46	9.62 214		9.66 404		0.33 596	9.95 810		14
		27		33			6	
47	9.62 241		9.66 437		0.33 563	9.95 804		13
		27		33			6	
48	9.62 268		9.66 470		0.33 530	9.95 798		12
		28		33			6	
49	9.62 296		9.66 503		0.33 497	9.95 792		11
		27		34			6	
50	9.62 323		9.66 537		0.33 463	9.95 786		**10**
		27		33			6	
51	9.62 350		9.66 570		0.33 430	9.95 780		9
		27		33			5	
52	9.62 377		9.66 603		0.33 397	9.95 775		8
		28		33			6	
53	9.62 405		9.66 636		0.33 364	9.95 769		7
		27		33			6	
54	9.62 432		9.66 669		0.33 331	9.95 763		6
		27		33			6	
55	9.62 459		9.66 702		0.33 298	9.95 757		5
		27		33			6	
56	9.62 486		9.66 735		0.33 265	9.95 751		4
		27		33			6	
57	9.62 513		9.66 768		0.33 232	9.95 745		3
		28		33			6	
58	9.62 541		9.66 801		0.33 199	9.95 739		2
		27		33			6	
59	9.62 568		9.66 834		0.33 166	9.95 733		1
		27		33			5	
60	9.62 595		9.66 867		0.33 133	9.95 728		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	34	33
1	3.4	3.3
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	**60**
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	33	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	32	0.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	**50**
11	9.62 892	27	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	26	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	33	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	32	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45
16	9.63 026	27	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	26	9.67 426	33	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	32	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	27	9.67 524	33	0.32 476	9.95 609	6	**40**
21	9.63 159	26	9.67 556	32	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	27	9.67 622	33	0.32 378	9.95 591	6	37
24	9.63 239	26	9.67 654	32	0.32 346	9.95 585	6	36
25	9.63 266	27	9.67 687	33	0.32 313	9.95 579	6	35
26	9.63 292	26	9.67 719	32	0.32 281	9.95 573	6	34
27	9.63 319	27	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	26	9.67 785	33	0.32 215	9.95 561	6	32
29	9.63 372	27	9.67 817	32	0.32 183	9.95 555	6	31
30	9.63 398	26	9.67 850	33	0.32 150	9.95 549	6	**30**
31	9.63 425	27	9.67 882	32	0.32 118	9.95 543	6	29
32	9.63 451	26	9.67 915	33	0.32 085	9.95 537	6	28
33	9.63 478	27	9.67 947	32	0.32 053	9.95 531	6	27
34	9.63 504	26	9.67 980	33	0.32 020	9.95 525	6	26
35	9.63 531	27	9.68 012	32	0.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	32	0.31 956	9.95 513	6	24
37	9.63 583	26	9.68 077	33	0.31 923	9.95 507	6	23
38	9.63 610	27	9.68 109	32	0.31 891	9.95 500	7	22
39	9.63 636	26	9.68 142	33	0.31 858	9.95 494	6	21
40	9.63 662	26	9.68 174	32	0.31 826	9.95 488	6	**20**
41	9.63 689	27	9.68 206	32	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	33	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	26	9.68 303	32	0.31 697	9.95 464	6	16
45	9.63 794	27	9.68 336	33	0.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	32	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	33	0.31 535	9.95 434	6	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	7	**10**
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	32	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	33	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	6	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	7	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	33	32
1	3.3	3.2
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	7	6	5
1	0.7	0.6	0.5
2	1.4	1.2	1.0
3	2.1	1.8	1.5
4	2.8	2.4	2.0
5	3.5	3.0	2.5
6	4.2	3.6	3.0
7	4.9	4.2	3.5
8	5.6	4.8	4.0
9	6.3	5.4	4.5

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	**60**
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	6	52
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	7	51
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	**50**
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	6	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	7	46
15	9.64 571	25	9.69 298	32	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	31	0.30 671	9.95 267	6	44
17	9.64 622	26	9.69 361	32	0.30 639	9.95 261	6	43
18	9.64 647	25	9.69 393	32	0.30 607	9.95 254	7	42
19	9.64 673	26	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	25	9.69 457	32	0.30 543	9.95 242	6	**40**
21	9.64 724	26	9.69 488	31	0.30 512	9.95 236	6	39
22	9.64 749	25	9.69 520	32	0.30 480	9.95 229	7	38
23	9.64 775	26	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	25	9.69 584	32	0.30 416	9.95 217	6	36
25	9.64 826	26	9.69 615	31	0.30 385	9.95 211	6	35
26	9.64 851	25	9.69 647	32	0.30 353	9.95 204	7	34
27	9.64 877	26	9.69 679	32	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	31	0.30 290	9.95 192	6	32
29	9.64 927	25	9.69 742	32	0.30 258	9.95 185	7	31
30	9.64 953	26	9.69 774	32	0.30 226	9.95 179	6	**30**
31	9.64 978	25	9.69 805	31	0.30 195	9.95 173	6	29
32	9.65 003	25	9.69 837	32	0.30 163	9.95 167	6	28
33	9.65 029	26	9.69 868	31	0.30 132	9.95 160	7	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	32	0.30 068	9.95 148	6	25
36	9.65 104	25	9.69 963	31	0.30 037	9.95 141	7	24
37	9.65 130	26	9.69 995	32	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	31	0.29 974	9.95 129	6	22
39	9.65 180	25	9.70 058	32	0.29 942	9.95 122	7	21
40	9.65 205	25	9.70 089	31	0.29 911	9.95 116	6	**20**
41	9.65 230	25	9.70 121	32	0.29 879	9.95 110	6	19
42	9.65 255	25	9.70 152	31	0.29 848	9.95 103	7	18
43	9.65 281	26	9.70 184	32	0.29 816	9.95 097	6	17
44	9.65 306	25	9.70 215	31	0.29 785	9.95 090	7	16
45	9.65 331	25	9.70 247	32	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14
47	9.65 381	25	9.70 309	31	0.29 691	9.95 071	7	13
48	9.65 406	25	9.70 341	32	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	31	0.29 628	9.95 059	6	11
50	9.65 456	25	9.70 404	32	0.29 596	9.95 052	7	**10**
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9
52	9.65 506	25	9.70 466	31	0.29 534	9.95 039	7	8
53	9.65 531	25	9.70 498	32	0.29 502	9.95 033	6	7
54	9.65 556	25	9.70 529	31	0.29 471	9.95 027	6	6
55	9.65 580	24	9.70 560	31	0.29 440	9.95 020	7	5
56	9.65 605	25	9.70 592	32	0.29 408	9.95 014	6	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	7	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	31	0.29 315	9.94 995	6	1
60	9.65 705	25	9.70 717	32	0.29 283	9.94 988	7	**0**
	L Cos	d	L Cot	c d	l. Tan	L Sin	d	′

P P

	32	31
1	3.2	3.1
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	26	25	24
1	2.6	2.5	2.4
2	5.2	5.0	4.8
3	7.8	7.5	7.2
4	10.4	10.0	9.6
5	13.0	12.5	12.0
6	15.6	15.0	14.4
7	18.2	17.5	16.8
8	20.8	20.0	19.2
9	23.4	22.5	21.6

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.65 705	24	9.70 717	31	0.29 283	9.94 988	6	**60**
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	25	9.70 841	31	0.29 159	9.94 962	6	56
5	9.65 828	24	9.70 873	32	0.29 127	9.94 956	7	55
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	25	9.70 935	31	0.29 065	9.94 943	7	53
8	9.65 902	24	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	**50**
11	9.65 976	24	9.71 059	31	0.28 941	9.94 917	7	49
12	9.66 001	25	9.71 090	31	0.28 910	9.94 911	6	48
13	9.66 025	24	9.71 121	31	0.28 879	9.94 904	7	47
14	9.66 050	25	9.71 153	32	0.28 847	9.94 898	6	46
15	9.66 075	25	9.71 184	31	0.28 816	9.94 891	7	45
16	9.66 099	24	9.71 215	31	0.28 785	9.94 885	6	44
17	9.66 124	25	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	24	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	25	9.71 308	31	0.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	**40**
21	9.66 221	24	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	25	9.71 401	31	0.28 599	9.94 845	6	38
23	9.66 270	24	9.71 431	30	0.28 569	9.94 839	7	37
24	9.66 295	25	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	35
26	9.66 343	24	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	25	9.71 555	31	0.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	24	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441	25	9.71 648	31	0.28 352	9.94 793	7	**30**
31	9.66 465	24	9.71 679	31	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	30	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	7	27
34	9.66 537	24	9.71 771	31	0.28 229	9.94 767	6	26
35	9.66 562	25	9.71 802	31	0.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	31	0.28 167	9.94 753	7	24
37	9.66 610	24	9.71 863	30	0.28 137	9.94 747	6	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	7	22
39	9.66 658	24	9.71 925	31	0.28 075	9.94 734	6	21
40	9.66 682	24	9.71 955	30	0.28 045	9.94 727	7	**20**
41	9.66 706	24	9.71 986	31	0.28 014	9.94 720	7	19
42	9.66 731	25	9.72 017	31	0.27 983	9.94 714	6	18
43	9.66 755	24	9.72 048	31	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	30	0.27 922	9.94 700	7	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	6	15
46	9.66 827	24	9.72 140	31	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	30	0.27 830	9.94 680	7	13
48	9.66 875	24	9.72 201	31	0.27 799	9.94 674	6	12
49	9.66 899	24	9.72 231	30	0.27 769	9.94 667	7	11
50	9.66 922	23	9.72 262	31	0.27 738	9.94 660	7	**10**
51	9.66 946	24	9.72 293	31	0.27 707	9.94 654	6	9
52	9.66 970	24	9.72 323	30	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	31	0.27 646	9.94 640	7	7
54	9.67 018	24	9.72 384	30	0.27 616	9.94 634	6	6
55	9.67 042	24	9.72 415	31	0.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	30	0.27 555	9.94 620	7	4
57	9.67 090	24	9.72 476	31	0.27 524	9.94 614	6	3
58	9.67 113	23	9.72 506	30	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	31	0.27 463	9.94 600	7	1
60	9.67 161	24	9.72 567	30	0.24 433	9.94 593	7	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	25	24	23
1	2.5	2.4	2.3
2	5.0	4.8	4.6
3	7.5	7.2	6.9
4	10.0	9.6	9.2
5	12.5	12.0	11.5
6	15.0	14.4	13.8
7	17.5	16.8	16.1
8	20.0	19.2	18.4
9	22.5	21.6	20.7

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		P P
0	9.67 161		9.72 567		0.27 433	9.94 593		**60**	
		24		31			6		
1	9.67 185		9.72 598		0.27 402	9.94 587		59	
		23		30			7		
2	9.67 208		9.72 628		0.27 372	9.94 580	7	58	
		24		31			6		
3	9.67 232		9.72 659		0.27 341	9.94 573	7	57	
		24		30			7		
4	9.67 256		9.72 689		0.27 311	9.94 567	7	56	
		24		31			7		
5	9.67 280		9.72 720		0.27 280	9.94 560	7	55	
		23		30			7		
6	9.67 303		9.72 750		0.27 250	9.94 553	7	54	
		24		30			7		
7	9.67 327		9.72 780		0.27 220	9.94 546	6	53	
		23		31			7		
8	9.67 350		9.72 811		0.27 189	9.94 540	7	52	
		24		30			7		
9	9.67 374		9.72 841		0.27 159	9.94 533	7	51	
		24		31			7		
10	9.67 398		9.72 872		0.27 128	9.94 526		**50**	
		23		30			7		
11	9.67 421		9.72 902		0.27 098	9.94 519	6	49	
		24		30			7		
12	9.67 445		9.72 932		0.27 068	9.94 513	7	48	
		23		31			7		
13	9.67 468		9.72 963		0.27 037	9.94 506	7	47	
		24		30			7		
14	9.67 492		9.72 993		0.27 007	9.94 499	7	46	
		23		30			7		
15	9.67 515		9.73 023		0.26 977	9.94 492	7	45	
		24		31			7		
16	9.67 539		9.73 054		0.26 946	9.94 485	6	44	
		23		30			7		
17	9.67 562		9.73 084		0.26 916	9.94 479	7	43	
		23		30			7		
18	9.67 586		9.73 114		0.26 886	9.94 472	7	42	
		23		30			7		
19	9.67 609		9.73 144		0.26 856	9.94 465	7	41	
		24		31			7		
20	9.67 633		9.73 175		0.26 825	9.94 458		**40**	
		23		30			7		
21	9.67 656		9.73 205		0.26 795	9.94 451	7	39	
		24		30			6		
22	9.67 680		9.73 235		0.26 765	9.94 445	7	38	
		23		30			7		
23	9.67 703		9.73 265		0.26 735	9.94 438	7	37	
		23		30			7		
24	9.67 726		9.73 295		0.26 705	9.94 431	7	36	
		24		31			7		
25	9.67 750		9.73 326		0.26 674	9.94 424	7	35	
		23		30			7		
26	9.67 773		9.73 356		0.26 644	9.94 417	7	34	
		23		30			7		
27	9.67 796		9.73 386		0.26 614	9.94 410	6	33	
		24		30			7		
28	9.67 820		9.73 416		0.26 584	9.94 404	7	32	
		23		30			7		
29	9.67 843		9.73 446		0.26 554	9.94 397	7	31	
		23		30			7		
30	9.67 866		9.73 476		0.26 524	9.94 390		**30**	
		24		31			7		
31	9.67 890		9.73 507		0.26 493	9.94 383	7	29	
		23		30			7		
32	9.67 913		9.73 537		0.26 463	9.94 376	7	28	
		23		30			7		
33	9.67 936		9.73 567		0.26 433	9.94 369	7	27	
		23		30			7		
34	9.67 959		9.73 597		0.26 403	9.94 362	7	26	
		23		30			7		
35	9.67 982		9.73 627		0.26 373	9.94 355	6	25	
		24		30			7		
36	9.68 006		9.73 657		0.26 343	9.94 349	7	24	
		23		30			7		
37	9.68 029		9.73 687		0.26 313	9.94 342	7	23	
		23		30			7		
38	9.68 052		9.73 717		0.26 283	9.94 335	7	22	
		23		30			7		
39	9.68 075		9.73 747		0.26 253	9.94 328	7	21	
		23		30			7		
40	9.68 098		9.73 777		0.26 223	9.94 321		**20**	
		23		30			7		
41	9.68 121		9.73 807		0.26 193	9.94 314	7	19	
		23		30			7		
42	9.68 144		9.73 837		0.26 163	9.94 307	7	18	
		23		30			7		
43	9.68 167		9.73 867		0.26 133	9.94 300	7	17	
		23		30			7		
44	9.68 190		9.73 897		0.26 103	9.94 293	7	16	
		23		30			7		
45	9.68 213		9.73 927		0.26 073	9.94 286	7	15	
		23		30			6		
46	9.68 237		9.73 957		0.26 043	9.94 279	7	14	
		23		30			7		
47	9.68 260		9.73 987		0.26 013	9.94 273	7	13	
		23		30			7		
48	9.68 283		9.74 017		0.25 983	9.94 266	7	12	
		22		30			7		
49	9.68 305		9.74 047		0.25 953	9.94 259	7	11	
		23		30			7		
50	9.68 328		9.74 077		0.25 923	9.94 252		**10**	
		23		30			7		
51	9.68 351		9.74 107		0.25 893	9.94 245	7	9	
		23		30			7		
52	9.68 374		9.74 137		0.25 863	9.94 238	7	8	
		23		29			7		
53	9.68 397		9.74 166		0.25 834	9.94 231	7	7	
		23		30			7		
54	9.68 420		9.74 196		0.25 804	9.94 224	7	6	
		23		30			7		
55	9.68 443		9.74 226		0.25 774	9.94 217	7	5	
		23		30			7		
56	9.68 466		9.74 256		0.25 744	9.94 210	7	4	
		23		30			7		
57	9.68 489		9.74 286		0.25 714	9.94 203	7	3	
		23		30			7		
58	9.68 512		9.74 316		0.25 684	9.94 196	7	2	
		22		29			7		
59	9.68 534		9.74 345		0.25 655	9.94 189	7	1	
		23		30			7		
60	9.68 557		9.74 375		0.25 625	9.94 182		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	P P

	31	30	29
1	3.1	3.0	2.9
2	6.2	6.0	5.8
3	9.3	9.0	8.7
4	12.4	12.0	11.6
5	15.5	15.0	14.5
6	18.6	18.0	17.4
7	21.7	21.0	20.3
8	24.8	24.0	23.2
9	27.9	27.0	26.1

	24	23	22
1	2.4	2.3	2.2
2	4.8	4.6	4.4
3	7.2	6.9	6.6
4	9.6	9.2	8.8
5	12.0	11.5	11.0
6	14.4	13.8	13.2
7	16.8	16.1	15.4
8	19.2	18.4	17.6
9	21.6	20.7	19.8

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

′	L Sin	d	L Tan	cd	L Cot	L Cos	d	
0	9.68 557		9.74 375		0.25 625	9.94 182		60
		23		30			7	
1	9.68 580		9.74 405		0.25 595	9.94 175		59
		23		30			7	
2	9.68 603		9.74 435		0.25 565	9.94 168		58
		22		30			7	
3	9.68 625		9.74 465		0.25 535	9.94 161		57
		23		29			7	
4	9.68 648		9.74 494		0.25 506	9.94 154		56
		23		30			7	
5	9.68 671		9.74 524		0.25 476	9.94 147		55
		23		30			7	
6	9.68 694		9.74 554		0.25 446	9.94 140		54
		22		29			7	
7	9.68 716		9.74 583		0.25 417	9.94 133		53
		23		30			7	
8	9.68 739		9.74 613		0.25 387	9.94 126		52
		23		30			7	
9	9.68 762		9.74 643		0.25 357	9.94 119		51
		22		30			7	
10	9.68 784		9.74 673		0.25 327	9.94 112		50
		23		29			7	
11	9.68 807		9.74 702		0.25 298	9.94 105		49
		22		30			7	
12	9.68 829		9.74 732		0.25 268	9.94 098		48
		23		30			8	
13	9.68 852		9.74 762		0.25 238	9.94 090		47
		23		29			7	
14	9.68 875		9.74 791		0.25 209	9.94 083		46
		22		30			7	
15	9.68 897		9.74 821		0.25 179	9.94 076		45
		22		30			7	
16	9.68 920		9.74 851		0.25 149	9.94 069		44
		22		29			7	
17	9.68 942		9.74 880		0.25 120	9.94 062		43
		23		30			7	
18	9.68 965		9.74 910		0.25 090	9.94 055		42
		22		29			7	
19	9.68 987		9.74 939		0.25 061	9.94 048		41
		23		30			7	
20	9.69 010		9.74 969		0.25 031	9.94 041		40
		22		29			7	
21	9.69 032		9.74 998		0.25 002	9.94 034		39
		23		30			7	
22	9.69 055		9.75 028		0.24 972	9.94 027		38
		22		30			7	
23	9.69 077		9.75 058		0.24 942	9.94 020		37
		23		29			8	
24	9.69 100		9.75 087		0.24 913	9.94 012		36
		22		30			7	
25	9.69 122		9.75 117		0.24 883	9.94 005		35
		22		29			7	
26	9.69 144		9.75 146		0.24 854	9.93 998		34
		23		30			7	
27	9.69 167		9.75 176		0.24 824	9.93 991		33
		22		29			7	
28	9.69 189		9.75 205		0.24 795	9.93 984		32
		23		30			7	
29	9.69 212		9.75 235		0.24 765	9.93 977		31
		22		29			7	
30	9.69 234		9.75 264		0.24 736	9.93 970		30
		22		30			7	
31	9.69 256		9.75 294		0.24 706	9.93 963		29
		23		29			8	
32	9.69 279		9.75 323		0.24 677	9.93 955		28
		22		30			7	
33	9.69 301		9.75 353		0.24 647	9.93 948		27
		22		29			7	
34	9.69 323		9.75 382		0.24 618	9.93 941		26
		22		29			7	
35	9.69 345		9.75 411		0.24 589	9.93 934		25
		23		30			7	
36	9.69 368		9.75 441		0.24 559	9.93 927		24
		22		29			7	
37	9.69 390		9.75 470		0.24 530	9.93 920		23
		22		30			8	
38	9.69 412		9.75 500		0.24 500	9.93 912		22
		22		29			7	
39	9.69 434		9.75 529		0.24 471	9.93 905		21
		22		29			7	
40	9.69 456		9.75 558		0.24 442	9.93 898		20
		23		30			7	
41	9.69 479		9.75 588		0.24 412	9.93 891		19
		22		29			7	
42	9.69 501		9.75 617		0.24 383	9.93 884		18
		22		30			8	
43	9.69 523		9.75 647		0.24 353	9.93 876		17
		22		29			7	
44	9.69 545		9.75 676		0.24 324	9.93 869		16
		22		29			7	
45	9.69 567		9.75 705		0.24 295	9.93 862		15
		22		30			7	
46	9.69 589		9.75 735		0.24 265	9.93 855		14
		22		29			8	
47	9.69 611		9.75 764		0.24 236	9.93 847		13
		22		29			7	
48	9.69 633		9.75 793		0.24 207	9.93 840		12
		22		29			7	
49	9.69 655		9.75 822		0.24 178	9.93 833		11
		22		30			7	
50	9.69 677		9.75 852		0.24 148	9.93 826		10
		22		29			7	
51	9.69 699		9.75 881		0.24 119	9.93 819		9
		22		29			8	
52	9.69 721		9.75 910		0.24 090	9.93 811		8
		22		29			7	
53	9.69 743		9.75 939		0.24 061	9.93 804		7
		22		30			7	
54	9.69 765		9.75 969		0.24 031	9.93 797		6
		22		29			8	
55	9.69 787		9.75 998		0.24 002	9.93 789		5
		22		29			7	
56	9.69 809		9.76 027		0.23 973	9.93 782		4
		22		29			7	
57	9.69 831		9.76 056		0.23 944	9.93 775		3
		22		30			7	
58	9.69 853		9.76 086		0.23 914	9.93 768		2
		22		29			8	
59	9.69 875		9.76 115		0.23 885	9.93 760		1
		22		29			7	
60	9.69 897		9.76 144		0.23 856	9.93 753		0
	L Cos	d	L Cot	cd	L Tan	L Sin	d	′

P P			
	30	29	23
1	3.0	2.9	2.3
2	6.0	5.8	4.6
3	9.0	8.7	6.9
4	12.0	11.6	9.2
5	15.0	14.5	11.5
6	18.0	17.4	13.8
7	21.0	20.3	16.1
8	24.0	23.2	18.4
9	27.0	26.1	20.7

P P			
	22	8	7
1	2.2	0.8	0.7
2	4.4	1.6	1.4
3	6.6	2.4	2.1
4	8.8	3.2	2.8
5	11.0	4.0	3.5
6	13.2	4.8	4.2
7	15.4	5.6	4.9
8	17.6	6.4	5.6
9	19.8	7.2	6.3

′	L Sin	d	L Tan	cd	L Cot	L Cos	d	′
0	9.69 897		9.76 144		0.23 856	9.93 753		**60**
		22		29			7	
1	9.69 919		9.76 173		0.23 827	9.93 746		59
		22		29			8	
2	9.69 941		9.76 202		0.23 798	9.93 738		58
		22		29			7	
3	9.69 963		9.76 231		0.23 769	9.93 731		57
		21		30			7	
4	9.69 984		9.76 261		0.23 739	9.93 724		56
		22		29			7	
5	9.70 006		9.76 290		0.23 710	9.93 717		55
		22		29			8	
6	9.70 028		9.76 319		0.23 681	9.93 709		54
		22		29			7	
7	9.70 050		9.76 348		0.23 652	9.93 702		53
		22		29			7	
8	9.70 072		9.76 377		0.23 623	9.93 695		52
		21		29			8	
9	9.70 093		9.76 406		0.23 594	9.93 687		51
		22		29			7	
10	9.70 115		9.76 435		0.23 565	9.93 680		**50**
		22		29			7	
11	9.70 137		9.76 464		0.23 536	9.93 673		49
		22		29			8	
12	9.70 159		9.76 493		0.23 507	9.93 665		48
		21		29			7	
13	9.70 180		9.76 522		0.23 478	9.93 658		47
		22		29			8	
14	9.70 202		9.76 551		0.23 449	9.93 650		46
		22		29			7	
15	9.70 224		9.76 580		0.23 420	9.93 643		45
		21		29			7	
16	9.70 245		9.76 609		0.23 391	9.93 636		44
		21		30			8	
17	9.70 267		9.76 639		0.23 361	9.93 628		43
		21		29			7	
18	9.70 288		9.76 668		0.23 332	9.93 621		42
		22		29			7	
19	9.70 310		9.76 697		0.23 303	9.93 614		41
		22		28			8	
20	9.70 332		9.76 725		0.23 275	9.93 606		**40**
		21		29			7	
21	9.70 353		9.76 754		0.23 246	9.93 599		39
		22		29			8	
22	9.70 375		9.76 783		0.23 217	9.93 591		38
		21		29			7	
23	9.70 396		9.76 812		0.23 188	9.93 584		37
		22		29			7	
24	9.70 418		9.76 841		0.23 159	9.93 577		36
		21		29			8	
25	9.70 439		9.76 870		0.23 130	9.93 569		35
		22		29			7	
26	9.70 461		9.76 899		0.23 101	9.93 562		34
		21		29			8	
27	9.70 482		9.76 928		0.23 072	9.93 554		33
		22		29			7	
28	9.70 504		9.76 957		0.23 043	9.93 547		32
		21		29			8	
29	9.70 525		9.76 986		0.23 014	9.93 539		31
		22		29			7	
30	9.70 547		9.77 015		0.22 985	9.93 532		**30**
		21		29			7	
31	9.70 568		9.77 044		0.22 956	9.93 525		29
		22		29			8	
32	9.70 590		9.77 073		0.22 927	9.93 517		28
		21		28			7	
33	9.70 611		9.77 101		0.22 899	9.93 510		27
		22		29			8	
34	9.70 633		9.77 130		0.22 870	9.93 502		26
		21		29			7	
35	9.70 654		9.77 159		0.22 841	9.93 495		25
		21		29			8	
36	9.70 675		9.77 188		0.22 812	9.93 487		24
		22		29			8	
37	9.70 697		9.77 217		0.22 783	9.93 480		23
		21		29			8	
38	9.70 718		9.77 246		0.22 754	9.93 472		22
		21		28			7	
39	9.70 739		9.77 274		0.22 726	9.93 465		21
		22		29			8	
40	9.70 761		9.77 303		0.22 697	9.93 457		**20**
		21		29			7	
41	9.70 782		9.77 332		0.22 668	9.93 450		19
		21		29			8	
42	9.70 803		9.77 361		0.22 639	9.93 442		18
		21		29			7	
43	9.70 824		9.77 390		0.22 610	9.93 435		17
		22		28			8	
44	9.70 846		9.77 418		0.22 582	9.93 427		16
		21		29			7	
45	9.70 867		9.77 447		0.22 553	9.93 420		15
		21		29			8	
46	9.70 888		9.77 476		0.22 524	9.93 412		14
		21		29			7	
47	9.70 909		9.77 505		0.22 495	9.93 405		13
		22		28			8	
48	9.70 931		9.77 533		0.22 467	9.93 397		12
		21		29			7	
49	9.70 952		9.77 562		0.22 438	9.93 390		11
		21		29			8	
50	9.70 973		9.77 591		0.22 409	9.93 382		**10**
		21		28			7	
51	9.70 994		9.77 619		0.22 381	9.93 375		9
		21		29			8	
52	9.71 015		9.77 648		0.22 352	9.93 367		8
		21		29			7	
53	9.71 036		9.77 677		0.22 323	9.93 360		7
		22		29			8	
54	9.71 058		9.77 706		0.22 294	9.93 352		6
		21		28			8	
55	9.71 079		9.77 734		0.22 266	9.93 344		5
		21		29			7	
56	9.71 100		9.77 763		0.22 237	9.93 337		4
		21		28			8	
57	9.71 121		9.77 791		0.22 209	9.93 329		3
		21		29			7	
58	9.71 142		9.77 820		0.22 180	9.93 322		2
		21		29			8	
59	9.71 163		9.77 849		0.22 151	9.93 314		1
		21		28			7	
60	9.71 184		9.77 877		0.22 123	9.93 307		**0**
	L Cos	d	L Cot	cd	L Tan	L Sin	d	′

P P			
	30	**29**	**28**
1	3.0	2.9	2.8
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	22	**21**
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	8	**7**
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

′	L Sin	d	L Tan	cd	L Cot	L Cos	d		P P
0	9.71 184		9.77 877		0.22 123	9.93 307		**60**	
		21		29			8		
1	9.71 205		9.77 906		0.22 094	9.93 299		59	
		21		29			8		
2	9.71 226		9.77 935		0.22 065	9.93 291		58	
		21		28			7		
3	9.71 247		9.77 963		0.22 037	9.93 284		57	
		21		29			8		
4	9.71 268		9.77 992		0.22 008	9.93 276		56	
		21		28			7		
5	9.71 289		9.78 020		0.21 980	9.93 269		55	
		21		29			8		
6	9.71 310		9.78 049		0.21 951	9.93 261		54	
		21		28			8		
7	9.71 331		9.78 077		0.21 923	9.93 253		53	
		21		29			7		
8	9.71 352		9.78 106		0.21 894	9.93 246		52	
		21		29			8		
9	9.71 373		9.78 135		0.21 865	9.93 238		51	
		20		28			8		
10	9.71 393		9.78 163		0.21 837	9.93 230		**50**	
		21		29			7		
11	9.71 414		9.78 192		0.21 808	9.93 223		49	
		21		28			8		
12	9.71 435		9.78 220		0.21 780	9.93 215		48	
		21		29			8		
13	9.71 456		9.78 249		0.21 751	9.93 207		47	
		21		28			7		
14	9.71 477		9.78 277		0.21 723	9.93 200		46	
		21		29			8		
15	9.71 498		9.78 306		0.21 694	9.93 192		45	
		21		28			8		
16	9.71 519		9.78 334		0.21 666	9.93 184		44	
		20		29			7		
17	9.71 539		9.78 363		0.21 637	9.93 177		43	
		21		28			8		
18	9.71 560		9.78 391		0.21 609	9.93 169		42	
		21		28			8		
19	9.71 581		9.78 419		0.21 581	9.93 161		41	
		21		29			7		
20	9.71 602		9.78 448		0.21 552	9.93 154		**40**	
		20		28			8		
21	9.71 622		9.78 476		0.21 524	9.93 146		39	
		21		29			8		
22	9.71 643		9.78 505		0.21 495	9.93 138		38	
		21		28			7		
23	9.71 664		9.78 533		0.21 467	9.93 131		37	
		21		29			8		
24	9.71 685		9.78 562		0.21 438	9.93 123		36	
		20		28			8		
25	9.71 705		9.78 590		0.21 410	9.93 115		35	
		21		28			7		
26	9.71 726		9.78 618		0.21 382	9.93 108		34	
		21		29			8		
27	9.71 747		9.78 647		0.21 353	9.93 100		33	
		20		28			8		
28	9.71 767		9.78 675		0.21 325	9.93 092		32	
		21		29			8		
29	9.71 788		9.78 704		0.21 296	9.93 084		31	
		21		28			7		
30	9.71 809		9.78 732		0.21 268	9.93 077		**30**	
		20		28			8		
31	9.71 829		9.78 760		0.21 240	9.93 069		29	
		21		29			8		
32	9.71 850		9.78 789		0.21 211	9.93 061		28	
		20		28			7		
33	9.71 870		9.78 817		0.21 183	9.93 053		27	
		21		28			8		
34	9.71 891		9.78 845		0.21 155	9.93 046		26	
		20		29			7		
35	9.71 911		9.78 874		0.21 126	9.93 038		25	
		21		28			8		
36	9.71 932		9.78 902		0.21 098	9.93 030		24	
		20		28			8		
37	9.71 952		9.78 930		0.21 070	9.93 022		23	
		21		29			8		
38	9.71 973		9.78 959		0.21 041	9.93 014		22	
		21		28			7		
39	9.71 994		9.78 987		0.21 013	9.93 007		21	
		20		28			8		
40	9.72 014		9.79 015		0.20 985	9.92 999		**20**	
		20		28			8		
41	9.72 034		9.79 043		0.20 957	9.92 991		19	
		21		29			8		
42	9.72 055		9.79 072		0.20 928	9.92 983		18	
		20		28			7		
43	9.72 075		9.79 100		0.20 900	9.92 976		17	
		21		28			8		
44	9.72 096		9.79 128		0.20 872	9.92 968		16	
		21		28			8		
45	9.72 116		9.79 156		0.20 844	9.92 960		15	
		21		29			8		
46	9.72 137		9.79 185		0.20 815	9.92 952		14	
		20		28			8		
47	9.72 157		9.79 213		0.20 787	9.92 944		13	
		20		28			8		
48	9.72 177		9.79 241		0.20 759	9.92 936		12	
		21		28			7		
49	9.72 198		9.79 269		0.20 731	9.92 929		11	
		20		28			8		
50	9.72 218		9.79 297		0.20 703	9.92 921		**10**	
		20		29			8		
51	9.72 238		9.79 326		0.20 674	9.92 913		9	
		21		28			8		
52	9.72 259		9.79 354		0.20 646	9.92 905		8	
		20		28			8		
53	9.72 279		9.79 382		0.20 618	9.92 897		7	
		20		28			8		
54	9.72 299		9.79 410		0.20 590	9.92 889		6	
		21		28			8		
55	9.72 320		9.79 438		0.20 562	9.92 881		5	
		20		28			7		
56	9.72 340		9.79 466		0.20 534	9.92 874		4	
		20		29			8		
57	9.72 360		9.79 495		0.20 505	9.92 866		3	
		21		28			8		
58	9.72 381		9.79 523		0.20 477	9.92 858		2	
		20		28			8		
59	9.72 401		9.79 551		0.20 449	9.92 850		1	
		20		28			8		
60	9.72 421		9.79 579		0.20 421	9.92 842		**0**	
	L Cos	d	L Cot	cd	L Tan	L Sin	d	′	P P

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	21	20
1	2.1	2.0
2	4.2	4.0
3	6.3	6.0
4	8.4	8.0
5	10.5	10.0
6	12.6	12.0
7	14.7	14.0
8	16.8	16.0
9	18.9	18.0

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.72 421		9.79 579		0.20 421	9.92 842		**60**
		20		28			8	
1	9.72 441		9.79 607		0.20 393	9.92 834		59
		20		28			8	
2	9.72 461		9.79 635		0.20 365	9.92 826		58
		21		28			8	
3	9.72 482		9.79 663		0.20 337	9.92 818		57
		20		28			8	
4	9.72 502		9.79 691		0.20 309	9.92 810		56
		20		28			7	
5	9.72 522		9.79 719		0.20 281	9.92 803		55
		20		28			8	
6	9.72 542		9.79 747		0.20 253	9.92 795		54
		20		29			8	
7	9.72 562		9.79 776		0.20 224	9.92 787		53
		20		28			8	
8	9.72 582		9.79 804		0.20 196	9.92 779		52
		20		28			8	
9	9.72 602		9.79 832		0.20 168	9.92 771		51
		20		28			8	
10	9.72 622		9.79 860		0.20 140	9.92 763		**50**
		21		28			8	
11	9.72 643		9.79 888		0.20 112	9.92 755		49
		20		28			8	
12	9.72 663		9.79 916		0.20 084	9.92 747		48
		20		28			8	
13	9.72 683		9.79 944		0.20 056	9.92 739		47
		20		28			8	
14	9.72 703		9.79 972		0.20 028	9.92 731		46
		20		28			8	
15	9.72 723		9.80 000		0.20 000	9.92 723		45
		20		28			8	
16	9.72 743		9.80 028		0.19 972	9.92 715		44
		20		28			8	
17	9.72 763		9.80 056		0.19 944	9.92 707		43
		20		28			8	
18	9.72 783		9.80 084		0.19 916	9.92 699		42
		20		28			8	
19	9.72 803		9.80 112		0.19 888	9.92 691		41
		20		28			8	
20	9.72 823		9.80 140		0.19 860	9.92 683		**40**
		20		28			8	
21	9.72 843		9.80 168		0.19 832	9.92 675		39
		20		27			8	
22	9.72 863		9.80 195		0.19 805	9.92 667		38
		20		28			8	
23	9.72 883		9.80 223		0.19 777	9.92 659		37
		19		28			8	
24	9.72 902		9.80 251		0.19 749	9.92 651		36
		20		28			8	
25	9.72 922		9.80 279		0.19 721	9.92 643		35
		20		28			8	
26	9.72 942		9.80 307		0.19 693	9.92 635		34
		20		28			8	
27	9.72 962		9.80 335		0.19 665	9.92 627		33
		20		28			8	
28	9.72 982		9.80 363		0.19 637	9.92 619		32
		20		28			8	
29	9.73 002		9.80 391		0.19 609	9.92 611		31
		20		28			8	
30	9.73 022		9.80 419		0.19 581	9.92 603		**30**
		19		28			8	
31	9.73 041		9.80 447		0.19 553	9.92 595		29
		20		27			8	
32	9.73 061		9.80 474		0.19 526	9.92 587		28
		20		28			8	
33	9.73 081		9.80 502		0.19 498	9.92 579		27
		20		28			8	
34	9.73 101		9.80 530		0.19 470	9.92 571		26
		20		28			8	
35	9.73 121		9.80 558		0.19 442	9.92 563		25
		19		28			8	
36	9.73 140		9.80 586		0.19 414	9.92 555		24
		20		28			9	
37	9.73 160		9.80 614		0.19 386	9.92 546		23
		20		28			8	
38	9.73 180		9.80 642		0.19 358	9.92 538		22
		20		27			8	
39	9.73 200		9.80 669		0.19 331	9.92 530		21
		19		28			8	
40	9.73 219		9.80 697		0.19 303	9.92 522		**20**
		20		28			8	
41	9.73 239		9.80 725		0.19 275	9.92 514		19
		20		28			8	
42	9.73 259		9.80 753		0.19 247	9.92 506		18
		19		28			8	
43	9.73 278		9.80 781		0.19 219	9.92 498		17
		20		27			8	
44	9.73 298		9.80 808		0.19 192	9.92 490		16
		20		28			8	
45	9.73 318		9.80 836		0.19 164	9.92 482		15
		19		28			9	
46	9.73 337		9.80 864		0.19 136	9.92 473		14
		20		28			8	
47	9.73 357		9.80 892		0.19 108	9.92 465		13
		20		27			8	
48	9.73 377		9.80 919		0.19 081	9.92 457		12
		19		28			8	
49	9.73 396		9.80 947		0.19 053	9.92 449		11
		20		28			8	
50	9.73 416		9.80 975		0.19 025	9.92 441		**10**
		19		28			8	
51	9.73 435		9.81 003		0.18 997	9.92 433		9
		20		27			8	
52	9.73 455		9.81 030		0.18 970	9.92 425		8
		19		28			9	
53	9.73 474		9.81 058		0.18 942	9.92 416		7
		20		28			8	
54	9.73 494		9.81 086		0.18 914	9.92 408		6
		19		27			8	
55	9.73 513		9.81 113		0.18 887	9.92 400		5
		20		28			8	
56	9.73 533		9.81 141		0.18 859	9.92 392		4
		19		28			8	
57	9.73 552		9.81 169		0.18 831	9.92 384		3
		20		27			8	
58	9.73 572		9.81 196		0.18 804	9.92 376		2
		19		28			9	
59	9.73 591		9.81 224		0.18 776	9.92 367		1
		20		28			8	
60	9.73 611		9.81 252		0.18 748	9.92 359		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P			
	29	**28**	**27**
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

P P			
	21	**20**	**19**
1	2.1	2.0	1.9
2	4.2	4.0	3.8
3	6.3	6.0	5.7
4	8.4	8.0	7.6
5	10.5	10.0	9.5
6	12.6	12.0	11.4
7	14.7	14.0	13.3
8	16.8	16.0	15.2
9	18.9	18.0	17.1

P P			
	9	**8**	**7**
1	0.9	0.8	0.7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	**60**
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	55
6	9.73 727	19	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	20	9.81 445	28	0.18 555	9.92 302	8	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	9	52
9	9.73 785	19	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	20	9.81 528	28	0.18 472	9.92 277	8	**50**
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	8	49
12	9.73 843	19	9.81 583	28	0.18 417	9.92 260	9	48
13	9.73 863	20	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	8	46
15	9.73 901	19	9.81 666	27	0.18 334	9.92 235	9	45
16	9.73 921	20	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	28	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	27	0.18 252	9.92 211	8	42
19	9.73 978	19	9.81 776	28	0.18 224	9.92 202	9	41
20	9.73 997	19	9.81 803	27	0.18 197	9.92 194	8	**40**
21	9.74 017	20	9.81 831	28	0.18 169	9.92 186	8	39
22	9.74 036	19	9.81 858	27	0.18 142	9.92 177	9	38
23	9.74 055	19	9.81 886	28	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	27	0.18 087	9.92 161	8	36
25	9.74 093	19	9.81 941	28	0.18 059	9.92 152	9	35
26	9.74 113	20	9.81 968	27	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	28	0.18 004	9.92 136	8	33
28	9.74 151	19	9.82 023	27	0.17 977	9.92 127	9	32
29	9.74 170	19	9.82 051	28	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	27	0.17 922	9.92 111	8	**30**
31	9.74 208	19	9.82 106	28	0.17 894	9.92 102	9	29
32	9.74 227	19	9.82 133	27	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	28	0.17 839	9.92 086	8	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	9	26
35	9.74 284	19	9.82 215	27	0.17 785	9.92 069	8	25
36	9.74 303	19	9.82 243	28	0.17 757	9.92 060	9	24
37	9.74 322	19	9.82 270	27	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	28	0.17 702	9.92 044	8	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	9	21
40	9.74 379	19	9.82 352	27	0.17 648	9.92 027	8	**20**
41	9.74 398	19	9.82 380	28	0.17 620	9.92 018	9	19
42	9.74 417	19	9.82 407	27	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	28	0.17 565	9.92 002	8	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	9	16
45	9.74 474	19	9.82 489	27	0.17 511	9.91 985	8	15
46	9.74 493	19	9.82 517	28	0.17 483	9.91 976	9	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	8	13
48	9.74 531	19	9.82 571	27	0.17 429	9.91 959	9	12
49	9.74 549	18	9.82 599	28	0.17 401	9.91 951	8	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	9	**10**
51	9.74 587	19	9.82 653	27	0.17 347	9.91 934	8	9
52	9.74 606	19	9.82 681	28	0.17 319	9.91 925	9	8
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	8	7
54	9.74 644	19	9.82 735	27	0.17 265	9.91 908	9	6
55	9.74 662	18	9.82 762	27	0.17 238	9.91 900	8	5
56	9.74 681	19	9.82 790	28	0.17 210	9.91 891	9	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	8	3
58	9.74 719	19	9.82 844	27	0.17 156	9.91 874	9	2
59	9.74 737	18	9.82 871	27	0.17 129	9.91 866	8	1
60	9.74 756	19	9.82 899	28	0.17 101	9.91 857	9	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P		
	28	**27**
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

P P		
	20	**19**	**18**
1	2.0	1.9	1.8
2	4.0	3.8	3.6
3	6.0	5.7	5.4
4	8.0	7.6	7.2
5	10.0	9.5	9.0
6	12.0	11.4	10.8
7	14.0	13.3	12.6
8	16.0	15.2	14.4
9	18.0	17.1	16.2

P P		
	9	**8**
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.74 756		9.82 899		0.17 101	9.91 857		**60**
		19		27			8	
1	9.74 775		9.82 926		0.17 074	9.91 849		59
2	9.74 794	19	9.82 953	27	0.17 047	9.91 840	9	58
3	9.74 812	18	9.82 980	27	0.17 020	9.91 832	8	57
4	9.74 831	19	9.83 008	28	0.16 992	9.91 823	9	56
5	9.74 850	19	9.83 035	27	0.16 965	9.91 815	8	55
6	9.74 868	18	9.83 062	27	0.16 938	9.91 806	9	54
7	9.74 887	19	9.83 089	27	0.16 911	9.91 798	8	53
8	9.74 906	19	9.83 117	28	0.16 883	9.91 789	9	52
9	9.74 924	18	9.83 144	27	0.16 856	9.91 781	8	51
10	9.74 943	19	9.83 171	27	0.16 829	9.91 772	9	**50**
		18		27			9	
11	9.74 961		9.83 198		0.16 802	9.91 763		49
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48
13	9.74 999	19	9.83 252	27	0.16 748	9.91 746	8	47
14	9.75 017	18	9.83 280	28	0.16 720	9.91 738	9	46
15	9.75 036	19	9.83 307	27	0.16 693	9.91 729	9	45
16	9.75 054	18	9.83 334	27	0.16 666	9.91 720	9	44
17	9.75 073	19	9.83 361	27	0.16 639	9.91 712	8	43
18	9.75 091	18	9.83 388	27	0.16 612	9.91 703	9	42
19	9.75 110	19	9.83 415	27	0.16 585	9.91 695	8	41
20	9.75 128	18	9.83 442	27	0.16 558	9.91 686	9	**40**
		19		28			9	
21	9.75 147		9.83 470		0.16 530	9.91 677		39
22	9.75 165	18	9.83 497	27	0.16 503	9.91 669	8	38
23	9.75 184	19	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	18	9.83 551	27	0.16 449	9.91 651	9	36
25	9.75 221	19	9.83 578	27	0.16 422	9.91 643	8	35
26	9.75 239	18	9.83 605	27	0.16 395	9.91 634	9	34
27	9.75 258	19	9.83 632	27	0.16 368	9.91 625	9	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	8	32
29	9.75 294	18	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313	19	9.83 713	27	0.16 287	9.91 599	9	**30**
		18		27			8	
31	9.75 331		9.83 740		0.16 260	9.91 591		29
32	9.75 350	19	9.83 768	28	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	9	27
34	9.75 386	18	9.83 822	27	0.16 178	9.91 565	8	26
35	9.75 405	19	9.83 849	27	0.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	9	23
38	9.75 459	18	9.83 930	27	0.16 070	9.91 530	8	22
39	9.75 478	19	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	9	**20**
		18		27			8	
41	9.75 514		9.84 011		0.15 989	9.91 504		19
42	9.75 533	19	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	9	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	8	15
46	9.75 605	18	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	19	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	9	11
50	9.75 678	18	9.84 254	27	0.15 746	9.91 425	8	**10**
		18		26			9	
51	9.75 696		9.84 280		0.15 720	9.91 416		9
52	9.75 714	18	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	19	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	9	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	8	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336	9	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	28	27	26
1	2.8	2.7	2.6
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	19	18
1	1.9	1.8
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	27	0.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	26	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	27	0.15 101	9.91 212	9	46
15	9.76 129	18	9.84 925	26	0.15 075	9.91 203	9	45
16	9.76 146	17	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	27	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	26	0.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	9	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	8	38
23	9.76 271	18	9.85 140	27	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	26	0.14 834	9.91 123	9	36
25	9.76 307	18	9.85 193	27	0.14 807	9.91 114	9	35
26	9.76 324	17	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	27	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	26	0.14 727	9.91 087	9	32
29	9.76 378	18	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395	17	9.85 327	27	0.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	27	0.14 646	9.91 060	9	29
32	9.76 431	18	9.85 380	26	0.14 620	9.91 051	9	28
33	9.76 448	17	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	27	0.14 566	9.91 033	9	26
35	9.76 484	18	9.85 460	26	0.14 540	9.91 023	10	25
36	9.76 501	17	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	27	0.14 486	9.91 005	9	23
38	9.76 537	18	9.85 540	26	0.14 460	9.90 996	9	22
39	9.76 554	17	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	27	0.14 406	9.90 978	9	**20**
41	9.76 590	18	9.85 620	26	0.14 380	9.90 969	9	19
42	9.76 607	17	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625	18	9.85 674	27	0.14 326	9.90 951	9	17
44	9.76 642	17	9.85 700	26	0.14 300	9.90 942	9	16
45	9.76 660	18	9.85 727	27	0.14 273	9.90 933	9	15
46	9.76 677	17	9.85 754	27	0.14 246	9.90 924	9	14
47	9.76 695	18	9.85 780	26	0.14 220	9.90 915	9	13
48	9.76 712	17	9.85 807	27	0.14 193	9.90 906	9	12
49	9.76 730	18	9.85 834	27	0.14 166	9.90 896	10	11
50	9.76 747	17	9.85 860	26	0.14 140	9.90 887	9	**10**
51	9.76 765	18	9.85 887	27	0.14 113	9.90 878	9	9
52	9.76 782	17	9.85 913	26	0.14 087	9.90 869	9	8
53	9.76 800	18	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	17	9.85 967	27	0.14 033	9.90 851	9	6
55	9.76 835	18	9.85 993	26	0.14 007	9.90 842	9	5
56	9.76 852	17	9.86 020	27	0.13 980	9.90 832	10	4
57	9.76 870	18	9.86 046	26	0.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	17	9.86 100	27	0.13 900	9.90 805	9	1
60	9.76 922	18	9.86 126	26	0.13 874	9.90 796	9	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		P P
0	9.76 922		9.86 126		0.13 874	9.90 796		60	
		17		27			9		
1	9.76 939		9.86 153		0.13 847	9.90 787		59	
		18		26			10		
2	9.76 957		9.86 179		0.13 821	9.90 777		58	
		17		27			9		
3	9.76 974		9.86 206		0.13 794	9.90 768		57	
		17		26			9		
4	9.76 991		9.86 232		0.13 768	9.90 759		56	
		18		27			9		
5	9.77 009		9.86 259		0.13 741	9.90 750		55	
		17		26			9		
6	9.77 026		9.86 285		0.13 715	9.90 741		54	
		17		27			10		
7	9.77 043		9.86 312		0.13 688	9.90 731		53	
		18		26			9		
8	9.77 061		9.86 338		0.13 662	9.90 722		52	
		17		27			9		
9	9.77 078		9.86 365		0.13 635	9.90 713		51	
		17		27			9		
10	9.77 095		9.86 392		0.13 608	9.90 704		50	
		17		26			10		
11	9.77 112		9.86 418		0.13 582	9.90 694		49	
		18		27			9		
12	9.77 130		9.86 445		0.13 555	9.90 685		48	
		17		26			9		
13	9.77 147		9.86 471		0.13 529	9.90 676		47	
		17		27			9		
14	9.77 164		9.86 498		0.13 502	9.90 667		46	
		17		26			10		
15	9.77 181		9.86 524		0.13 476	9.90 657		45	
		18		27			9		
16	9.77 199		9.86 551		0.13 449	9.90 648		44	
		17		26			9		
17	9.77 216		9.86 577		0.13 423	9.90 639		43	
		17		26			9		
18	9.77 233		9.86 603		0.13 397	9.90 630		42	
		17		27			10		
19	9.77 250		9.86 630		0.13 370	9.90 620		41	
		18		26			9		
20	9.77 268		9.86 656		0.13 344	9.90 611		40	
		17		27			9		
21	9.77 285		9.86 683		0.13 317	9.90 602		39	
		17		26			10		
22	9.77 302		9.86 709		0.13 291	9.90 592		38	
		17		27			9		
23	9.77 319		9.86 736		0.13 264	9.90 583		37	
		17		26			9		
24	9.77 336		9.86 762		0.13 238	9.90 574		36	
		17		27			9		
25	9.77 353		9.86 789		0.13 211	9.90 565		35	
		17		26			10		
26	9.77 370		9.86 815		0.13 185	9.90 555		34	
		17		27			9		
27	9.77 387		9.86 842		0.13 158	9.90 546		33	
		18		26			9		
28	9.77 405		9.86 868		0.13 132	9.90 537		32	
		17		26			10		
29	9.77 422		9.86 894		0.13 106	9.90 527		31	
		17		27			9		
30	9.77 439		9.86 921		0.13 079	9.90 518		30	
		17		26			9		
31	9.77 456		9.86 947		0.13 053	9.90 509		29	
		17		27			10		
32	9.77 473		9.86 974		0.13 026	9.90 499		28	
		17		26			9		
33	9.77 490		9.87 000		0.13 000	9.90 490		27	
		17		27			10		
34	9.77 507		9.87 027		0.12 973	9.90 480		26	
		17		26			9		
35	9.77 524		9.87 053		0.12 947	9.90 471		25	
		17		26			9		
36	9.77 541		9.87 079		0.12 921	9.90 462		24	
		17		27			10		
37	9.77 558		9.87 106		0.12 894	9.90 452		23	
		17		26			9		
38	9.77 575		9.87 132		0.12 868	9.90 443		22	
		17		26			9		
39	9.77 592		9.87 158		0.12 842	9.90 434		21	
		17		27			10		
40	9.77 609		9.87 185		0.12 815	9.90 424		20	
		17		26			9		
41	9.77 626		9.87 211		0.12 789	9.90 415		19	
		17		27			10		
42	9.77 643		9.87 238		0.12 762	9.90 405		18	
		17		26			9		
43	9.77 660		9.87 264		0.12 736	9.90 396		17	
		17		26			10		
44	9.77 677		9.87 290		0.12 710	9.90 386		16	
		17		27			9		
45	9.77 694		9.87 317		0.12 683	9.90 377		15	
		17		26			9		
46	9.77 711		9.87 343		0.12 657	9.90 368		14	
		17		26			10		
47	9.77 728		9.87 369		0.12 631	9.90 358		13	
		16		27			9		
48	9.77 744		9.87 396		0.12 604	9.90 349		12	
		17		26			10		
49	9.77 761		9.87 422		0.12 578	9.90 339		11	
		17		26			9		
50	9.77 778		9.87 448		0.12 552	9.90 330		10	
		17		27			10		
51	9.77 795		9.87 475		0.12 525	9.90 320		9	
		17		26			9		
52	9.77 812		9.87 501		0.12 499	9.90 311		8	
		17		26			10		
53	9.77 829		9.87 527		0.12 473	9.90 301		7	
		17		27			9		
54	9.77 846		9.87 554		0.12 446	9.90 292		6	
		17		26			10		
55	9.77 862		9.87 580		0.12 420	9.90 282		5	
		16		26			9		
56	9.77 879		9.87 606		0.12 394	9.90 273		4	
		17		27			10		
57	9.77 896		9.87 633		0.12 367	9.90 263		3	
		17		26			9		
58	9.77 913		9.87 659		0.12 341	9.90 254		2	
		17		26			10		
59	9.77 930		9.87 685		0.12 315	9.90 244		1	
		16		26			9		
60	9.77 946		9.87 711		0.12 289	9.90 235		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	P P

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17	16
1	1.8	1.7	1.6
2	3.6	3.4	3.2
3	5.4	5.1	4.8
4	7.2	6.8	6.4
5	9.0	8.5	8.0
6	10.8	10.2	9.6
7	12.6	11.9	11.2
8	14.4	13.6	12.8
9	16.2	15.3	14.4

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.77 946		9.87 711		0.12 289	9.90 235		**60**
		17		27			10	
1	9.77 963		9.87 738		0.12 262	9.90 225		59
		17		26			9	
2	9.77 980		9.87 764		0.12 236	9.90 216		58
		17		26			10	
3	9.77 997		9.87 790		0.12 210	9.90 206		57
		16		27			9	
4	9.78 013		9.87 817		0.12 183	9.90 197		56
		17		26			10	
5	9.78 030		9.87 843		0.12 157	9.90 187		55
		17		26			9	
6	9.78 047		9.87 869		0.12 131	9.90 178		54
		16		26			10	
7	9.78 063		9.87 895		0.12 105	9.90 168		53
		17		27			9	
8	9.78 080		9.87 922		0.12 078	9.90 159		52
		17		26			10	
9	9.78 097		9.87 948		0.12 052	9.90 149		51
		16		26			10	
10	9.78 113		9.87 974		0.12 026	9.90 139		**50**
		17		26			9	
11	9.78 130		9.88 000		0.12 000	9.90 130		49
		17		27			10	
12	9.78 147		9.88 027		0.11 973	9.90 120		48
		16		26			9	
13	9.78 163		9.88 053		0.11 947	9.90 111		47
		17		26			10	
14	9.78 180		9.88 079		0.11 921	9.90 101		46
		17		26			10	
15	9.78 197		9.88 105		0.11 895	9.90 091		45
		16		26			9	
16	9.78 213		9.88 131		0.11 869	9.90 082		44
		17		27			10	
17	9.78 230		9.88 158		0.11 842	9.90 072		43
		16		26			9	
18	9.78 246		9.88 184		0.11 816	9.90 063		42
		17		26			10	
19	9.78 263		9.88 210		0.11 790	9.90 053		41
		17		26			10	
20	9.78 280		9.88 236		0.11 764	9.90 043		**40**
		16		26			9	
21	9.78 296		9.88 262		0.11 738	9.90 034		39
		17		27			10	
22	9.78 313		9.88 289		0.11 711	9.90 024		38
		16		26			10	
23	9.78 329		9.88 315		0.11 685	9.90 014		37
		17		26			9	
24	9.78 346		9.88 341		0.11 659	9.90 005		36
		16		26			10	
25	9.78 362		9.88 367		0.11 633	9.89 995		35
		17		26			10	
26	9.78 379		9.88 393		0.11 607	9.89 985		34
		16		27			9	
27	9.78 395		9.88 420		0.11 580	9.89 976		33
		17		26			10	
28	9.78 412		9.88 446		0.11 554	9.89 966		32
		16		26			10	
29	9.78 428		9.88 472		0.11 528	9.89 956		31
		17		26			9	
30	9.78 445		9.88 498		0.11 502	9.89 947		**30**
		16		26			10	
31	9.78 461		9.88 524		0.11 476	9.89 937		29
		17		26			10	
32	9.78 478		9.88 550		0.11 450	9.89 927		28
		16		27			9	
33	9.78 494		9.88 577		0.11 423	9.89 918		27
		16		26			10	
34	9.78 510		9.88 603		0.11 397	9.89 908		26
		17		26			10	
35	9.78 527		9.88 629		0.11 371	9.89 898		25
		16		26			10	
36	9.78 543		9.88 655		0.11 345	9.89 888		24
		17		26			9	
37	9.78 560		9.88 681		0.11 319	9.89 879		23
		16		26			10	
38	9.78 576		9.88 707		0.11 293	9.89 869		22
		16		26			10	
39	9.78 592		9.88 733		0.11 267	9.89 859		21
		17		26			10	
40	9.78 609		9.88 759		0.11 241	9.89 849		**20**
		16		27			9	
41	9.78 625		9.88 786		0.11 214	9.89 840		19
		17		26			10	
42	9.78 642		9.88 812		0.11 188	9.89 830		18
		16		26			10	
43	9.78 658		9.88 838		0.11 162	9.89 820		17
		16		26			10	
44	9.78 674		9.88 864		0.11 136	9.89 810		16
		17		26			9	
45	9.78 691		9.88 890		0.11 110	9.89 801		15
		16		26			10	
46	9.78 707		9.88 916		0.11 084	9.89 791		14
		16		26			10	
47	9.78 723		9.88 942		0.11 058	9.89 781		13
		16		26			10	
48	9.78 739		9.88 968		0.11 032	9.89 771		12
		17		26			10	
49	9.78 756		9.88 994		0.11 006	9.89 761		11
		16		26			9	
50	9.78 772		9.89 020		0.10 980	9.89 752		**10**
		16		26			10	
51	9.78 788		9.89 046		0.10 954	9.89 742		9
		17		27			10	
52	9.78 805		9.89 073		0.10 927	9.89 732		8
		16		26			10	
53	9.78 821		9.89 099		0.10 901	9.89 722		7
		16		26			10	
54	9.78 837		9.89 125		0.10 875	9.89 712		6
		16		26			10	
55	9.78 853		9.89 151		0.10 849	9.89 702		5
		16		26			9	
56	9.78 869		9.89 177		0.10 823	9.89 693		4
		17		26			10	
57	9.78 886		9.89 203		0.10 797	9.89 683		3
		16		26			10	
58	9.78 902		9.89 229		0.10 771	9.89 673		2
		16		26			10	
59	9.78 918		9.89 255		0.10 745	9.89 663		1
		16		26			10	
60	9.78 934		9.89 281		0.10 719	9.89 653		**0**
′	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P		
	27	**26**
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

P P		
	17	**16**
1	1.7	1.6
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

P P		
	10	**9**
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	60
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	10	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	9	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	50
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	10	45
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	9	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	40
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	35
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	10	32
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	11	31
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	30
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	16	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	15	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	16	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	15	9.90 320	26	0.09 680	9.89 254	10	20
41	9.79 589	16	9.90 346	26	0.09 654	9.89 244	10	19
42	9.79 605	16	9.90 371	25	0.09 629	9.89 233	11	18
43	9.79 621	16	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 684	16	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	15	9.90 527	26	0.09 473	9.89 173	10	12
49	9.79 715	16	9.90 553	26	0.09 447	9.89 162	11	11
50	9.79 731	15	9.90 578	25	0.09 422	9.89 152	10	10
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	16	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	15	9.90 682	26	0.09 318	9.89 112	10	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	11	5
56	9.79 825	16	9.90 734	26	0.09 266	9.89 091	10	4
57	9.79 840	15	9.90 759	25	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	10	2
59	9.79 872	16	9.90 811	26	0.09 189	9.89 060	11	1
60	9.79 887	15	9.90 837	26	0.09 163	9.89 050	10	0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	17	16	15
1	1.7	1.6	1.5
2	3.4	3.2	3.0
3	5.1	4.8	4.5
4	6.8	6.4	6.0
5	8.5	8.0	7.5
6	10.2	9.6	9.0
7	11.9	11.2	10.5
8	13.6	12.8	12.0
9	15.3	14.4	13.5

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	**60**
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 981	15	9.90 992	26	0.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	0.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	**50**
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	0.08 724	9.88 875	10	43
18	9.80 166	15	9.91 301	26	0.08 699	9.88 865	10	42
19	9.80 182	16	9.91 327	26	0.08 673	9.88 855	11	41
20	9.80 197	15	9.91 353	26	0.08 647	9.88 844	10	**40**
21	9.80 213	16	9.91 379	25	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	26	0.08 596	9.88 824	11	38
23	9.80 244	16	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	15	9.91 482	25	0.08 518	9.88 793	11	35
26	9.80 290	16	9.91 507	26	0.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33
28	9.80 320	15	9.91 559	26	0.08 441	9.88 761	10	32
29	9.80 336	16	9.91 585	25	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	26	0.08 390	9.88 741	11	**30**
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 382	16	9.91 662	26	0.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27
34	9.80 412	15	9.91 713	26	0.08 287	9.88 699	11	26
35	9.80 428	16	9.91 739	26	0.08 261	9.88 688	10	25
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	26	0.08 209	9.88 668	11	23
38	9.80 473	15	9.91 816	25	0.08 184	9.88 657	10	22
39	9.80 489	16	9.91 842	26	0.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	26	0.08 132	9.88 636	10	**20**
41	9.80 519	15	9.91 893	25	0.08 107	9.88 626	11	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 550	16	9.91 945	26	0.08 055	9.88 605	10	17
44	9.80 565	15	9.91 971	26	0.08 029	9.88 594	11	16
45	9.80 580	15	9.91 996	25	0.08 004	9.88 584	10	15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	11	14
47	9.80 610	15	9.92 048	26	0.07 952	9.88 563	10	13
48	9.80 625	15	9.92 073	25	0.07 927	9.88 552	11	12
49	9.80 641	16	9.92 099	26	0.07 901	9.88 542	10	11
50	9.80 656	15	9.92 125	26	0.07 875	9.88 531	11	**10**
51	9.80 671	15	9.92 150	25	0.07 850	9.88 521	10	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	26	0.07 798	9.88 499	11	7
54	9.80 716	15	9.92 227	25	0.07 773	9.88 489	10	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	11	5
56	9.80 746	15	9.92 279	26	0.07 721	9.88 468	10	4
57	9.80 762	16	9.92 304	25	0.07 696	9.88 457	11	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	10	2
59	9.80 792	15	9.92 356	26	0.07 644	9.88 436	11	1
60	9.80 807	15	9.92 381	25	0.07 619	9.88 425	11	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P		
	26	**25**
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

P P		
	16	**15**
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

P P		
	11	**10**
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

40°

139°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.80 807		9.92 381		0.07 619	9.88 425		**60**
		15		26			10	
1	9.80 822		9.92 407		0.07 593	9.88 415		59
2	9.80 837	15	9.92 433	26	0.07 567	9.88 404	11	58
3	9.80 852	15	9.92 458	25	0.07 542	9.88 394	10	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	26	0.07 490	9.88 372	11	55
6	9.80 897	15	9.92 535	25	0.07 465	9.88 362	10	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	26	0.07 413	9.88 340	11	52
9	9.80 942	15	9.92 612	25	0.07 388	9.88 330	10	51
10	9.80 957		9.92 638	26	0.07 362	9.88 319	11	**50**
		15		25			11	
11	9.80 972		9.92 663		0.07 337	9.88 308		49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	10	48
13	9.81 002	15	9.92 715	26	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	25	0.07 260	9.88 276	11	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	10	45
16	9.81 047	15	9.92 792	26	0.07 208	9.88 255	11	44
17	9.81 061	14	9.92 817	25	0.07 183	9.88 244	11	43
18	9.81 076	15	9.92 843	26	0.07 157	9.88 234	10	42
19	9.81 091	15	9.92 868	25	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	26	0.07 080	9.88 201	11	39
22	9.81 136	15	9.92 945	25	0.07 055	9.88 191	10	38
23	9.81 151	15	9.92 971	26	0.07 029	9.88 180	11	37
24	9.81 166	15	9.92 996	25	0.07 004	9.88 169	11	36
25	9.81 180	14	9.93 022	26	0.06 978	9.88 158	11	35
26	9.81 195	15	9.93 048	26	0.06 952	9.88 148	10	34
27	9.81 210	15	9.93 073	25	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	26	0.06 901	9.88 126	11	32
29	9.81 240	15	9.93 124	25	0.06 876	9.88 115	11	31
30	9.81 254	14	9.93 150	26	0.06 850	9.88 105	10	**30**
31	9.81 269	15	9.93 175	25	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	26	0.06 773	9.88 072	11	27
34	9.81 314	15	9.93 252	25	0.06 748	9.88 061	11	26
35	9.81 328	14	9.93 278	26	0.06 722	9.88 051	10	25
36	9.81 343	15	9.93 303	25	0.06 697	9.88 040	11	24
37	9.81 358	15	9.93 329	26	0.06 671	9.88 029	11	23
38	9.81 372	14	9.93 354	25	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	26	0.06 594	9.87 996	11	**20**
41	9.81 417	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	14	9.93 457	26	0.06 543	9.87 975	10	18
43	9.81 446	15	9.93 482	25	0.06 518	9.87 964	11	17
44	9.81 461	15	9.93 508	26	0.06 492	9.87 953	11	16
45	9.81 475	14	9.93 533	25	0.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	26	0.06 441	9.87 931	11	14
47	9.81 505	15	9.93 584	25	0.06 416	9.87 920	11	13
48	9.81 519	14	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	26	0.06 364	9.87 898	11	11
50	9.81 549	15	9.93 661	25	0.06 339	9.87 887	11	**10**
51	9.81 563	14	9.93 687	26	0.06 313	9.87 877	10	9
52	9.81 578	15	9.93 712	25	0.06 288	9.87 866	11	8
53	9.81 592	14	9.93 738	26	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 622	15	9.93 789	26	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	25	0.06 186	9.87 822	11	4
57	9.81 651	15	9.93 840	26	0.06 160	9.87 811	11	3
58	9.81 665	14	9.93 865	25	0.06 135	9.87 800	11	2
59	9.81 680	15	9.93 891	26	0.06 109	9.87 789	11	1
60	9.81 694	14	9.93 916	25	0.06 084	9.87 778	11	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.81 694		9.93 916		0.06 084	9.87 778		**60**
		15		26			11	
1	9.81 709		9.93 942		0.06 058	9.87 767		59
		14		25			11	
2	9.81 723		9.93 967		0.06 033	9.87 756		58
		15		26			11	
3	9.81 738		9.93 993		0.06 007	9.87 745		57
		14		25			11	
4	9.81 752		9.94 018		0.05 982	9.87 734		56
		15		26			11	
5	9.81 767		9.94 044		0.05 956	9.87 723		55
		14		25			11	
6	9.81 781		9.94 069		0.05 931	9.87 712		54
		15		26			11	
7	9.81 796		9.94 095		0.05 905	9.87 701		53
		14		25			11	
8	9.81 810		9.94 120		0.05 880	9.87 690		52
		15		26			11	
9	9.81 825		9.94 146		0.05 854	9.87 679		51
		14		25			11	
10	9.81 839		9.94 171		0.05 829	9.87 668		**50**
		15		26			11	
11	9.81 854		9.94 197		0.05 803	9.87 657		49
		14		25			11	
12	9.81 868		9.94 222		0.05 778	9.87 646		48
		14		26			11	
13	9.81 882		9.94 248		0.05 752	9.87 635		47
		15		25			11	
14	9.81 897		9.94 273		0.05 727	9.87 624		46
		14		26			11	
15	9.81 911		9.94 299		0.05 701	9.87 613		45
		15		25			12	
16	9.81 926		9.94 324		0.05 676	9.87 601		44
		14		26			11	
17	9.81 940		9.94 350		0.05 650	9.87 590		43
		15		25			11	
18	9.81 955		9.94 375		0.05 625	9.87 579		42
		14		26			11	
19	9.81 969		9.94 401		0.05 599	9.87 568		41
		14		25			11	
20	9.81 983		9.94 426		0.05 574	9.87 557		**40**
		15		26			11	
21	9.81 998		9.94 452		0.05 548	9.87 546		39
		14		25			11	
22	9.82 012		9.94 477		0.05 523	9.87 535		38
		14		26			11	
23	9.82 026		9.94 503		0.05 497	9.87 524		37
		15		25			11	
24	9.82 041		9.94 528		0.05 472	9.87 513		36
		14		26			12	
25	9.82 055		9.94 554		0.05 446	9.87 501		35
		14		25			11	
26	9.82 069		9.94 579		0.05 421	9.87 490		34
		15		25			11	
27	9.82 084		9.94 604		0.05 396	9.87 479		33
		14		26			11	
28	9.82 098		9.94 630		0.05 370	9.87 468		32
		14		25			11	
29	9.82 112		9.94 655		0.05 345	9.87 457		31
		14		26			11	
30	9.82 126		9.94 681		0.05 319	9.87 446		**30**
		15		25			12	
31	9.82 141		9.94 706		0.05 294	9.87 434		29
		14		26			11	
32	9.82 155		9.94 732		0.05 268	9.87 423		28
		14		25			11	
33	9.82 169		9.94 757		0.05 243	9.87 412		27
		15		26			11	
34	9.82 184		9.94 783		0.05 217	9.87 401		26
		14		25			11	
35	9.82 198		9.94 808		0.05 192	9.87 390		25
		14		26			12	
36	9.82 212		9.94 834		0.05 166	9.87 378		24
		14		25			11	
37	9.82 226		9.94 859		0.05 141	9.87 367		23
		14		25			11	
38	9.82 240		9.94 884		0.05 116	9.87 356		22
		15		26			11	
39	9.82 255		9.94 910		0.05 090	9.87 345		21
		14		25			11	
40	9.82 269		9.94 935		0.05 065	9.87 334		**20**
		14		26			12	
41	9.82 283		9.94 961		0.05 039	9.87 322		19
		14		25			11	
42	9.82 297		9.94 986		0.05 014	9.87 311		18
		14		26			11	
43	9.82 311		9.95 012		0.04 988	9.87 300		17
		15		25			12	
44	9.82 326		9.95 037		0.04 963	9.87 288		16
		14		25			11	
45	9.82 340		9.95 062		0.04 938	9.87 277		15
		14		26			11	
46	9.82 354		9.95 088		0.04 912	9.87 266		14
		14		25			11	
47	9.82 368		9.95 113		0.04 887	9.87 255		13
		14		26			12	
48	9.82 382		9.95 139		0.04 861	9.87 243		12
		14		25			11	
49	9.82 396		9.95 164		0.04 836	9.87 232		11
		14		26			11	
50	9.82 410		9.95 190		0.04 810	9.87 221		**10**
		14		25			12	
51	9.82 424		9.95 215		0.04 785	9.87 209		9
		15		25			11	
52	9.82 439		9.95 240		0.04 760	9.87 198		8
		14		26			11	
53	9.82 453		9.95 266		0.04 734	9.87 187		7
		14		25			12	
54	9.82 467		9.95 291		0.04 709	9.87 175		6
		14		26			11	
55	9.82 481		9.95 317		0.04 683	9.87 164		5
		14		25			11	
56	9.82 495		9.95 342		0.04 658	9.87 153		4
		14		26			12	
57	9.82 509		9.95 368		0.04 632	9.87 141		3
		14		25			11	
58	9.82 523		9.95 393		0.04 607	9.87 130		2
		14		25			11	
59	9.82 537		9.95 418		0.04 582	9.87 119		1
		14		26			12	
60	9.82 551		9.95 444		0.04 556	9.87 107		**0**
′	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	12	11
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		P P
0	9.82 551		9.95 444		0.04 556	9.87 107		60	
		14		25			11		
1	9.82 565		9.95 469		0.04 531	9.87 096		59	
		14		26			11		
2	9.82 579		9.95 495		0.04 505	9.87 085		58	
		14		25			12		
3	9.82 593		9.95 520		0.04 480	9.87 073		57	
		14		25			11		
4	9.82 607		9.95 545		0.04 455	9.87 062		56	
		14		26			12		
5	9.82 621		9.95 571		0.04 429	9.87 050		55	
		14		25			11		
6	9.82 635		9.95 596		0.04 404	9.87 039		54	
		14		26			11		
7	9.82 649		9.95 622		0.04 378	9.87 028		53	
		14		25			12		
8	9.82 663		9.95 647		0.04 353	9.87 016		52	
		14		25			11		
9	9.82 677		9.95 672		0.04 328	9.87 005		51	
		14		26			12		**26 25**
10	9.82 691		9.95 698		0.04 302	9.86 993		50	1 2.6 2.5
		14		25			11		2 5.2 5.0
11	9.82 705		9.95 723		0.04 277	9.86 982		49	3 7.8 7.5
		14		25			12		4 10.4 10.0
12	9.82 719		9.95 748		0.04 252	9.86 970		48	5 13.0 12.5
		14		26			11		6 15.6 15.0
13	9.82 733		9.95 774		0.04 226	9.86 959		47	7 18.2 17.5
		14		25			12		8 20.8 20.0
14	9.82 747		9.95 799		0.04 201	9.86 947		46	9 23.4 22.5
		14		26			11		
15	9.82 761		9.95 825		0.04 175	9.86 936		45	
		14		25			12		
16	9.82 775		9.95 850		0.04 150	9.86 924		44	
		13		25			11		
17	9.82 788		9.95 875		0.04 125	9.86 913		43	
		14		26			11		
18	9.82 802		9.95 901		0.04 099	9.86 902		42	
		14		25			12		
19	9.82 816		9.95 926		0.04 074	9.86 890		41	
		14		26			11		
20	9.82 830		9.95 952		0.04 048	9.86 879		40	
		14		25			12		
21	9.82 844		9.95 977		0.04 023	9.86 867		39	
		14		25			12		
22	9.82 858		9.96 002		0.03 998	9.86 855		38	
		14		26			11		
23	9.82 872		9.96 028		0.03 972	9.86 844		37	
		13		25			12		
24	9.82 885		9.96 053		0.03 947	9.86 832		36	
		14		25			11		
25	9.82 899		9.96 078		0.03 922	9.86 821		35	
		14		26			12		
26	9.82 913		9.96 104		0.03 896	9.86 809		34	**14 13**
		14		25			11		
27	9.82 927		9.96 129		0.03 871	9.86 798		33	1 1.4 1.3
		14		26			12		
28	9.82 941		9.96 155		0.03 845	9.86 786		32	2 2.8 2.6
		14		25			11		
29	9.82 955		9.96 180		0.03 820	9.86 775		31	3 4.2 3.9
		13		25			12		
30	9.82 968		9.96 205		0.03 795	9.86 763		30	4 5.6 5.2
		14		26			11		
31	9.82 982		9.96 231		0.03 769	9.86 752		29	5 7.0 6.5
		14		25			12		
32	9.82 996		9.96 256		0.03 744	9.86 740		28	6 8.4 7.8
		14		25			12		
33	9.83 010		9.96 281		0.03 719	9.86 728		27	7 9.8 9.1
		13		26			11		
34	9.83 023		9.96 307		0.03 693	9.86 717		26	8 11.2 10.4
		14		25			12		
35	9.83 037		9.96 332		0.03 668	9.86 705		25	9 12.6 11.7
		14		25			11		
36	9.83 051		9.96 357		0.03 643	9.86 694		24	
		14		26			12		
37	9.83 065		9.96 383		0.03 617	9.86 682		23	
		13		25			12		
38	9.83 078		9.96 408		0.03 592	9.86 670		22	
		14		25			11		
39	9.83 092		9.96 433		0.03 567	9.86 659		21	
		14		26			12		
40	9.83 106		9.96 459		0.03 541	9.86 647		20	
		14		25			12		
41	9.83 120		9.96 484		0.03 516	9.86 635		19	
		13		26			11		
42	9.83 133		9.96 510		0.03 490	9.86 624		18	
		14		25			12		
43	9.83 147		9.96 535		0.03 465	9.86 612		17	**12 11**
		14		25			12		
44	9.83 161		9.96 560		0.03 440	9.86 600		16	1 1.2 1.1
		13		26			11		
45	9.83 174		9.96 586		0.03 414	9.86 589		15	2 2.4 2.2
		14		25			12		
46	9.83 188		9.96 611		0.03 389	9.86 577		14	3 3.6 3.3
		14		25			12		
47	9.83 202		9.96 636		0.03 364	9.86 565		13	4 4.8 4.4
		13		26			11		
48	9.83 215		9.96 662		0.03 338	9.86 554		12	5 6.0 5.5
		14		25			12		
49	9.83 229		9.96 687		0.03 313	9.86 542		11	6 7.2 6.6
		13		25			12		
50	9.83 242		9.96 712		0.03 288	9.86 530		10	7 8.4 7.7
		14		26			12		
51	9.83 256		9.96 738		0.03 262	9.86 518		9	8 9.6 8.8
		14		25			11		
52	9.83 270		9.96 763		0.03 237	9.86 507		8	9 10.8 9.9
		13		25			12		
53	9.83 283		9.96 788		0.03 212	9.86 495		7	
		14		26			12		
54	9.83 297		9.96 814		0.03 186	9.86 483		6	
		13		25			11		
55	9.83 310		9.96 839		0.03 161	9.86 472		5	
		14		25			12		
56	9.83 324		9.96 864		0.03 136	9.86 460		4	
		14		26			12		
57	9.83 338		9.96 890		0.03 110	9.86 448		3	
		13		25			12		
58	9.83 351		9.96 915		0.03 085	9.86 436		2	
		14		25			11		
59	9.83 365		9.96 940		0.03 060	9.86 425		1	
		13		26			12		
60	9.83 378		9.96 966		0.03 034	9.86 413		0	
′	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	P P

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		P P	
0	9.83 378		9.96 966		0.03 034	9.86 413		60		
		14		25			12			
1	9.83 392		9.96 991		0.03 009	9.86 401		59		
		13		25			12			
2	9.83 405		9.97 016		0.02 984	9.86 389		58		
		14		26			12			
3	9.83 419		9.97 042		0.02 958	9.86 377		57		
		13		25			11			
4	9.83 432		9.97 067		0.02 933	9.86 366		56		
		14		25			12			
5	9.83 446		9.97 092		0.02 908	9.86 354		55		
		13		26			12			
6	9.83 459		9.97 118		0.02 882	9.86 342		54		
		14		25			12			
7	9.83 473		9.97 143		0.02 857	9.86 330		53		
		13		25			12			
8	9.83 486		9.97 168		0.02 832	9.86 318		52		
		14		25			12			
9	9.83 500		9.97 193		0.02 807	9.86 306		51		
		13		26			11			
10	9.83 513		9.97 219		0.02 781	9.86 295		50		
		14		25			12			
11	9.83 527		9.97 244		0.02 756	9.86 283		49		
		13		25			12			
12	9.83 540		9.97 269		0.02 731	9.86 271		48		
		14		26			12			
13	9.83 554		9.97 295		0.02 705	9.86 259		47		
		13		25			12			
14	9.83 567		9.97 320		0.02 680	9.86 247		46		
		14		25			12			
15	9.83 581		9.97 345		0.02 655	9.86 235		45		
		13		26			12			
16	9.83 594		9.97 371		0.02 629	9.86 223		44		
		14		25			12			
17	9.83 608		9.97 396		0.02 604	9.86 211		43		
		13		25			11			
18	9.83 621		9.97 421		0.02 579	9.86 200		42		
		13		26			12			
19	9.83 634		9.97 447		0.02 553	9.86 188		41		
		14		25			12			
20	9.83 648		9.97 472		0.02 528	9.86 176		40		
		13		25			12			
21	9.83 661		9.97 497		0.02 503	9.86 164		39		
		13		26			12			
22	9.83 674		9.97 523		0.02 477	9.86 152		38		
		14		25			12			
23	9.83 688		9.97 548		0.02 452	9.86 140		37		
		13		25			12			
24	9.83 701		9.97 573		0.02 427	9.86 128		36		
		14		25			12			
25	9.83 715		9.97 598		0.02 402	9.86 116		35		
		13		26			12			
26	9.83 728		9.97 624		0.02 376	9.86 104		34		
		13		25			12			
27	9.83 741		9.97 649		0.02 351	9.86 092		33		
		14		25			12			
28	9.83 755		9.97 674		0.02 326	9.86 080		32		
		13		26			12			
29	9.83 768		9.97 700		0.02 300	9.86 068		31		
		13		25			12			
30	9.83 781		9.97 725		0.02 275	9.86 056		30		
		14		25			12			
31	9.83 795		9.97 750		0.02 250	9.86 044		29		
		13		26			12			
32	9.83 808		9.97 776		0.02 224	9.86 032		28		
		13		25			12			
33	9.83 821		9.97 801		0.02 199	9.86 020		27		
		13		25			12			
34	9.83 834		9.97 826		0.02 174	9.86 008		26		
		14		25			12			
35	9.83 848		9.97 851		0.02 149	9.85 996		25		
		13		26			12			
36	9.83 861		9.97 877		0.02 123	9.85 984		24		
		13		25			12			
37	9.83 874		9.97 902		0.02 098	9.85 972		23		
		13		25			12			
38	9.83 887		9.97 927		0.02 073	9.85 960		22		
		14		26			12			
39	9.83 901		9.97 953		0.02 047	9.85 948		21		
		13		25			12			
40	9.83 914		9.97 978		0.02 022	9.85 936		20		
		13		25			12			
41	9.83 927		9.98 003		0.01 997	9.85 924		19		
		13		26			12			
42	9.83 940		9.98 029		0.01 971	9.85 912		18		
		14		25			12			
43	9.83 954		9.98 054		0.01 946	9.85 900		17		
		13		25			12			
44	9.83 967		9.98 079		0.01 921	9.85 888		16		
		13		25			12			
45	9.83 980		9.98 104		0.01 896	9.85 876		15		
		13		26			12			
46	9.83 993		9.98 130		0.01 870	9.85 864		14		
		13		25			13			
47	9.84 006		9.98 155		0.01 845	9.85 851		13		
		14		25			12			
48	9.84 020		9.98 180		0.01 820	9.85 839		12		
		13		26			12			
49	9.84 033		9.98 206		0.01 794	9.85 827		11		
		13		25			12			
50	9.84 046		9.98 231		0.01 769	9.85 815		10		
		13		25			12			
51	9.84 059		9.98 256		0.01 744	9.85 803		9		
		13		25			12			
52	9.84 072		9.98 281		0.01 719	9.85 791		8		
		13		26			12			
53	9.84 085		9.98 307		0.01 693	9.85 779		7		
		13		25			13			
54	9.84 098		9.98 332		0.01 668	9.85 766		6		
		14		25			12			
55	9.84 112		9.98 357		0.01 643	9.85 754		5		
		13		26			12			
56	9.84 125		9.98 383		0.01 617	9.85 742		4		
		13		25			12			
57	9.84 138		9.98 408		0.01 592	9.85 730		3		
		13		25			12			
58	9.84 151		9.98 433		0.01 567	9.85 718		2		
		13		25			12			
59	9.84 164		9.98 458		0.01 542	9.85 706		1		
		13		26			13			
60	9.84 177		9.98 484		0.01 516	9.85 693		0		
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	P P	

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	13
1	1.4	1.3
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	11
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.84 177		9.98 484		0.01 516	9.85 693		**60**
		13		25			12	
1	9.84 190		9.98 509		0.01 491	9.85 681		59
		13		25			12	
2	9.84 203		9.98 534		0.01 466	9.85 669		58
		13		26			12	
3	9.84 216		9.98 560		0.01 440	9.85 657		57
		13		25			12	
4	9.84 229		9.98 585		0.01 415	9.85 645		56
		13		25			13	
5	9.84 242		9.98 610		0.01 390	9.85 632		55
		13		25			12	
6	9.84 255		9.98 635		0.01 365	9.85 620		54
		14		26			12	
7	9.84 269		9.98 661		0.01 339	9.85 608		53
		13		25			12	
8	9.84 282		9.98 686		0.01 314	9.85 596		52
		13		25			13	
9	9.84 295		9.98 711		0.01 289	9.85 583		51
		13		26			12	
10	9.84 308		9.98 737		0.01 263	9.85 571		**50**
		13		25			12	
11	9.84 321		9.98 762		0.01 238	9.85 559		49
		13		25			12	
12	9.84 334		9.98 787		0.01 213	9.85 547		48
		13		25			13	
13	9.84 347		9.98 812		0.01 188	9.85 534		47
		13		26			12	
14	9.84 360		9.98 838		0.01 162	9.85 522		46
		13		25			12	
15	9.84 373		9.98 863		0.01 137	9.85 510		45
		12		25			13	
16	9.84 385		9.98 888		0.01 112	9.85 497		44
		13		25			12	
17	9.84 398		9.98 913		0.01 087	9.85 485		43
		13		26			12	
18	9.84 411		9.98 939		0.01 061	9.85 473		42
		13		25			13	
19	9.84 424		9.98 964		0.01 036	9.85 460		41
		13		25			12	
20	9.84 437		9.98 989		0.01 011	9.85 448		**40**
		13		26			12	
21	9.84 450		9.99 015		0.00 985	9.85 436		39
		13		25			13	
22	9.84 463		9.99 040		0.00 960	9.85 423		38
		13		25			12	
23	9.84 476		9.99 065		0.00 935	9.85 411		37
		13		25			12	
24	9.84 489		9.99 090		0.00 910	9.85 399		36
		13		26			13	
25	9.84 502		9.99 116		0.00 884	9.85 386		35
		13		25			12	
26	9.84 515		9.99 141		0.00 859	9.85 374		34
		13		25			13	
27	9.84 528		9.99 166		0.00 834	9.85 361		33
		12		25			12	
28	9.84 540		9.99 191		0.00 809	9.85 349		32
		13		26			12	
29	9.84 553		9.99 217		0.00 783	9.85 337		31
		13		25			13	
30	9.84 566		9.99 242		0.00 758	9.85 324		**30**
		13		25			12	
31	9.84 579		9.99 267		0.00 733	9.85 312		29
		13		26			13	
32	9.84 592		9.99 293		0.00 707	9.85 299		28
		13		25			12	
33	9.84 605		9.99 318		0.00 682	9.85 287		27
		13		25			13	
34	9.84 618		9.99 343		0.00 657	9.85 274		26
		12		25			12	
35	9.84 630		9.99 368		0.00 632	9.85 262		25
		13		26			12	
36	9.84 643		9.99 394		0.00 606	9.85 250		24
		13		25			13	
37	9.84 656		9.99 419		0.00 581	9.85 237		23
		13		25			12	
38	9.84 669		9.99 444		0.00 556	9.85 225		22
		13		25			13	
39	9.84 682		9.99 469		0.00 531	9.85 212		21
		12		26			12	
40	9.84 694		9.99 495		0.00 505	9.85 200		**20**
		13		25			13	
41	9.84 707		9.99 520		0.00 480	9.85 187		19
		13		25			12	
42	9.84 720		9.99 545		0.00 455	9.85 175		18
		13		25			13	
43	9.84 733		9.99 570		0.00 430	9.85 162		17
		12		26			12	
44	9.84 745		9.99 596		0.00 404	9.85 150		16
		13		25			13	
45	9.84 758		9.99 621		0.00 379	9.85 137		15
		13		25			12	
46	9.84 771		9.99 646		0.00 354	9.85 125		14
		13		26			13	
47	9.84 784		9.99 672		0.00 328	9.85 112		13
		12		25			12	
48	9.84 796		9.99 697		0.00 303	9.85 100		12
		13		25			13	
49	9.84 809		9.99 722		0.00 278	9.85 087		11
		13		25			13	
50	9.84 822		9.99 747		0.00 253	9.85 074		**10**
		13		26			12	
51	9.84 835		9.99 773		0.00 227	9.85 062		9
		12		25			13	
52	9.84 847		9.99 798		0.00 202	9.85 049		8
		13		25			12	
53	9.84 860		9.99 823		0.00 177	9.85 037		7
		13		25			13	
54	9.84 873		9.99 848		0.00 152	9.85 024		6
		12		26			12	
55	9.84 885		9.99 874		0.00 126	9.85 012		5
		13		25			13	
56	9.84 898		9.99 899		0.00 101	9.84 999		4
		13		25			13	
57	9.84 911		9.99 924		0.00 076	9.84 986		3
		12		25			12	
58	9.84 923		9.99 949		0.00 051	9.84 974		2
		13		26			13	
59	9.84 936		9.99 975		0.00 025	9.84 961		1
		13		25			12	
60	9.84 949		0.00 000		0.00 000	9.84 949		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

P P

	26	25	14
1	2.6	2.5	1.4
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

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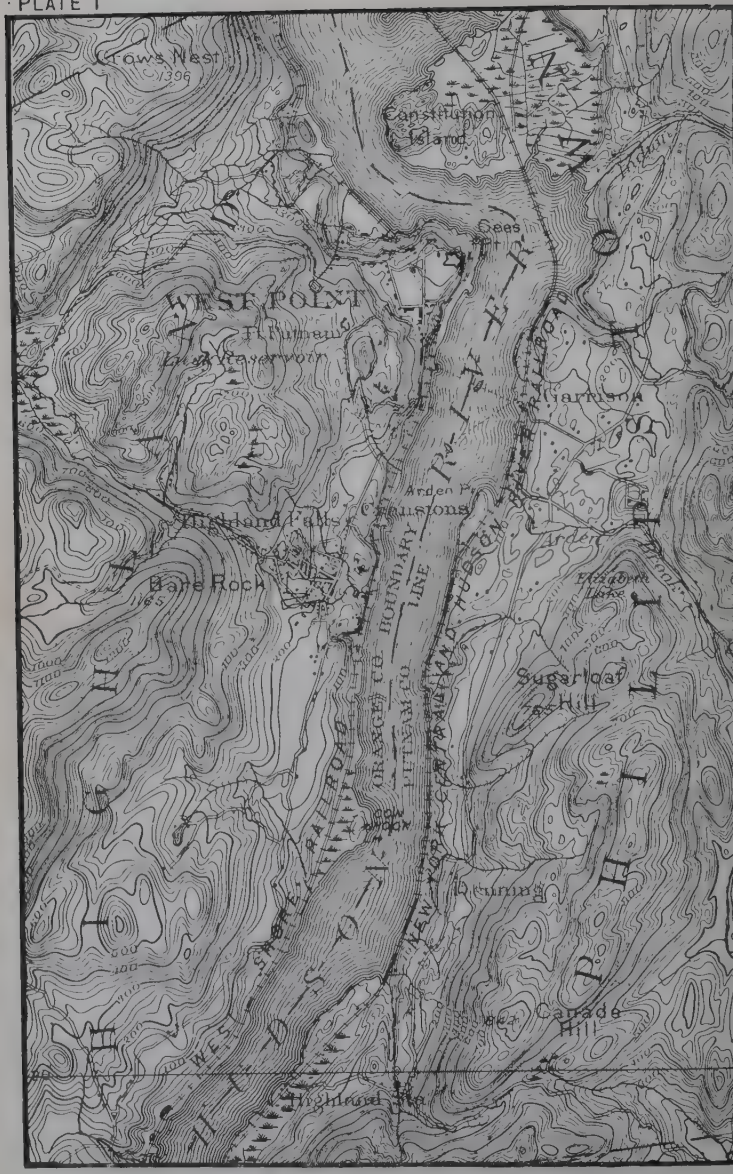
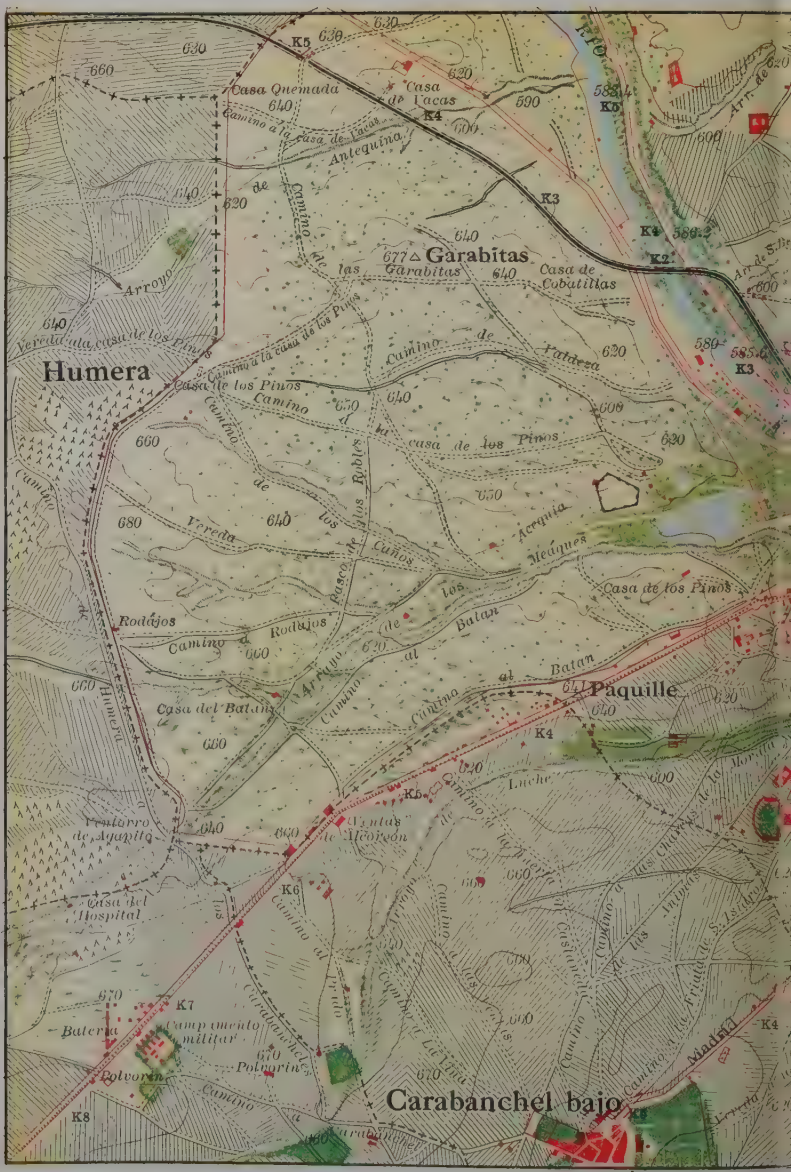
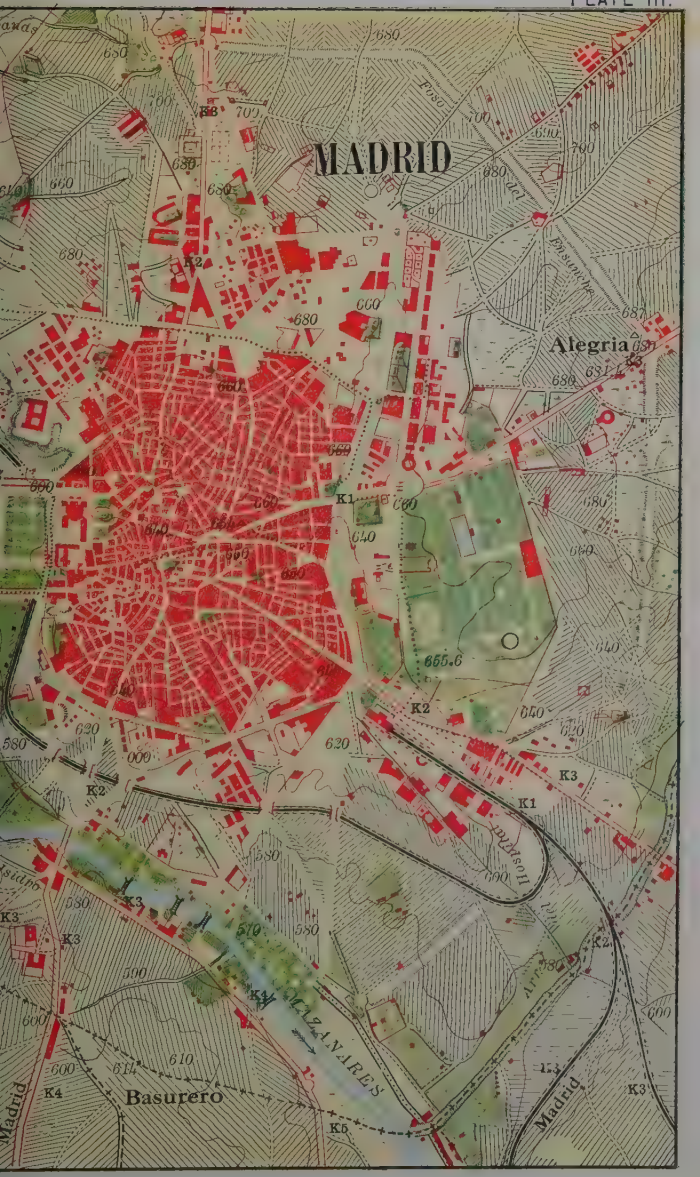
PLATES I-VII

PLATE I


Crows Nest
1396
Constitution Island
Gees Pt
WEST POINT
Ft Putnam
Lusk Reservoir
Garrison
Arden Pt
Cranstons
Highland Falls
Arden
Bare Rock
1165
Elizabeth Lake
Sugarloaf Hill
ORANGE CO
PUTNAM CO
BOUNDARY LINE
NEW YORK CENTRAL HUDSON RIVER RAILROAD
Deming
Canada Hill
Highland Sta
1 ½ 0 1 Mile

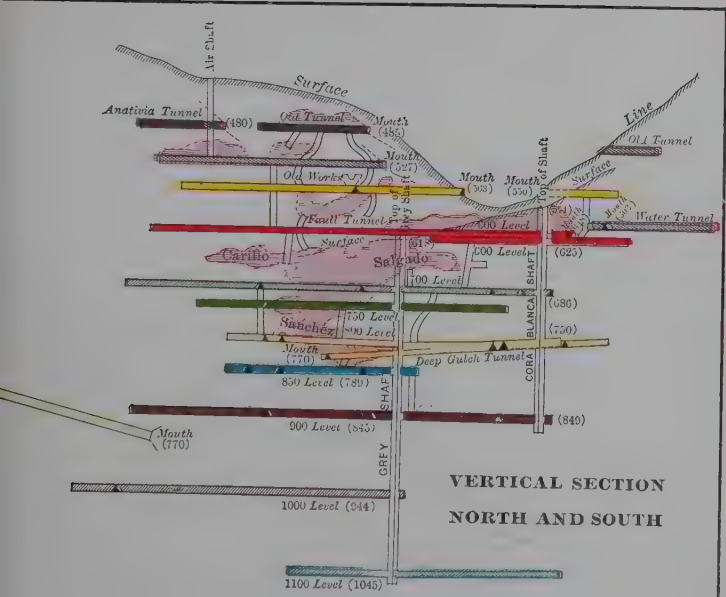

Humera
Garabitas
Casa Quemada
Casa de Lacas
Camino a la Casa de Lacas
Antequina
Arroyo
Vereda de la casa de los Pinos
Camino a la casa de los Pinos
Casa de los Pinos
Camino de Valdeza
Casa de Cobatillas
Camino de la casa de los Pinos
Camino de los Robles
Vereda
Camino de los Cuños
Acequia
Melques
Casa de los Pinos
Rodajos
Camino a Rodajos
Camino de Batan
Paquille
Casa del Batan
Camino de Humera
Polvorin de Aguila
Casa del Hospital
Batería
Campamento militar
Polvorin
Carabanchel bajo
Camino de Madrid
Camino de las Chozas
Camino de la Fuente de S. Isidro
Camino de las Imperiales
Camino de la Florida
Camino de la Moncloa
Camino de la Casa de Campo
Camino de la Puerta de Hierro
Camino de la Ermita
Camino de Carabanchel
Arroyo de la Vaguada
Arroyo de S. Pedro
Arroyo de la Florida
Arroyo de la Moncloa
Arroyo de la Puerta de Hierro
Arroyo de la Ermita
Arroyo de Carabanchel
Arroyo de la Vaguada


MADRID
Alegria
Basurero
MANZANARES
Madrid
Madrid
K1
K2
K3
K4
K5
665.6

VERTICAL SECTION

EAST AND WEST

L.L. POATES ENGR'G CO., N.Y.



Plan and Vertical Sections

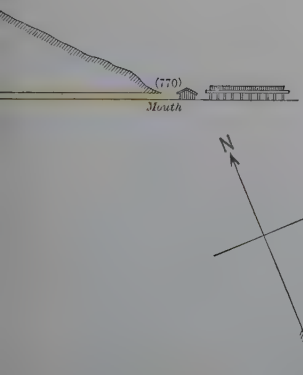
of the

UNDERGROUND WORKS

of the

CORA BLANCA MINE

NEW ALMADEN, CALIFORNIA



Scale in feet

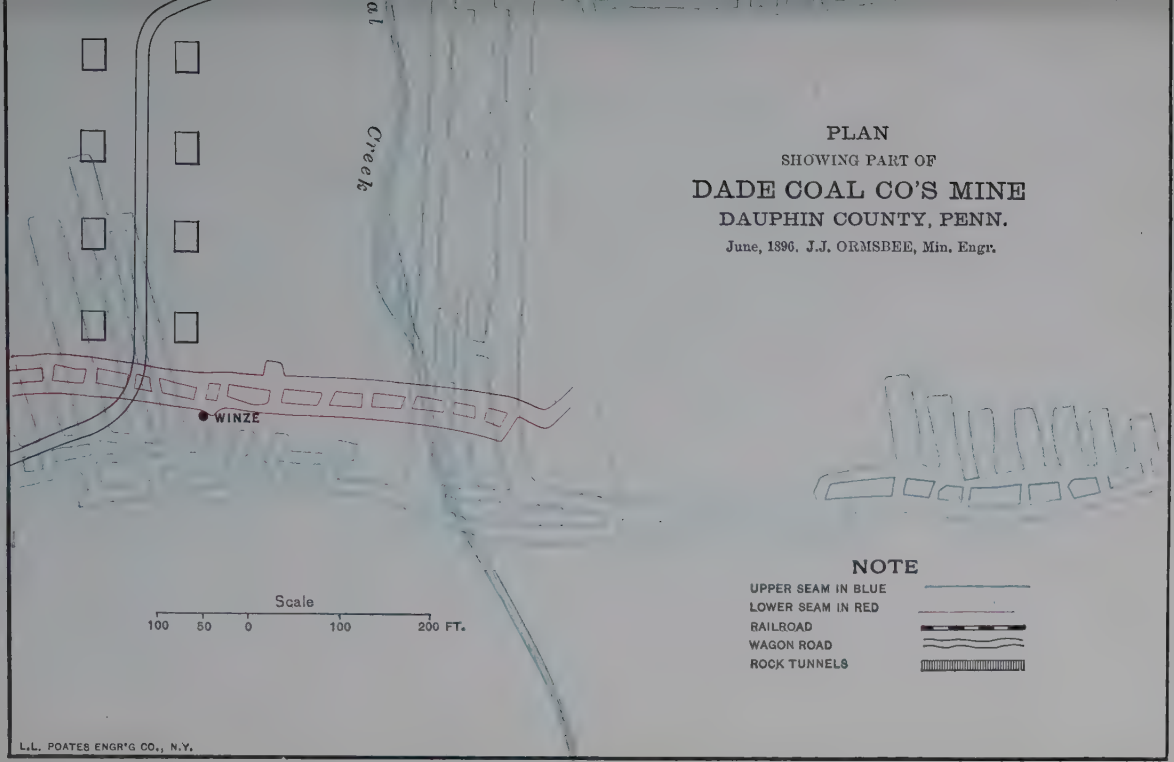
0 50 100 150 200 250 300

April, 1896

Note:- The Figures in brackets, thus (510) indicate the vertical distance in feet below the datum point, a monument on the summit of Mine Hill.

The shafts and levels on the Sections are projected upon one plane in order to show their connection and relative elevations.

JUNE 1896
510
509
506
508
503
507
El-1236
505
504
502
501
El. 1228
WINZE



PLAN

SHOWING PART OF

DADE COAL CO'S MINE

DAUPHIN COUNTY, PENN.

June, 1896. J.J. ORMSBEE, Min. Engr.

L.L. POATES ENGR'G CO., N.Y.

PLATE V.

ISOGONIC CHART
OF THE
UNITED STATES
FOR THE EPOCH, JANUARY, 1910
SCALE OF MILES
0 100 200 300 400
Longitude West from Greenwich
EAST DECLINATION
WEST DECLINATION
GULF OF MEXICO
PACIFIC OCEAN
ATLANTIC OCEAN
Annual Change
MINNESOTA
NORTH DAKOTA
SOUTH DAKOTA
MONTANA
WYOMING
NEBRASKA
KANSAS
COLORADO
UTAH
NEVADA
ARIZONA
NEW MEXICO
OKLAHOMA
TEXAS
LOUISIANA
ARKANSAS
MISSOURI
ILLINOIS
WISCONSIN
MICHIGAN
INDIANA
OHIO
KENTUCKY
TENNESSEE
MISSISSIPPI
ALABAMA
GEORGIA
SOUTH CAROLINA
NORTH CAROLINA
VIRGINIA
WEST VIRGINIA
PENNSYLVANIA
NEW YORK
MAINE
VERMONT
NEW HAMPSHIRE
MASSACHUSETTS
RHODE ISLAND
CONNECTICUT
NEW JERSEY
DELAWARE
MARYLAND
OREGON
WASHINGTON
CALIFORNIA

CIVIL DIVISIONS

States, Counties, Townships, Capitals and Principal Cities.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Towns and Villages

abcdefghijklmnopqrstuvwxyz

HYDROGRAPHY

Lakes, Rivers and Bays

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Creeks, Brooks, Springs, small Lakes, Ponds, Marshes and Glaciers.

abcdefghijklmnopqrstuvwxyz

PUBLIC WORKS

Railroads, Tunnels, Bridges. Ferries, Wagon-roads, Trails, Fords and Dams.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Thickness of letter ⅓ of height.
Slope of letter 3 parts of base to 8 of height.

TOPOGRAPHY

Mountains, Plateaus, Lines of Cliffs and Canyons.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Peaks, small Valleys, Canyons, Islands and Points.

abcdefghijklmnopqrstuvwxyz

For Plans, Lot Surveys, Notes, etc.

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

1234567890

Scale

PLATE VII

Slightly altered from a plate in Wilson's Topographical Surveying. By permission of H.M. Wilson